

Eduqas
Physics
for A Level Year 1 & AS

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How to use this book

This book has been written to support the Eduqas AS Physics specification and the first half of the A level specification. The layout of the book matches that of components 1 and 2 respectively of the AS Physics specification. The same material is specified for the A level course and, because the topics are distributed differently in the AS and A level courses, a matching grid is provided on page 5.

Example: A level component 3 unit 1 (3.1), the nature of waves, is the same as AS component 2 unit 4 (2.4).

There are also colour flashes on each right hand page that allow you to identify the topic number in both AS and A Level specifications. The flash for AS is blue and for Year 1 of the 2 year course, the flash is orange, e.g.: **2.4** **3.1**

It provides you with information which covers the content requirements of the course as well as plenty of practice questions to allow you to keep track of your progress and to prepare successfully for your AS and A level examinations.

This book covers all three of the Assessment Objectives (AOs) required for your Eduqas course. The AOs are:

- **AO1, Knowledge and understanding** of physics ideas and practice. This comprises 35% of the AS examination (30% of the A level) including the specified practical activities.
- **AO2, Applying knowledge and understanding** of physics ideas and practice, which comprises 45% of both the AS and A level examinations.
- **AO3, Analysing, interpreting and evaluating scientific information, ideas and evidence** which is 20% of the AS examination (25% of the A level).

The main chapters in the book are the AS components 1 and 2.

- Component 1 covers Motion, Matter and Energy
- Component 2 covers Electricity and Light

Additional chapters are

- Chapter 3 – Practical skills
- Chapter 4 – Mathematical skills

The component 1 and 2 sections include much practical and mathematical material in context. Chapters 3 and 4 cover aspects of these skills which are more appropriately dealt with separately.

Level of coverage

This book contains material which is examined at both AS and A level. It is expected that, whilst some users will take AS Physics only, a large percentage of its users will proceed to take the full A level, with or without having taken the AS qualification. Because of this, the level of coverage of both the material and some of the practice questions is higher than required for AS.

Practice questions

As well as the self-test questions in the margin of the main text, each section in components 1 and 2 ends with an exercise of

practice questions. As well as containing material relating to the content of the sections, the exercises contain data analysis questions around the specified practical work for the unit.

Some questions also relate to the content of more than one unit: both the AS and A level examinations require candidates to answer such synoptic questions, which bring together a range of ideas from the physics course. The solutions to these exercises as well as the self-test questions are to be found at the end of the book. Unless the question asks for the reasoning or working, the solutions to mathematical questions are generally limited to a final answer rather than showing the way of reaching the answer.

Examination style questions

At the end of each of components 1 and 2, you will find a set of examination style questions. As the style and level of demand of the new AS and A level examinations have changed significantly, these questions are not past-paper questions but have been specially written to reflect the changed demands. Similarly to the exercises at the ends of the sections, parts of these questions have also been written with A level candidates in mind and some parts draw together material from different sections of the specification. The solutions to these questions take the form of model answers and which, in many cases, more than provide the minimum answer to achieve full marks. In some cases alternative, equally valid, answers are given.

Margin features

The text is supplemented by a number of margin features:

Terms & definitions

These are physics terms and laws that you need to be able to quote without further information.

For example:

Terms & definitions

The **refractive index**, n , of a material is defined by $n = \frac{c}{v}$, where v and c are the speeds of light in the material and a vacuum respectively.

Physics examination papers always contain a few marks for defining terms or stating a physical law.

Study point

Some ideas from the main text are further developed in Study points. These are used for material which is important for you to understand but is tangential to the flow of the text. Some Study point boxes include material designed to extend your understanding beyond the requirements of the specification.

For example:

Study point

The equation $V = \frac{ER}{R+r}$ can be interpreted as the pd across the output resistor, R , for a potential divider with input voltage E .

Self-test

1.1.1

Self-test

Questions to enable you to check your understanding of the subject at that point in the text. They are often short calculations (though some are longer), which require you to apply equations developed in the main text. The answers to these Self-test questions are given at the end of the book.

Stretch & challenge

S&C

Stretch & Challenge

Material developed in these is designed to make you think more deeply about the subject. They usually include questions for you to answer and will be useful for students who will progress to the full A level and to those intending to progress to higher study in physics or engineering. Unlike the self-test marginalia, the answers to these questions are not provided.

Maths check / Practical tip

MATHS CHECK

PRACTICAL TIP

These refer to particular techniques and often direct you to Chapters 3 and 4 for a fuller treatment.

Exam tip

Exam tip

These are to help you avoid common unnecessary mistakes in answering examination questions.

Matching A level to AS units

		AS Units														
		1.1 page 10	1.2 page 24	1.3 page 36	1.4 page 50	1.5 page 58	1.6 page 70	1.7 page 80	2.1 page 94	2.2 page 100	2.3 page 112	2.4 page 124	2.5 page 130	2.6 page 144	2.7 page 154	2.8 page 166
A2 Units	1.1	✓														
	1.2		✓													
	1.3			✓												
	1.4				✓											
	2.1								✓							
	2.2									✓						
	2.3										✓					
	2.5					✓										
	2.7						✓									
	3.1											✓				
	3.2												✓			
	3.3													✓		
	3.4														✓	
	3.5															✓
	3.7							✓								

The AS examination

The Eduqas AS Physics aims to encourage students to

- develop essential knowledge and understanding of different areas of the subject and how they relate to each other
- develop and demonstrate a deep appreciation of the skills, knowledge and understanding of scientific methods
- develop competence and confidence in a variety of practical, mathematical and problem-solving skills
- develop their interest in and enthusiasm for the subject, including developing an interest in further study and careers associated with the subject
- understand how society makes decisions about scientific issues and how the sciences contribute to the success of the economy and society.

Examination questions are written to reflect the assessment objectives (AOs) as laid out in the specification. Learners must meet the following AOs in the context of content detailed in the specification:

Assessment objective 1

Learners must: Demonstrate knowledge and understanding of scientific ideas, processes, techniques and procedures

35% of the marks for questions set on the examination papers are for AO1. As well as pure recall, such as stating laws and definitions, this includes knowing which equations to use, substituting into equations and describing experimental techniques.

Assessment objective 2

Learners must: Apply knowledge and understanding of scientific ideas, processes, techniques and procedures:

- in a theoretical context
- in a practical context
- when handling qualitative data
- when handling quantitative data.

45% of the marks for questions set on the examination papers are for AO2. Bringing together ideas to explain phenomena, solving mathematical problems and performing calculations using experimental results and graphs are categorised as AO2. Application involves using the skills you have acquired in situations you have not previously encountered, e.g. in synoptic questions.

Assessment objective 3

Learners must: Analyse, interpret and evaluate scientific information, ideas and evidence, including in relation to issues, to:

- make judgements and reach conclusions
- develop and refine practical design and procedures.

20% of the marks for questions set on the examination papers are for AO3. These marks include determining quantities using experimental results and also responding to data to draw conclusions.

AS Physics Components 1 and 2 written papers

(2 × 1 hour 30 minutes)

Both papers consist of approximately 7 structured questions, each of which contains several parts, with a total of 75 marks. The questions comprise a mixture of short and extended answer parts.

Quality of extended writing (QER)

Some questions assess how well you can present a detailed argument. These are called Quality of Extended Response questions and are indicated by QER next to the mark allocation. These questions are worth 6 marks and the examiner will assess how well you communicate your physics as well as the standard of the physics itself.

Synoptic questions

To quote from the specification, ‘Learners’ understanding of the connections between the different elements of the subject and their holistic understanding of the subject is a requirement of all AS specifications. In practice, this means that in each component learners will be required to demonstrate their ability to draw together different areas of knowledge and understanding from across the full course of study.’ This is known as synoptic assessment. Similarly A level students will be expected to be familiar with AS material.

Questions in one AS component will not focus specifically on content from the other component but some questions in each exam will draw upon skills and knowledge acquired in studying for the other component. For example, energies of fundamental particles (component 1) could be expressed in electron-volts (eV) (from component 2).

Mark allocations

The mark allocation for each question part is given in square brackets, e.g.

State the principle of conservation of momentum. [2]

The mark allocation gives a good clue to the detail required in your answer. The [2] is a good clue that there are two aspects to the answer. In this case the examiner would expect:

1. a statement that the (vector) sum of the momenta remains the same, and
2. the conditions, i.e. in a closed system or if no resultant external force acts.

On some occasions, an examiner might decide to allocate only one mark to the statement, *but still demand both points to be made.*

Questions involving calculations

Depending on their complexity, questions which require the calculation of a result will generally be allocated more than one mark. Unless the question specifically asks for the working to be shown, full marks will normally be awarded for just a correct answer, consisting of a number, a unit and (in the case of vector quantities) a direction. However, incorrect answers will only be given credit for correct stages in the calculation, so you are advised always to show your working.

Error carried forward (ecf)

This is also referred to as *consequential marking*. The principle is that the results of a calculation, in one part of a structured question, is treated as correct if it is used in a subsequent part of the same question. It is often not applied *within* a question part. See the Study and Revision Guide for further discussion of the application of ecf.

Command words used in Eduqas exam questions

Examination 'questions' are not usually phrased as questions. They are usually instructions to do some work. Examiners choose the command words carefully so that you understand the sort of answer required. This is a list of the most common command words used.

State

Give a brief, concise answer with no explanation.

For example:

State the acceleration at time $t = 0$.

For questions which require the value of a quantity to be stated, it is expected that no calculation (or at most a trivial one) is needed.

Describe

Write a short account with no explanation.

For example:

Describe the motion shown in the $v-t$ graph.

Experimental methods can also be asked for using this command word.

Explain

This requires reasons to be given. Depending on the question a description might also be required.

For example:

Explain which of these stars appears brightest.

In this example the brightest star has first to be identified (which might not itself be allocated a mark). The explanation might need to include a calculation.

Suggest

This command word often occurs in the last part of a structured question. There might not be a definite answer but you are expected to put forward a sensible suggestion based upon your physics knowledge.

For example:

Suggest how you could investigate whether the assumption was correct.

Calculate

Use one or more equations to find the value of an unknown quantity.

For example:

Calculate the power dissipated in the $20\ \Omega$ resistor.

Determine

This command word is often used instead of calculate. Give a numerical answer by manipulating data you have been given. There is no absolute difference between the words but determine tends to be used in situations where an additional process is required beyond a calculation.

For example:

Use the graph to determine the acceleration of the rocket at $15.0\ \text{s}$.

In this case you might first need to draw a tangent to the velocity–time graph.

Compare

For example:

Compare the appearance of the two stars.

Make sure that you make a comparison rather than two separate descriptions, e.g. 'Star A is brighter than star B.'

Discuss

This is often used for questions on the practical application of scientific ideas or technological developments.

For example:

Discuss whether this remote community should install a wind farm or an undersea turbine.

As with suggest there is no correct answer to this type of question. The examiner is looking for reasoned arguments based upon data and physics principles.

Overview: Component 1 Motion, Energy and Matter

Basic physics

p10

- The 6 base SI units (kg, m, s, A, mol, K), representing other units in terms of them and checking equations for homogeneity.
- Scalar and vector quantities, manipulating coplanar vectors and resolution of vectors.
- Calculations of density.
- The moment of a force, the principle of moments.
- Stability and the centre of gravity of an object.
- Conditions for equilibrium of an object.

PRACTICAL WORK

- Measurement of the density of solids.
- Determination of unknown masses by using the principle of moments.

Kinematics

p24

- Displacement, speed, velocity and acceleration and their graphical representation.
- Interpret speed and displacement-time graphs for uniform and non-uniform acceleration.
- Derivation and use of equations for linear uniformly accelerated motion.
- Motion under gravity; terminal velocity.
- 2D motion; the independence of vertical and horizontal motion of a body; projectiles.

PRACTICAL WORK

- Measurement of g by free fall.

Dynamics

p36

- Force and Newton's 3rd law of motion.
- Free body diagrams.
- The relationship $\Sigma F = ma$ for constant mass.
- Linear momentum; the principle of conservation of momentum and its application to elastic and inelastic collisions.
- Force as the rate of change of momentum; applying this in situations where mass is constant.

PRACTICAL WORK

- Investigation of Newton's 2nd law.

Energy concepts

p50

- Work and energy transfer, including situations in which the force and motion are in different directions.
- The principle of conservation of energy applied to transfers including gravitational and elastic potential energy and kinetic energy.
- Power as the rate of energy transfer.
- Dissipative forces, e.g. drag and friction, reduce the overall efficiency of the system; calculations of efficiency.

Solids under stress

p58

- Hooke's law; the spring constant.
- Stress, strain and the Young modulus.
- Work done and the area under a force-extension curve.
- Classification of materials as crystalline, amorphous and polymeric.
- The features of a force-extension (or stress-strain) graph for a ductile metal; explanation in terms of dislocations and grain boundaries.
- The features of a force-extension (or stress-strain) graph for a brittle material; breaking by crack propagation and its control.
- The features of a force-extension (or stress-strain) graph for rubber; non-linearity, low Young modulus, straightening of long-chain molecules, hysteresis.

PRACTICAL WORK

- Determination of Young modulus of a metal in the form of a wire.
- Investigation of the force-extension relationship for rubber.

Radiation and stars

p70

- Stellar emission and absorption spectra and their origin.
- Black bodies; stars as good approximations to black bodies.
- The black body spectrum; Wien's displacement law, Stefan's law.
- The inverse square law.
- Investigating stellar luminosity, size, temperature and distance.
- Multiwavelength astronomy.

Particles and nuclear structure

p80

- Matter is composed of quarks and leptons.
- Antiparticles and their properties; symbols for antiparticles of electrons, quarks and hadrons.
- The quark / antiquark structure of hadrons, including baryons, antibaryons and mesons.
- Recall the quark compositions of the neutron and proton and suggest quark composition of other first generation hadrons by applying conservation rules to given reactions.
- The properties of the strong, weak, electromagnetic and gravitational interactions.
- Neutrino involvement and quark flavour changes are exclusive to weak interactions.

Motion, Energy and Matter

Component 1

This foundation component of the AS Physics course builds upon concepts developed in Key Stage 4 in addition to introducing entirely new material.

- The initial unit, Basic Physics, examines the language of physics in terms of quantities and units, which are now written in the standard manner of the scientific community using negative indices where appropriate.
- The core of Component 1 comprises the concepts of motion and energy. These are explored in greater depth than in earlier courses, the vector aspects of motion being examined and the mathematical relationships between quantities of motion investigated.
- Engineers and materials scientists rely on knowledge of the properties of materials to be able to make buildings and machines. These properties are investigated and explained in terms of the behaviour of the constituent molecules.
- Electromagnetic radiation is used to explore the nature of the universe and its constituent parts – stars, galaxies and the cosmic microwave background radiation. Using the whole range of the e-m spectrum allows a much fuller picture of the universe to be obtained than by visible light alone.
- The familiar structures of the material world, atoms and molecules, are seen to be built upon combinations of the fundamental particles of nature, leptons, quarks and antiquarks, interacting by four fundamental forces. The rules of particle interactions are explored.

Content

- 1.1 Basic Physics
- 1.2 Kinematics
- 1.3 Dynamics
- 1.4 Energy concepts
- 1.5 Solids under stress
- 1.6 Using radiation to investigate stars
- 1.7 Particles and nuclear structure

Practical work

Practical work is integral to any physics course. Component 1 provides a wealth of opportunities for students to hone their practical skills as well as to develop their understanding of the contents.

1.1 Basic Physics

Study point

Don't think of a theory as just a hunch or a guess. In order to be called a *theory* an explanation needs a lot of experimental evidence to support it and it has to explain a range of phenomena. The statement, 'It's just a theory!' misses the point.

Study point

Since 1983, the metre has been defined as 'the length of the path travelled by light in a vacuum during a time interval of $1/299,792,458$ of a second'.

The history of the definition of the metre is worth investigating.

Exam tip

Luminous intensity is not used in AS or A level physics.

Study point

The symbols for quantities are printed in *italics*, e.g. m , T . The symbols for units are printed in plain print, e.g. kg, K.

1.1.1

Self-test

Simplify the following:

- $6a + 2a$
- $6a \times 3a$
- $6a \div 3b$
- $(6a)^2$

1.1.2

Self-test

Derive the unit of volume by considering a cube.

Physics is an experimental science. It involves making measurements of quantities, such as pressure, speed, electric current and temperature and discovering laws, which concern relationships between quantities, and formulating theories, to explain why natural phenomena occur. This unit includes some details of how to handle physical quantities, which are covered in level 2 courses such as GCSE Physics. Some of its contents will therefore be familiar but they are taken to a higher level.

1.1.1 Quantities and units

(a) Base quantities and units

In order to measure a quantity, such as length, we need to have a defined standard to compare the length with. In our system of units, *Le système international d'unités* (abbreviated to SI), the defined unit of length is the metre with the abbreviation **m**. What does a reported length of, say, 53.7 m mean?

$$53.7 \text{ m} = 53.7 \times \text{the defined unit length;}$$

in other words the distance light can travel in $53.7 / 299,792,458$ of a second!

Table 1.1.1 shows the 7 base quantities with their SI units.

Table 1.1.1 SI quantities and units

Quantity		Unit	
Name of quantity	Symbol	Name of unit	abbreviation
mass	m	kilogram	kg
length	l	metre	m
time	t	second	s
electric current	I	ampere	A
temperature	T	kelvin	K
amount of substance	n	mole	mol
luminous intensity	L	candela	cd

Notice that the definition of the metre depends upon another definition (the second) as well as a physical property (the speed of light). The table also has common symbols for the quantities, e.g. t for time and l for length. Note that other symbols can be used, e.g. x and r for lengths and M for mass.

(b) Derived quantities and units

Most of the time physicists work with quantities other than the base quantities, e.g. area, volume, pressure, power. They use the base units in combination to express these. In order to derive these units we treat them as algebraic letters and remember some simple algebraic rules. To remind yourself of them see Self-test 1.1.1.

The easiest way of understanding how to derive a unit is to look at some examples:

1. **Unit of area.** We start with a defining equation:

Area of a rectangle = length $\times$ breadth

$\therefore$ Unit of area = unit of length $\times$ unit of breadth

But length and breadth are both distances so they both have **m** as their unit.

$\therefore$ Unit of area = **m** $\times$ **m** = **m**².

2. Unit of change of speed (or velocity).

The unit of speed (or velocity) is **m s**⁻¹. If the speed of a car changes from 15 **m s**⁻¹ to 33 **m s**⁻¹ then

change of speed = final speed – initial speed

$$= 33 \text{ m s}^{-1} - 15 \text{ m s}^{-1}$$

$$= 18 \text{ m s}^{-1} \text{ [remember that, in algebra, } 33a - 15a = 18a]$$

So the unit of a change of speed is the same as the unit of speed.

3. Unit of acceleration. Again we start with a defining equation.

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}} \quad \text{or } a = \frac{\Delta v}{t}$$

$$\therefore [a] = \frac{[\Delta v]}{[t]} = \frac{\text{m s}^{-1}}{\text{s}} = \text{m s}^{-2}$$

Some derived units are used very frequently and it is useful to learn how to express them in terms of the base SI units. Using the following equations:

$$\text{Force} = \text{mass} \times \text{acceleration}; \text{Work} = \text{Force} \times \text{distance}; \text{Power} = \frac{\text{Work}}{\text{time}}$$

you should be able to show that **N** = **kg m s**⁻², **J** = **kg m**² **s**⁻² and **W** = **kg m**² **s**⁻³.

Example

The drag force, F_D , on a sphere moving through a fluid is given by Stokes' formula, $F_D = 6\pi\eta av$, where a is the radius of the sphere, v the velocity and η [eta] is the *coefficient of viscosity* of the fluid. Find the unit of η in terms of the base SI units.

$$\text{Rearranging the equation } \eta = \frac{F_D}{6\pi av}. \text{ 6 and } \pi \text{ have no units, so } [\eta] = \frac{[F_D]}{[a][v]}$$

$$[F_D] = \text{kg m s}^{-2}, [a] = \text{m} \text{ and } [v] = \text{m s}^{-1} \therefore [\eta] = \frac{\text{kg m s}^{-2}}{\text{m}^2 \text{ s}^{-1}} = \text{kg m}^{-1} \text{ s}^{-1}.$$

(c) Using SI multipliers and standard form

Many problems arise in which the quantities are either much larger or much smaller than the basic quantities. The data is thus given either in *standard form* or using SI multipliers. This example has data in mixed forms.

Example

Calculate the energy transmitted by a 44 kV power cable in one day if it carries a current of 2.5×10^2 A. [Use $P = IV$ and $E = Pt$]

From the two equations, $E = IVt$.

$$\therefore \text{Converting to the basic units: } E = 2.5 \times 10^2 \text{ A} \times 44 \times 10^3 \text{ V} \times 86\,400 \text{ s} \\ = 9.5 \times 10^{11} \text{ J (2 s.f.)}$$

Study point

It is rather tedious to keep writing *unit of*, so we use square brackets to stand for this:

Thus [length] = **m**

and [area] = **m**².

Terms & definitions

A useful symbol for a change of something is Δ (delta). So

Δv = change of velocity.

Exam tip

Learn the expressions for **N**, **J** and **W** in terms of **kg**, **m** and **s**; learn also how to derive them.

Self-test

1.1.3

The unit of the coefficient of viscosity, η , is usually written as 'Pa s' (pascal second) where Pa is the unit of pressure, defined by

$$\text{pressure} = \frac{\text{force}}{\text{area}}.$$

Show that this unit is the same as that derived in the example.

MATHS CHECK

See Section 4.1.1 (c) and (d) for SI multipliers and standard form.

Exam tip

In the example, we could have written 44 kV as 4.4×10^4 V. However, to avoid mistakes it is easier to write it as 44×10^3 and let the calculator handle it!

Homogeneity – Rule 1

Two quantities a and b can only be added together if they have the same units – and then the answer has the same units.

The same goes for subtraction.

Homogeneity – Rule 2

An equation is homogeneous only if the units of the two sides are the same.

1.1.4**Self-test**

Show that the equation

$$x = ut + \frac{1}{2}at^2$$

is homogeneous.

(Remember that $\frac{1}{2}$ has no units.)

1.1.2 Checking equations for homogeneity

Consider the equation: $v^2 = u^2 + 2ax$, where u and v are the initial and final velocities, a the acceleration and x the displacement (distance) of a uniformly accelerating object. We're going to take this equation apart and look at the units of its various bits.

1. The u^2 term: Now $[u] = \text{m s}^{-1}$, so $[u^2] = (\text{m s}^{-1})^2 = \text{m}^2 \text{s}^{-2}$.

2. The $2ax$ term: $[2ax] = [a] \times [x] = \text{m s}^{-2} \times \text{m} = \text{m}^2 \text{s}^{-2}$

Let's just stop here for a moment: the u^2 term and the $2ax$ term **have the same units!** Why is this important? Because it means that they **can** be added together. See rule 1. This means that the unit of the right-hand side of the equation is $\text{m}^2 \text{s}^{-2}$.

3. The v^2 term: $[v] = \text{m s}^{-1}$, so $[v^2] = (\text{m s}^{-1})^2 = \text{m}^2 \text{s}^{-2}$

Notice that **the left-hand side has the same unit as the right-hand side**. Why is this important? Two things can only be equal if they have the same units; 53 V can never be equal to 53 A – similarly 1 day and 1 cm could never be the same!

We say that this equation is **homogeneous** – only terms with the same units are added or subtracted and the units of the two sides are the same. If the 'equation' isn't homogeneous is cannot be right – you must have remembered it wrongly.

Warning

Just because an equation is homogeneous doesn't mean that it is right, e.g. $v^2 = u^2 + 3as$ is homogeneous and incorrect!

Terms & definitions

A vector quantity has magnitude and direction. A scalar quantity just has magnitude.

1.1.3 Scalar and vector quantities

(a) Forces and how to add them

The effect of a force on an object depends upon the direction in which it acts. Think about the effects of the two forces on the sledge in Fig. 1.1.1(a) and (b).

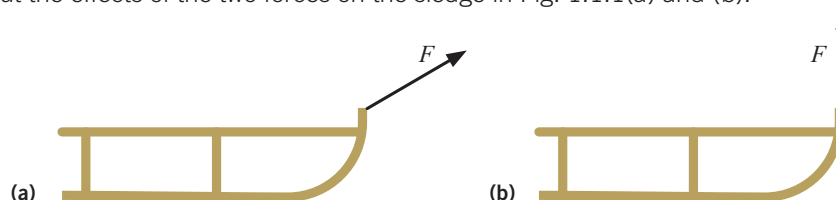


Fig. 1.1.1 Forces on a sledge

Force is known as a *vector* quantity. Other quantities, such as mass or density, are fully specified by their magnitude (size) and are referred to as *scalar* quantities.

Scalar quantities are easy to add, e.g.

$$3 \text{ kg} + 4 \text{ kg} = 7 \text{ kg}$$

and similarly, subtract

$$4 \text{ kg} - 3 \text{ kg} = 1 \text{ kg}.$$

We just use the normal rules of arithmetic.

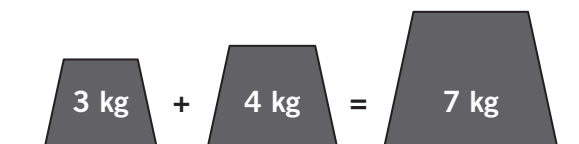


Fig. 1.1.2 Adding masses

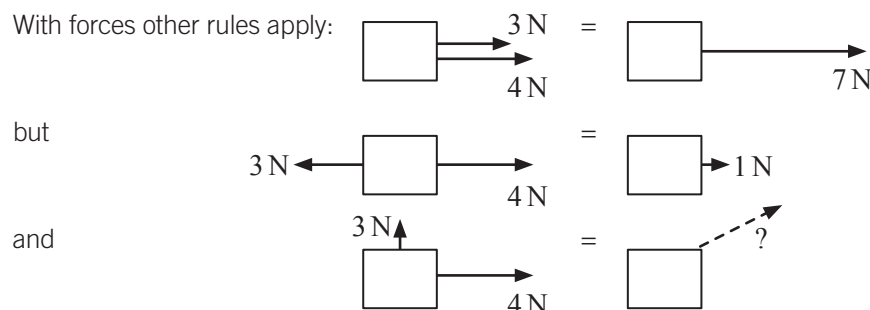


Fig. 1.1.3 Adding forces

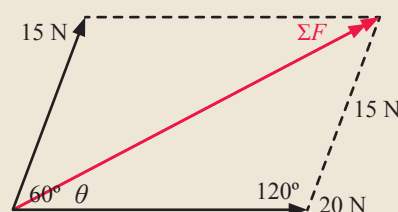
The combined effect of two (or more) forces is called the resultant force, F_{res} . We can calculate the sum of two forces using the *parallelogram law of vector addition*, as shown in Fig. 1.1.4. You *could* find the resultant force, ΣF , by drawing a scale diagram but it is more accurate to calculate it using trigonometry, e.g. the cosine rule or Pythagoras' theorem (if the forces are at right angles).

Note that for the AS exam you only need to be able to add two forces at right angles but if you are going on to higher studies you will need to be able to cope with other angles.

Example

Two forces, of magnitudes 20 N and 15 N act on an object at an angle of 60° to each other. Find the resultant force.

Answer First draw a sketch diagram and add the extra information, e.g. the 120° angle.



Using the cosine rule: $(\Sigma F)^2 = 20^2 + 15^2 - 2 \times 20 \times 15 \cos 120^\circ = 925$

$$\therefore \Sigma F = \sqrt{925} = 30.4 \text{ N}$$

Next calculate θ using the sine rule: $\frac{\sin \theta}{15} = \frac{\sin 120^\circ}{30.4}$, $\therefore \sin \theta = \frac{15 \sin 120^\circ}{30.4} = 0.427$

$$\therefore \theta = \sin^{-1} 0.427 = 25.3^\circ.$$

So the resultant force is 30.4 N at 25.3° to the 20 N force.

Most of the time you will only have to combine two vectors at right angles, which you can do using Pythagoras' theorem and simple trig, such as $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

Self-test 1.1.5 is an example.

(b) Motion scalars and vectors

Like length, distance is a scalar quantity. The question, 'What is the distance between Aberystwyth and Bangor?' doesn't ask about direction. If you know the answer, it doesn't help you to navigate from A to B. However, 'Bangor is 91 km due north of Aberystwyth' would enable a pilot to fly from one to the other. This quantity, which includes direction as well as distance, is called **displacement**. The displacement of Bangor from Aberystwyth is 91 km north. Similarly, Flint is 65 km east of Bangor.

Study point

What Fig. 1.1.3 means is that the combined effect of a 3 N and a 4 N force on the motion of an object is the same as a single force of magnitude 7 N (if they are in the same direction), 1 N (if they are in opposing directions) or anything in between if they are at different angles.

Study point

The resultant force, F_{res} , is often written ΣF (read as 'sigma F'). Sigma, Σ , means 'the sum of' so ΣF means 'the sum of the forces'.

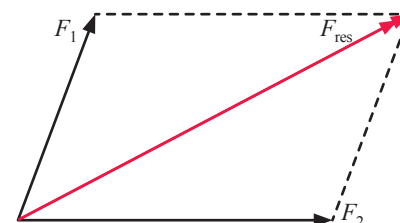


Fig. 1.1.4 The parallelogram law of vector addition

Exam tip

A force has a direction as well as a magnitude so, if the examiner asks you to calculate a force, you need to specify the direction. Make sure you make it clear what angle you mean.

Exam tip

If the two vectors are at 90° you can add them using simple trig, e.g. $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ and Pythagoras' theorem.

Self-test

1.1.5

Calculate the resultant force of a 3 N and a 4 N force acting at right angles (as in Fig. 1.1.3).

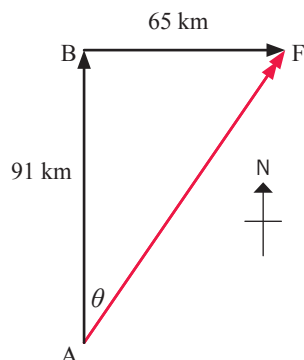


Fig. 1.1.6 Adding displacements

Terms & definitions

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

MATHS CHECK

If you look at Figs 1.1.4 and 1.1.6 we seem to have used two different ways of adding vectors.

In fact they give the same answer.

Here is 1.1.6 drawn like 1.1.4:

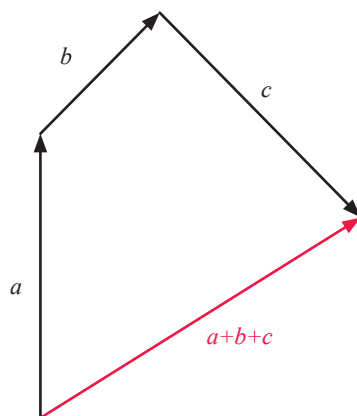
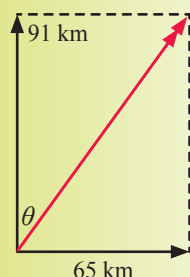


Fig. 1.1.7 Adding more than two vectors

What is the displacement from Aberystwyth to Flint? Fig. 1.1.6 shows how we can add the displacements $\vec{AB}$ and $\vec{BF}$ to give the resultant $\vec{AF}$ displacement (shown in red). You should be able to show that $\vec{AF} \sim 112 \text{ km}$ at $35.5^\circ \text{ E of N}$.

We calculate speed (strictly, mean speed) using $\text{speed} = \frac{\text{distance}}{\text{time}}$. Distance is a scalar so

speed is as well. The vector equivalent of speed is **velocity**, which is defined on the left. The next example shows the distinction between the two.

Example

A light aircraft flies from Aberystwyth to Bangor and then to Flint in a time of two hours. Using the data above, calculate (a) the mean speed and (b) the mean velocity.

(a) Distance travelled = $AB + BF = 91 + 65 = 156 \text{ km}$

$$\begin{aligned} \therefore \text{Mean speed} &= \frac{156 \text{ km}}{2 \text{ h}} \\ &= 78 \text{ km h}^{-1} \end{aligned}$$

(b) Displacement $\vec{AF} \sim 112 \text{ km}$ at $35.5^\circ \text{ E of N}$

$$\begin{aligned} \therefore \text{Mean velocity} &= \frac{112 \text{ km}}{2 \text{ h}} \\ &= 56 \text{ km h}^{-1} \text{ at } 35.5^\circ \text{ E of N. [Note: direction!]} \end{aligned}$$

Similarly **acceleration** is defined as the change in **velocity** per unit time and is also a vector, i.e.

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

$$\text{or, in symbols, } a = \frac{\Delta v}{\Delta t}$$

(c) Lists of scalar and vector quantities

These lists include most of the scalar and vector quantities that you'll encounter in AS/A level physics. The ones in italics are only in the full A level course.

Scalars – density, mass, volume, area, distance, length, speed, work, energy (all forms), power, time, resistance, temperature, potential (or pd or voltage), electric charge, *capacitance*, *activity*, pressure

Vectors – displacement, velocity, acceleration, force, momentum, *electric field strength*, *magnetic field strength* (or *magnetic flux density*), *gravitational field strength*.

(d) Adding more than two vectors

We have seen how to add two vectors using either the parallelogram method (see Fig. 1.1.4) or the nose-to-tail method (Fig. 1.1.6). Fig. 1.1.7 shows how we can extend the latter way of addition to more than two vectors. An alternative method is given in Section 1.1.4.

1.1.4 Working with vectors

(a) Subtracting vectors

To calculate an acceleration we need first to find a change in velocity, Δv . If the first velocity is v_1 and the second is v_2 , the $\Delta v = v_2 - v_1$. We know how to subtract scalars. How does it work for vectors?

For numbers we know that we can rewrite $53 - 45$ as $(-45) + 53$. We do the same thing with vectors. To find $v_2 - v_1$, in Fig. 1.1.8(a), we **add** $-v_1$ to v_2 . We can do this using the nose-to-tail method in (b) or the parallelogram method in (c). It doesn't matter which one you use: $v_2 - v_1$, the red vector, is obviously the same length and direction in (b) and (c). Diagram (b) is probably easier to draw but it is easier to make a mistake with the direction of $v_2 - v_1$.

Example

A car changes velocity from 25 m s^{-1} due E, to 20 m s^{-1} due N in 8.0 seconds. Calculate the mean acceleration.

Step 1: Draw the diagram. Take care with the direction of $v_2 - v_1$: Go backwards along the v_1 vector ($-v_1$) then forwards along the v_2 ($+v_2$).

Step 2: Use Pythagoras to calculate Δv .

$$(\Delta v)^2 = 25^2 + 20^2 = 1025. \therefore \Delta v = 32.0 \text{ m s}^{-1}.$$

Step 3: Calculate θ . $\tan \theta = \frac{25}{20} = 1.25. \therefore \theta = 51.3^\circ$

Step 4: Calculate a . $a = \frac{\Delta v}{\Delta t} = \frac{32.0}{8.0} = 4.0 \text{ m s}^{-2}$ at 51.3° W of N. [NB direction!]

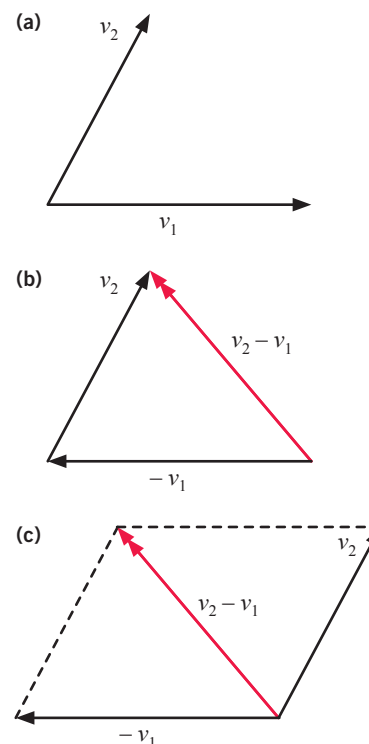
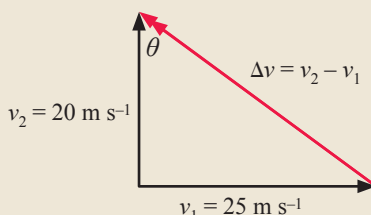


Fig. 1.1.8 Subtracting vectors

Exam tip

At AS, you will only have to subtract vectors at right angles.

(b) Components of vectors

Look back at Fig. 1.1.1(a). How much of the force F is pulling the sledge forwards and how much of it is lifting the sledge? In other words, if a force, F , acts at an angle θ to the horizontal, what are its horizontal and vertical components, F_h and F_v ? Fig. 1.1.9 clarifies the question: F_h and F_v are the horizontal and vertical forces which add together to give F as the resultant.

Using elementary trigonometry: $F_h = F \cos \theta$

$$F_v = F \sin \theta$$

$$\text{and } F = \sqrt{F_h^2 + F_v^2}$$

Why is this a useful technique? For all sorts of reasons. Here are just two:

1. If the motion is horizontal (like the sledge) the horizontal component of the force multiplied by the distance moved gives the work done, i.e. the energy transferred.
2. When adding several (i.e. more than two) vectors, it is often easier to find the horizontal and vertical components of each and add them.

Sometimes it is useful to find the components in directions other than horizontal and vertical, e.g. for the forces on a car on a slope the sensible directions to calculate components of the forces or velocity are parallel to and at right angles to the slope.

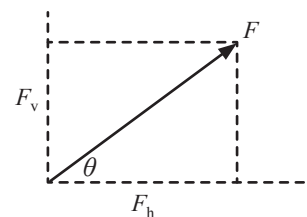


Fig. 1.1.9 Force components

1.1.6

Self-test

In Fig. 1.1.9, $F = 150 \text{ N}$ and $\theta = 30^\circ$. Calculate F_v and F_h .

Terms & definitions

Finding the component of a force is called **resolving**: we **resolve** a force into its horizontal and vertical components.

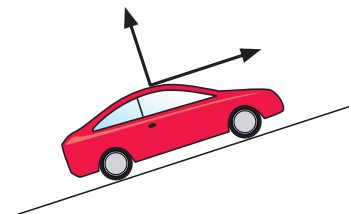


Fig. 1.1.10 Component directions on a slope

1.1.7

Self-test

What is the component of B in the y direction?

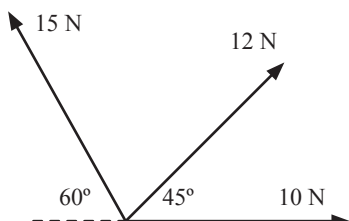
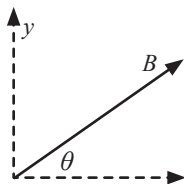


Fig. 1.1.12 Example

Study point

In the example, the angle between the 15 N force and the right horizontal line is 120° .

Terms & definitions

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}; \rho = \frac{M}{V}$$

UNIT

If mass is measured in kg and volume in m^3 , the unit of density is kg m^{-3} . For mass in g and volume in cm^3 , density is in g cm^{-3} .
 $1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$.

SYMBOL

The usual symbol for density is the Greek letter ρ (rho).

We will meet this sort of situation often in the AS course. The important thing to remember is that the component of a vector, A , in a direction at angle θ to the direction of the vector is always $A \cos \theta$.

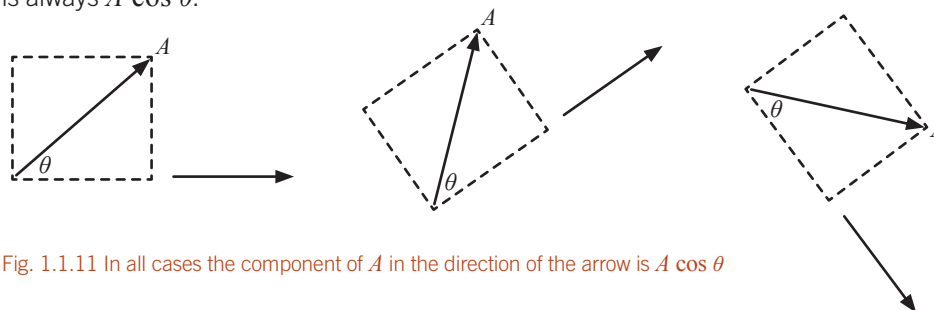


Fig. 1.1.11 In all cases the component of A in the direction of the arrow is $A \cos \theta$

Example

Use components to find the resultant of the forces in Fig. 1.1.12.

$$\begin{aligned} \text{Total horizontal component} &= 10 + 12 \cos 45^\circ + 15 \cos 120^\circ \text{ [see Study point]} \\ \text{(to the right)} &= 10 + 8.485 - 7.5 \text{ [Note the '-' sign]} \\ &= 10.99 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Total vertical component} &= 15 \sin 60^\circ + 12 \sin 45^\circ \\ \text{(upwards)} &= 21.48 \text{ N} \end{aligned}$$

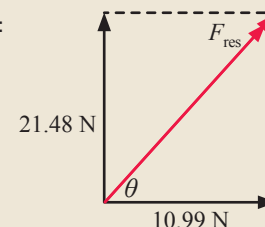
Combine the two components using Pythagoras' theorem:

$$F_{\text{res}}^2 = 21.48^2 + 10.99^2$$

$$\therefore F_{\text{res}} = 24.1 \text{ N.}$$

$$\text{And } \theta = \tan^{-1} \frac{21.48}{10.99} = 62.9^\circ$$

$\therefore$ The resultant force is 24.1 N at 62.9° to the horizontal.



1.1.5 Density

For a material of uniform composition, the mass of a sample is directly proportional to its volume. Hence the ratio of mass to volume is a constant, which is characteristic of the material. We call this constant the **density**.

Table 1.1.2 contains the densities of some common materials. You should make sure you can make a reasonable estimate of densities if you have to in the exams.

Table 1.1.2 Densities

Material	$\rho / \text{kg m}^{-3}$	$\rho / \text{g cm}^{-3}$	Material	$\rho / \text{kg m}^{-3}$	$\rho / \text{g cm}^{-3}$
Air*	1.29	0.001 29	Steel	7900	7.90
Water	1 000	1.00	Aluminium	2800	2.8
Brick	2 300	2.30	Mercury	13 600	13.6
Petrol	880	0.88	Gold	19 300	19.3

* At 0°C and atmospheric pressure.

The range of densities of materials on the Earth is quite large. The drawings in Fig. 1.1.13 all represent 1 tonne (10^3 kg) of material. But this range pales into insignificance compared with the range of densities in the universe. For comparison, Table 1.1.3 gives the size of 1 tonne cubes of various materials outside the Earth. The range of densities shown in the table is $\sim 10^{33}$.

Table 1.1.3 Densities in the universe	
material	width of 1 t cube
Interstellar space	10^6 km
Red giant star	100 m
The sun	0.89 m
White dwarf	8.9 mm
Neutron star	15 μm

Fig. 1.1.14 The Cat's Eye nebula
The white dot in the middle is a white dwarf
star with a density of $\sim 10^9 \text{ kg m}^{-3}$.

Problems involving density will usually require you to convert units. Either the density or the volume often needs to be converted. If the volume is given in cm^3 and the density in kg m^{-3} then:

either convert the volume using $1 \text{ cm}^3 = 1 \times 10^{-6} \text{ m}^3$

or convert the density using $1000 \text{ kg m}^{-3} = 1 \text{ g cm}^{-3}$

Example

A rectangular block of steel, of density 7900 kg m^{-3} , has length 10.0 cm , width 5.0 cm and height 4.0 cm . Calculate its mass.

Equation first: $\rho = \frac{M}{V} \therefore M = \rho V$. We'll use kg and m^3 as the units.

$$\begin{aligned}
 \text{Mass} &= 7900 \text{ kg m}^{-3} (10 \times 10^{-2} \text{ m} \times 5 \times 10^{-2} \text{ m} \times 4 \times 10^{-2} \text{ m}) \\
 &= 7900 \text{ kg m}^{-3} \times 2 \times 10^{-4} \text{ m}^3 \\
 &= 1.58 \text{ kg}
 \end{aligned}$$

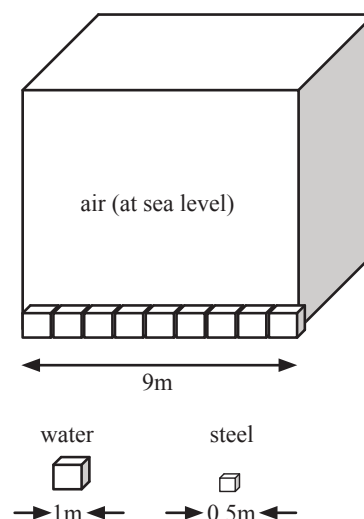


Fig. 1.1.13 Range of densities

Self-test

1.1.8

Repeat the calculation in the example using g and cm^{-3} . Remember to convert the density from kg m^{-3} to g cm^{-3} .

1.1.6 Moments of forces

(a) The turning effect of a force

Sometimes a force causes things to accelerate. Sometimes they stretch or compress an object or make it rotate.

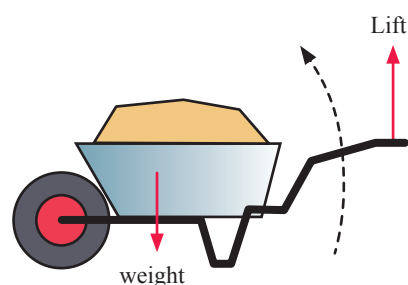


Fig. 1.1.15 Forces causing rotation

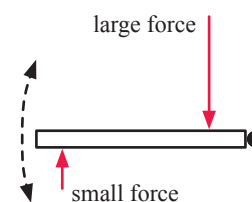
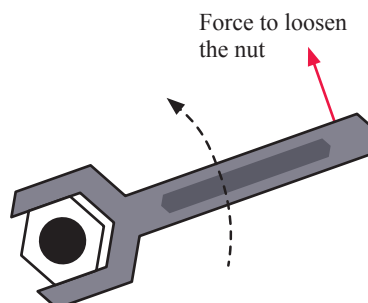


Fig. 1.1.16 Forces on a door

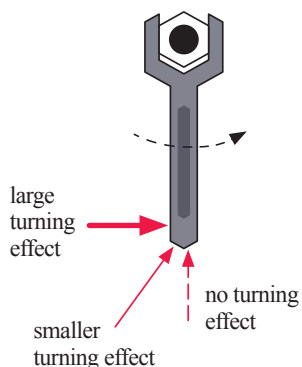


Fig. 1.1.17 Direction matters

Study point

The moment of a force is also called its *torque*. The symbol τ is sometimes used for moment (or torque).

UNIT

The SI unit of moment is N m . It can also be expressed in kN m , N cm , etc.

1.1.9

Self-test

Identify the moment of each of the forces in Fig. 1.1.15 as CM (clockwise) or ACM.

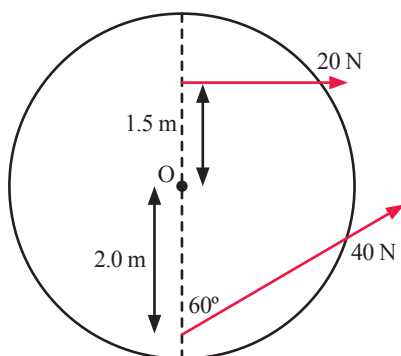


Fig. 1.1.19 Example

The forces (red arrows) in Fig. 1.1.15 cause the wheelbarrow, the spanner and nut and the bicycle sprocket wheel to turn about the pivot. Anyone who has used a spanner knows that the longer the handle, the easier it is to undo a nut. In other words, the turning effect of the force is bigger if it is applied further away from the pivot.

An easy experiment to show this difference in turning effect is for two people to push on either side of a door. It is easy for a child to hold a door shut against an adult – if the adult pushes close to the hinge! (See Fig. 1.1.16)

(b) The principle of moments

The turning effect of a force about a point depends upon its direction as well as the distance from a point – see Fig. 1.1.17. We take account of this when we define the **moment** of a force, which is the mathematical expression of its turning effect:

The **moment** of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

This is clarified in Fig. 1.1.18. The force F is applied at a distance x from P . But the perpendicular distance from P to the line of action of F is d . So:

$$\text{Moment of } F \text{ about } P = Fd.$$

Looking at Fig. 1.1.18 again we notice that $d = x \cos \theta$.

$$\therefore \text{Moment of } F \text{ about } P = Fx \cos \theta.$$

We can also write this as $(F \cos \theta)x$.

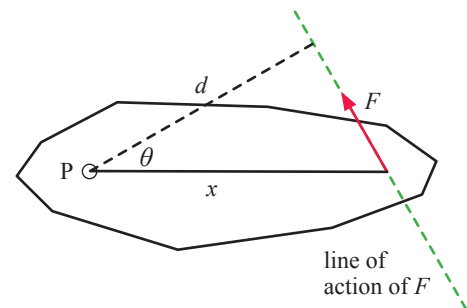


Fig. 1.1.18 Moment of F about $P = Fd$

But $F \cos \theta$ is the component of F perpendicular to the line joining P and the point of application of F , so this is another way of calculating the moment.

If we look back at Fig. 1.1.16, we see that the two forces are acting in opposite senses: the small force tends to make the door move clockwise about the hinge; the large force, anti-clockwise. We say that the small force has a **clockwise moment** (CM) and the large force has an **anti-clockwise moment** (ACM).

Example

Calculate the moment about O of each of the forces in Fig. 1.1.19

(a) The perpendicular distance of the line of action of the 20 N force is 1.5 m from O .

$$\therefore \text{The clockwise moment of the 20 N force about } O = 20 \text{ N} \times 1.5 \text{ m} = 30 \text{ N m}.$$

(b) **Either:** The perpendicular distance of the line of action of the 40 N force is $2.0 \sin 60^\circ$ from O .

$$\therefore \text{The ACM of the 40 N force about } O = 2.0 \sin 60^\circ \times 40 \text{ N m} = 69.3 \text{ N m}$$

Or: The perpendicular component of the 40 N force to the 2.0 m displacement is $40 \sin 60^\circ$

$$\therefore \text{The ACM of the 40 N force about } O = 40 \sin 60^\circ \times 2.0 \text{ N m} = 69.3 \text{ N m}$$

Now, if those are the only forces acting, the disc in the example will start turning anticlockwise – the anticlockwise moment is larger than the clockwise moment. There is a **resultant anticlockwise moment** of $69.3 - 30.0 = 39.3 \text{ N m}$. What happens after that is not clear because we don't know how the forces are applied to the disc; will they stay the same in magnitude, direction and position of application? But it does lead us to an important principle: the **Principle of Moments** (see the definition). For the moment we'll ignore the phrases 'about any point' and 'about the same point'. We'll come back to them in Section 1.1.7.

With the aid of the PoM, we are in a position to solve a real-life problem!

Example

Where must the fat cat sit to balance the other two on the see-saw?

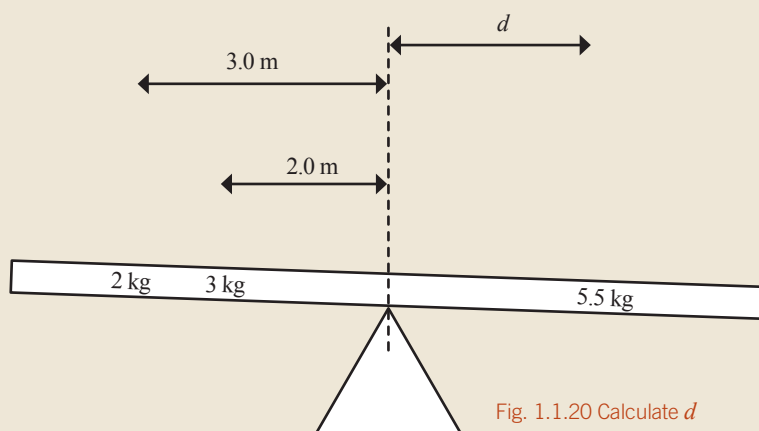


Fig. 1.1.20 Calculate d

The weights of the cats are (using mg), 19.62 N, 29.43 N and 53.96 N respectively. Using PoM, the resultant moment about the pivot must be zero.

$$\therefore \text{Taking clockwise as positive: } 53.96d - 29.43 \times 2.0 - 19.62 \times 3.0 = 0$$

$$\therefore \text{Solving this equation } d = 2.18 \text{ m}$$

$\therefore$ The fat cat must sit 2.18 m from the pivot.

(c) The centre of gravity (C of G)

In the last example, we treated the cats (of all sizes) as though they were point masses. This is obviously not true – they are spread out. However, for any object we can identify a point at which we can consider all its weight to act. This is called its **centre of gravity**. In a uniform gravitational field (which will always be the case in AS physics) the C of G of a symmetric body, of uniform density, will lie on any plane of symmetry. Fig. 1.1.21 has the examples you are likely to meet:

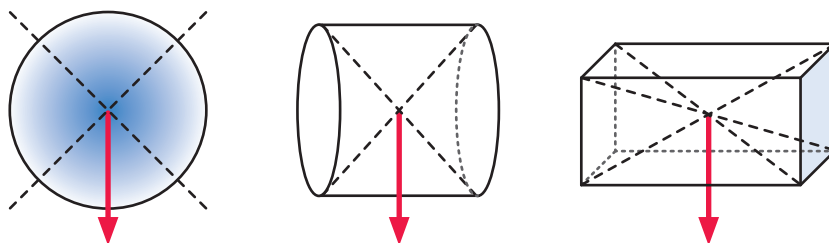


Fig. 1.1.21 Centres of gravity

Terms & definitions

The **principle of moments**: For a body to be in equilibrium under the action of a number of forces, the resultant moment about any point is zero (alternatively: the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about the same point).

Study point

Notice that the see-saw in the Example is pivoted at the centre. We'll deal with more difficult situations in Section 1.1.7.

Self-test

1.1.10

If the mass of the see-saw plank in the example is 10 kg, show that the pivot must exert an upward force of ~200 N on the plank.

Stretch & Challenge

For a rigid body, the position of the C of G is fixed within the body; but not for flexible bodies. the position moves. Where is the high jumper's C of G?

Fig. 1.1.22

For a standing object, such as a bus or a racing car, the lower the centre of gravity and the wider the base, the more stable it is. That means that objects with low centres of gravity can be tipped more before they topple over.

Fig. 1.1.23 shows the principle of this using a tall rectangular block. The block is just on the edge of tipping – it could go either way because the C of G is just over the balance point. From the geometry,

$$\tan \theta = \frac{w}{h}$$

where h is the height of the C of G and

w is half the width of the block.

Racing cars have a very wide wheel base and are very low on the ground, so their centres of gravity are also low. Fig. 1.1.24 shows a tilt test on an F1 car.

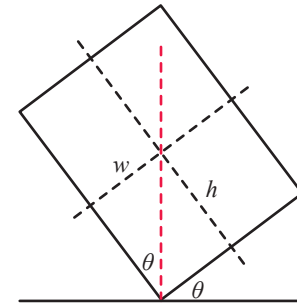


Fig. 1.1.23 Stability of a block

Fig. 1.1.24 Stability testing

Terms & definitions

For a body to be in **equilibrium** under the action of a number of forces ... (see main text)

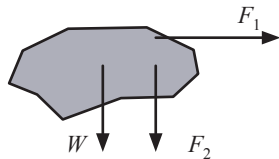


Fig. 1.1.25 This object is not in equilibrium

Study point

In Fig. 1.1.26, there are two unknown quantities (F and d), so we need two equations to calculate them.

1.1.7 Conditions for equilibrium

A body is said to be **in equilibrium** if it is moving and rotating at a constant rate. In many cases, especially when applied to engineering objects, such as bridges and buildings, this means it is not moving at all. In order for this to happen:

1. The resultant force on the object must be zero, and
2. The resultant moment (about any point) must be zero (the principle of moments).

The object in Fig. 1.1.25 is clearly **not** in equilibrium: the resultant force is downwards and to the right and the resultant moment is clockwise (about the centre of gravity).

What about the metre rule arrangement in Fig. 1.1.26? The rule weighs 1 N and we can consider this weight to act at the 50 cm mark, the centre of gravity. Assuming it balances, what are the values of d and F ?

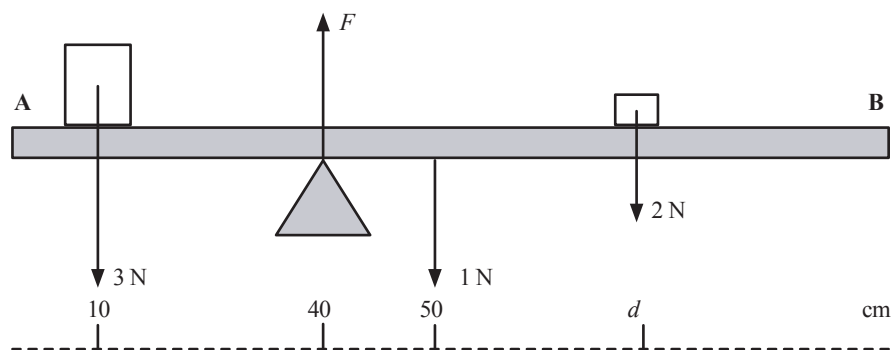


Fig. 1.1.26 Balanced rule

1.1.11

Self-test

Find d in Fig. 1.1.26 by assuming $F = 6$ N, (from applying $\Sigma F = 0$) and taking moments about **B**.

1.1.12

Self-test

Find d and F in Fig. 1.1.26 by taking moments, about **A**, then about **B** and solving the simultaneous equations.

Applying condition 1: The resultant force = 0

$$\therefore F = 3 \text{ N} + 1 \text{ N} + 2 \text{ N} = 6 \text{ N},$$

$\therefore$ The pivot exerts an upward force of 6 N for the ruler to be in equilibrium.

Applying condition 2: The resultant moment = 0 (about any point).

Let's take moments about end **A**: The 3, 1 and 2 N forces each have a clockwise moment about **A**; F (= 6 N) has an anticlockwise moment. Taking CM as positive:

$$\therefore 3 \text{ N} \times 10 \text{ cm} - 6 \text{ N} \times 40 \text{ cm} + 1 \text{ N} \times 50 \text{ cm} + 2 \text{ N} \times d = 0$$

$\therefore$ (simplifying) $2d = 160 \text{ cm}$. $\therefore$ The 2 N weight must be at the 80 cm mark.

The last problem we are going to look at is, how to find an unknown force if there are forces at different angles. For example, what force F must we apply in Fig. 1.1.27 so that the forces are in equilibrium? We don't need to worry about rotations because all the forces pass through the same point. We have a choice of three techniques – but two of them are essentially the same!

- Add the 10 N and 6 N. Then F must be equal and opposite to the resultant.
- Add the 10 N, the 6 N and F as in Fig. 1.1.7, so that the resultant is 0.

The labels are left for you to complete!

- Resolve in two directions – horizontally and vertically are the obvious ones:

Vertically: $F \cos \theta = 10 \text{ N}$ [1] Remember: through the angle $\rightarrow \cos$

Horizontally: $F \sin \theta = 6 \text{ N}$ [2] Through $90^\circ - \theta \rightarrow \sin$

Dividing [2] by [1] and remembering that $\frac{\sin \theta}{\cos \theta} = \tan \theta \rightarrow \tan \theta = \frac{10}{6}$.

This lets us calculate θ and then we can use [1] or [2] to calculate F .

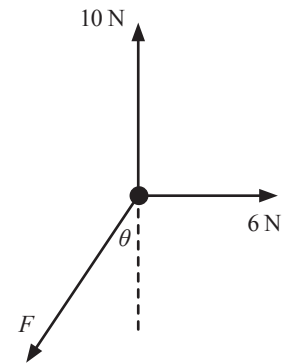
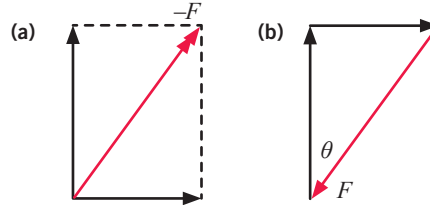


Fig. 1.1.27 Find the equilibrant

1.1.13

Self-test

Find F and θ in Fig. 1.1.27 using methods (a), (b) and finish off (c).

We finish off with two more difficult examples. The first shows that the principle of moments applies even when there isn't a pivot: if something doesn't start to rotate, the moments of the forces about any point must add to zero, even for a bridge! The second is more difficult because the forces are not parallel.

Example

Fig. 1.1.28 shows a load on a bridge. Calculate the forces, F_1 and F_2 , provided by the supports.

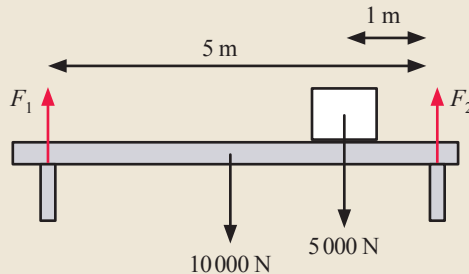


Fig. 1.1.28 Forces on a bridge

Apply the conditions for equilibrium:

- Resultant force = 0

$$\begin{aligned} \therefore F_1 + F_2 &= 10\,000 + 5\,000 \\ &= 15\,000 \text{ N} \quad [1] \end{aligned}$$

- Resultant moment about any point = 0

Take moments about the left-hand support with clockwise being positive.

$$\therefore 10\,000 \text{ N} \times 2.5 \text{ m} + 5\,000 \text{ N} \times 4 \text{ m} - F_2 \times 5 \text{ m} = 0$$

$$\therefore 5F_2 = 45\,000 \text{ N}$$

$$\therefore F_2 = 9\,000 \text{ N} \quad [2]$$

Substitute the value for F_2 in equation [1] to calculate $F_1 \rightarrow F_1 = 6\,000 \text{ N}$

Stretch & Challenge

If a body in equilibrium has **three** non-parallel forces acting on it, all the lines of action of the forces must pass through a single point.

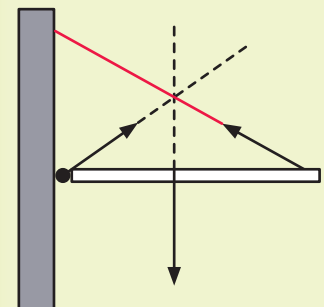


Fig. 1.1.29

The force exerted by the hinge on the rod must pass through a point on the cord vertically above the C of M of the rod.

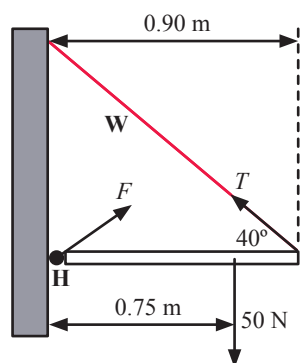


Fig. 1.1.30 Pub sign

1.1.14

Self-test

Calculate the magnitude and direction of the force F in Fig. 1.1.30.

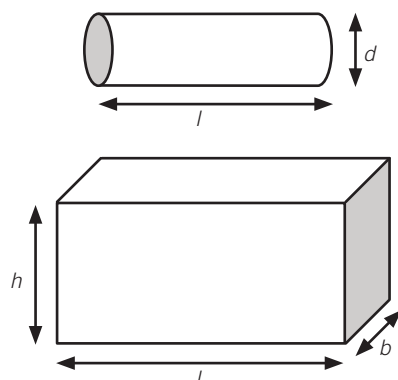


Fig. 1.1.31 Regular solids

PRACTICAL TIP

See Chapter 3 for combining and reducing uncertainties.

Exam tip

Remember that you measure the *diameter* of a wire, not the *radius*!

Example

A pub sign is supported on a vertical wall by a hinge, H and a wire, W , as shown in Fig. 1.1.30. Calculate the tension, T , in W .

Take moments about H . ACM positive. Using the principle of moments:

$$T \sin 40^\circ \times 0.90 \text{ m} - 50 \text{ N} \times 0.75 \text{ m} = 0 \therefore T = 64.8 \text{ N}$$

Following on from this example, we can also use the equilibrium conditions to find the force exerted by the hinge on the pub sign bar – F in Fig. 1.1.30. We could do this using the triangle of forces:

The three forces on the sign have a 0 resultant. Using the result for the tension, the triangle looks like this.

We can use the cosine rule to find F and θ .

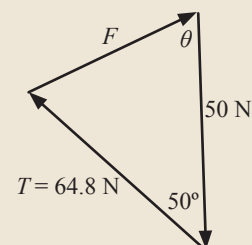
Alternatively we use components and the fact that the resultant horizontal and vertical components are zero:

$$\therefore \text{Horizontally: } F \sin \theta = T \cos 40^\circ \therefore F \sin \theta = 49.6 \text{ N} \quad [1]$$

$$\text{and vertically } F \cos \theta + T \sin 40^\circ = 50 \therefore F \cos \theta = 8.35 \text{ N} \quad [2]$$

$$\text{If we then divide equation [1] by equation [2] we get } \tan \theta = \frac{49.6}{8.35} = 5.94.$$

So we can calculate θ and hence F .



1.1.8 Practical work

(a) Measuring the density of solids

Finding the density of a substance involves measuring the mass and the volume and dividing mass by volume. This practical is often used to test understanding of uncertainties and how to combine them. The mass is usually determined from a single reading electronic balance, so the absolute uncertainty is taken as ± 1 in the last digit of the reading;

e.g. a reading of 159.73 g would be taken to be $(159.73 \pm 0.01) \text{ g}$.

The uncertainty in mass is often not as significant as that in the volume.

How the volume is determined depends upon whether the solid object has a regular shape, e.g. a cuboid, e.g. a microscope slide, or a cylinder, e.g. a wire.

(i) Regular solids

$$\text{Volume of a cuboid} = lbh; \text{ volume of a cylinder} = Al = \pi r^2 l = \frac{\pi d^2 l}{4}$$

For lengths up to $\sim 15 \text{ cm}$, digital callipers, with a resolution of 0.01 mm are normally used. It is important when using them to check the zero reading, i.e. close the jaws and take a reading. Any reading should be subtracted from the reading with the object being measured. For lengths $> 15 \text{ cm}$ a metre rule with a resolution of 1.0 mm is normally used.

The precision can be improved for a set of identical objects by laying them end to end, e.g. laying 10 microscope slides end to end gives a length of $\sim 75 \text{ cm}$; using a mm scale to measure the length gives a % uncertainty of 0.13%.

(ii) Irregular solids

The solid, e.g. a rock, is suspended from a thread and lowered into a measuring cylinder of water until it is completely submerged. The increase in volume reading is the volume of the solid. If the solid is too large for a measuring cylinder, a *displacement can* is used

(Fig. 1.1.32) and the water overflow captured in a measuring cylinder. Disadvantages of this method are: (a) the resolution of the measuring cylinder is quite large (typically $1\text{--}2\text{ cm}^3$ for a 100 cm^3 cylinder) and (b) the volume of water overflowing is not necessarily exactly the same as the volume of the object.

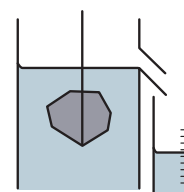


Fig. 1.1.32 Measuring the volume of a rock

(b) Measuring mass using the principle of moments

In Fig. 1.1.33, the long bar is a $\frac{1}{2}$ metre or metre rule. The triangle is any pivot – it could be as simple as an outstretched finger. The pivot is placed at the centre of gravity of the rule.

When the bar balances, the ACM and the CM about the pivot are equal.

$$\therefore Mgx = mgy$$

$$\therefore Mx = my.$$

We can also use this technique to find the mass of the bar itself. The position of the C of G is found as in the Study point.

A known mass, m , is hung near one end of the bar. The pivot point is found and distances x and y measured.

Then, as above: $Mx = my$.

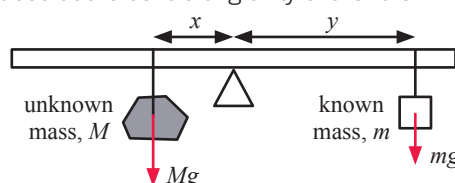


Fig. 1.1.33 Finding an unknown mass

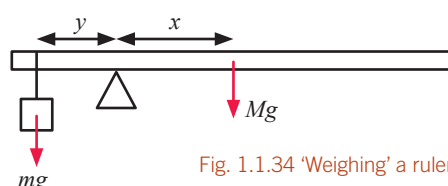


Fig. 1.1.34 'Weighing' a ruler

Study point

The centre of gravity is not necessarily at the midpoint of the scale. It is a good idea to do a preliminary experiment to find it:



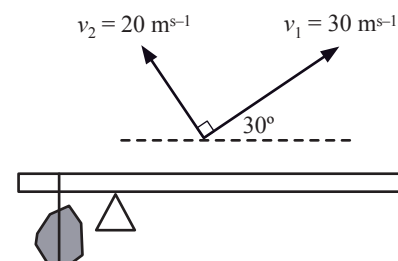
The C of G is above the balance point.

PRACTICAL TIP

The results will be most accurate if both x and y are as large as possible. Thus, in Fig. 1.1.33, the known mass should be similar to the unknown mass.

Exercise 1.1

- Newton's law of gravitation states that two small bodies of masses, of mass M_1 and M_2 , separated by a distance d attract each other with a force, F , given by: $F = \frac{GM_1M_2}{d^2}$, where G is the universal gravitational constant.
 - Show that $[G] = \text{N m}^{-2} \text{ kg}^{-2}$.
 - Express $[G]$ in terms of the base SI units, **m**, **kg** and **s**.
- The power, P , dissipated by a resistor, R , with a pd, V , across it and a current, I , is given by $P = IV$.
 - Express the unit of pd (the volt, **V**) in terms of the base SI units, **m**, **kg**, **s**, **A**.
 - By using a suitable equation express the ohm, Ω , in terms of the base SI units.
- A cylinder of length **1.5 m** and diameter **60 mm** is made from iron of density 7900 kg m^{-3} . Calculate its mass.
- The radius of the orbit of Mars is **220 million km**. Express this in metres in standard form.
- Estimate the mass of a **1 mm** diameter aluminium ($\rho = 2.7\text{ g cm}^{-3}$) wire which stretches once around the Earth.
- Find the horizontal and vertical components of each of $\mathbf{v}_1$ and $\mathbf{v}_2$.
- Use components to find
 - the sum and
 - the difference ($\mathbf{v}_2 - \mathbf{v}_1$) of the two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ and express the direction of the answers as an angle to the dotted line.
- Repeat the calculations of Q7 using parallelograms of vectors.
- A rock balances a uniform **50 g** metre rule when it is hung from the **10 cm** mark and the fulcrum at the **20 cm** mark. When it is lowered into a measuring cylinder the water level rises from **150 cm**³ to **190 cm**³. Calculate the density of the rock.
- The fulcrum in Q9 is moved to the **40 cm** mark and the ruler rebalanced by adding an additional mass to the **80 cm** mark. Calculate (a) the required mass and (b) the upward force exerted on the ruler by the fulcrum.



1.2 Kinematics

Kinematics is to do with motion and its mathematical description. It is the study of how things move without considering why they move, which is covered in the next topic, Dynamics.

Study point

Kinematics comes from the Greek word κίνημα (kinema) which means *motion*. From this we also get *kinetic energy* and *cinema* (motion pictures).

Terms & definitions

$$\text{Mean velocity} = \frac{\text{displacement}}{\text{time taken}}$$

SYMBOL

We use the symbol x for displacement. Many textbooks use the symbol s .

Study point

An upward-sloping x - t graph means that x is increasing, i.e. a positive velocity. If the graph slopes down, the direction of motion is reversed.

MATHS CHECK

See Chapter 4 for help with finding the gradient of a graph for straight and curved graphs.

1.2.1

Self-test

Calculate the mean velocity in this time shown in the first 6 seconds of Fig. 1.2.1.

1.2.2

Self-test

Over which periods in Fig. 1.2.1 is the velocity constant?

1.2.3

Self-test

Describe the motion in Fig. 1.2.1 qualitatively.

1.2.1 Speed and velocity

(a) Motion in a straight line – displacement–time graphs

Looking at motion in a straight line is not as restrictive as it might appear. To start with, road and rail transport (especially the latter) are restricted to motion along linear pathways. Even the fact that the routes wriggle about somewhat needn't concern us as we are not investigating the causes of motion. Also, when we consider motion in two (and three) dimensions, we'll often look at the components of the motion: essentially, 3D motion consists of three sets of motion in straight lines!

The displacement–time graph, Fig. 1.2.1 is of a car moving along a road. We'll use it to help understand certain terms. The displacement is the distance along the road measured to the right.

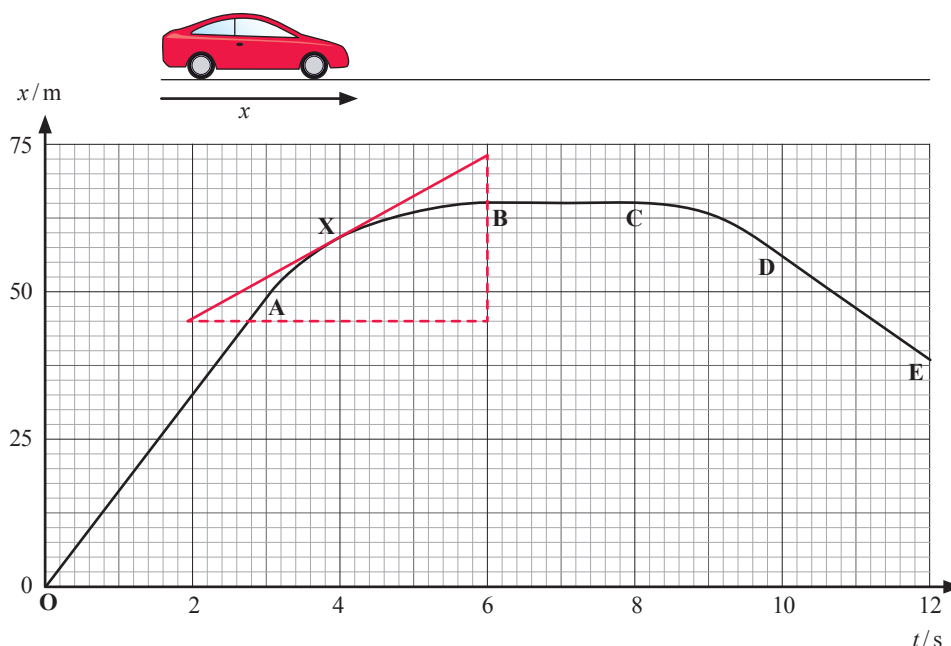


Fig. 1.2.1 x - t graph for a car

1. An upward slope (between **O** and **B**) represents an *increase* in x with time, so a movement in the positive x direction; downward slope (**D** to **E**) is moving in the negative direction; horizontal (**B** to **C**) means stationary.
2. The *gradient* of the graph represents the increase in displacement per second, so the gradient of a displacement–time graph is the velocity. If the graph is a straight line, the gradient is constant and so the velocity is constant.

Interrogating Fig. 1.2.1 mathematically:

$$\text{The mean velocity between 3 and 8 seconds [A to C]} = \frac{\Delta x}{\Delta t} = \frac{(65-50) \text{ m}}{5 \text{ s}} = 3 \text{ m s}^{-1}.$$

From **D** to **E** the velocity is constant. $v_{DE} = \frac{\Delta x}{\Delta t} = \frac{(37-59) \text{ m}}{2.4 \text{ s}} = \frac{-22}{2.4} = -8.3 \text{ m s}^{-1}$.

The *instantaneous velocity* at **X** is the gradient of the tangent at **X**:

$$v_X = \frac{\Delta s}{\Delta t} = \frac{(73-46) \text{ m}}{4.0 \text{ s}} = \frac{27}{4.0} = 6.8 \text{ m s}^{-1}.$$

The displacement, x , for the whole journey is 37 m but the distance, d , travelled by the car is 93 m. These are made up as follows:

$$x_{OB} + x_{BC} + x_{CE} = 65 \text{ m} + 0 \text{ m} + (-28 \text{ m}) = 37 \text{ m}$$

$$d_{OB} + d_{BC} + d_{CE} = 65 \text{ m} + 0 \text{ m} + 28 \text{ m} = 93 \text{ m}$$

So the *mean speed* for the whole journey = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{93}{12} = 7.8 \text{ m s}^{-1}$.

(b) Motion in two dimensions

Again we'll look at a specific example to see how to understand the terms.

The Earth orbits the Sun in a (nearly) circular path at a (nearly) steady speed. The radius of the orbit is 149.6 million km and the orbital period is 365.25 days. We'll use this information and Fig. 1.2.2 to distinguish between various terms. You might want to look back to Section 1.1.3.

First we'll work out the *mean speed* of the Earth in its orbit:

$$\text{Mean speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

In 1 orbit, distance travelled = $2\pi \times 149.6 \times 10^6 \text{ km}$.

Time for 1 orbit = 365.25 days = $3.156 \times 10^7 \text{ s}$

$$\therefore \text{Mean speed} = \frac{2\pi \times 149.6 \times 10^6 \text{ km}}{3.156 \times 10^7 \text{ s}} = 29.8 \text{ km s}^{-1}.$$

We'll assume that this is also the *instantaneous speed* at any moment because the speed is nearly constant.

What about velocity? The *instantaneous velocity* at position 1, v_1 , is its speed in direction x :

$$\therefore v_1 = 29.8 \text{ km s}^{-1} \text{ in direction } x.$$

and $v_2 = 29.8 \text{ km s}^{-1}$ 90° from x [measuring angles anticlockwise from x].

What about v_3 ? We could write: $v_3 = 29.8 \text{ km s}^{-1}$ 180° from x . We could write this as $v_3 = -29.8 \text{ km s}^{-1}$ in direction x . [Note the minus sign.]

The *mean velocity* is defined by:

$$\text{Mean velocity} = \frac{\text{displacement}}{\text{time taken}}$$

so to calculate the mean velocity from position 1 to 2, $\langle v_{1,2} \rangle$, we first need to know the displacement Δx .

Using Pythagoras' theorem you should be able to show that $\Delta x = 211.6$ million km at 45° from the x direction. Also the time $1 \rightarrow 2$ is $\frac{1}{4}$ of a year.

$$\therefore \langle v_{1,2} \rangle = \frac{\Delta x}{\Delta t} = \frac{211.6 \times 10^6 \text{ km}}{0.789 \times 10^7 \text{ s}} = 26.8 \text{ km s}^{-1} \text{ at } 45^\circ \text{ from } x.$$

Study point

Δ (delta) means *change of*, so Δx means *change of displacement*.

If x changes from x_1 to x_2 then $\Delta x = (x_2 - x_1)$.

Study point

The *speed* of an object takes no account of direction, so the speed between **D** and **E** is 8.3 m s^{-1} .

Self-test

1.2.4

Calculate the mean *velocity* for the journey in Fig. 1.2.1.

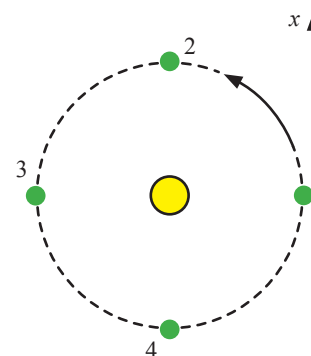


Fig. 1.2.2 Earth in orbit

Self-test

1.2.5

What is the instantaneous velocity at position 4?

Exam tip

Know how to convert, hours, days and years into s:

$$1 \text{ h} = 3600 \text{ s}$$

$$1 \text{ d} = 24 \times 3600 = 86400 \text{ s}$$

$$1 \text{ yr} = 365.25 \times 86400 \text{ s}$$

Self-test

1.2.6

For Fig. 1.2.2 determine

(a) v_4 , (b) $\langle v_{1,3} \rangle$ (c) $\langle v_{3,4} \rangle$

Self-test

1.2.7

For Fig. 1.2.2 what is the velocity, v , 1 month (a) before and (b) after position 1?

1.2.2 Acceleration

Study point

If the velocity changes from v_1 to v_2 , then $\Delta v = v_2 - v_1$.

Study point

The unit of acceleration is m s^{-2} .

An acceleration of 5 m s^{-2} means that the velocity increases by 5 m s^{-1} every second, e.g. velocities of 2, 7, 12, 17... m s^{-1} at 1 second intervals.

1.2.8

Self-test

Describe the journey in Fig. 1.2.3 between 20 and 68 s.

Terms & definitions

The gradient of a velocity–time graph is the acceleration.

So a horizontal v – t line represents constant velocity.

Terms & definitions

Deceleration is negative acceleration; e.g. an acceleration of -1.5 m s^{-2} is a deceleration of 1.5 m s^{-2} .

1.2.9

Self-test

In Fig. 1.2.3, calculate the mean acceleration between 10 and 35 seconds.

Study point

The displacement is often said to be the area *under* the graph, but if the velocity is negative it will be the area under the t -axis and above the graph.

(a) Motion in a straight line – velocity–time graphs

Sections **AB** and **CD** of Fig. 1.2.1 represent changing velocity, i.e. acceleration. The mean acceleration, over a time interval Δt , is defined as:

$$\text{Mean acceleration} = \frac{\Delta v}{\Delta t}$$

An approximation to the *instantaneous acceleration* at a particular time is given by the same expression but with the time interval, Δt , made very small.

Again it is convenient to look at this using a graph: this time a graph of velocity against time (a v – t graph). This time, in Fig. 1.2.3, we'll imagine a local train, e.g. an underground train, moving between stations. It starts off at rest, accelerates uniformly for 5 s, coasts for 15 s, etc., before finally decelerating to rest at 68 s.

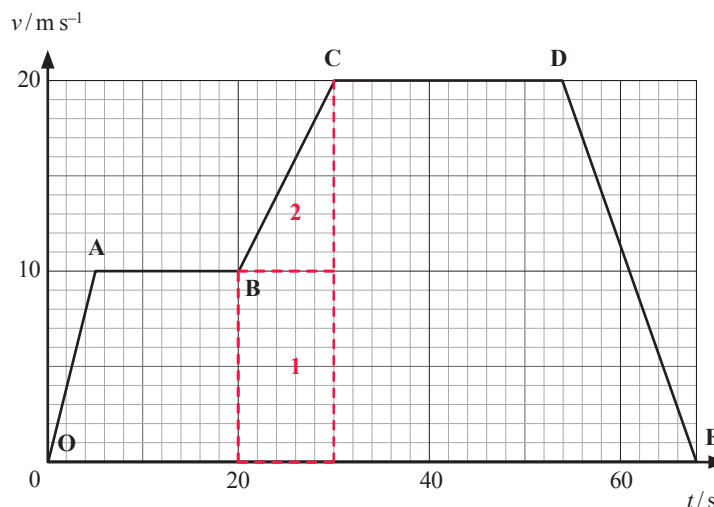


Fig. 1.2.3 v – t graph for an underground train

$$\text{The acceleration of the train in OA} = \frac{\Delta v}{\Delta t} = \frac{10 \text{ m s}^{-1}}{5.0 \text{ s}} = 2.0 \text{ m s}^{-2}.$$

$$\text{The acceleration between D and E} = \frac{\Delta v}{\Delta t} = \frac{(0 - 20) \text{ m s}^{-1}}{14 \text{ s}} = -1.43 \text{ m s}^{-2}.$$

The v – t graph allows us to calculate the displacement: it is the 'area' between the graph and the t -axis. For example (from Fig. 1.2.3)

$$\begin{aligned} \text{Displacement between B and C} &= \text{area between the graph and the } t\text{-axis} \\ &= \text{area 1} + \text{area 2 [in red on the graph]} \\ &= 10 \text{ s} \times 10 \text{ m s}^{-1} + \frac{1}{2} (10 \text{ s} \times 10 \text{ m s}^{-1}) \\ &= 150 \text{ m} \end{aligned}$$

The displacement for the whole journey is 945 m [see Self-test 1.2.10]. From this we can find the mean velocity for the trip:

$$\text{Mean velocity} = \frac{\Delta x}{\Delta t} = \frac{945 \text{ m}}{70 \text{ s}} = 13.5 \text{ m s}^{-1}.$$

Example

The v - t graph is of a bullet as it is fired into a tank of water.

Use the graph to calculate:

- (a) The deceleration at 2 ms,
(b) The displacement by the bullet in coming to rest.

Answer

- (a) Acceleration is the gradient of the tangent at 2 ms.

From the red triangle

$$a = \frac{\Delta v}{\Delta t} = \frac{(10 - 190) \text{ m s}^{-1}}{3.2 \text{ ms}}$$

$$= -56.2 \times 10^3 \text{ m s}^{-2}$$

$\therefore$ Deceleration is 56.2 km s^{-2} .

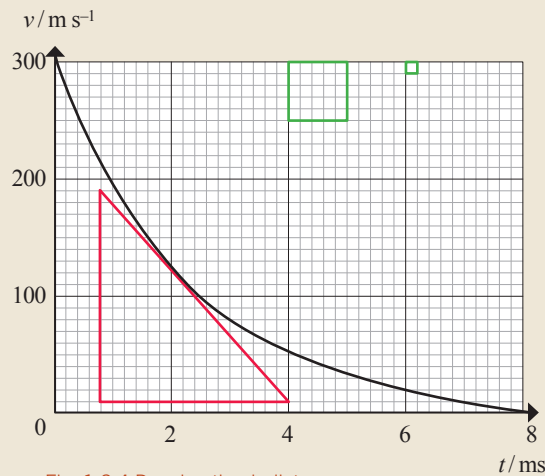


Fig. 1.2.4 Decelerating bullet

- (b) Displacement = area under graph. Using the trapezoidal rule (see Maths check), with $\Delta t = 1.0 \text{ ms}$:

$$\text{Area under graph} = (150 + 195 + 128 + 82 + 52 + 33 + 20 + 10 + 0) \text{ m s}^{-1} \times 0.001 \text{ s}$$

$$= 0.670 \text{ m}$$

$\therefore$ Displacement of bullet when coming to rest = 0.67 m (2 s.f.)

Self-test

1.2.10

By dividing the area under the graph in Fig. 1.2.3 suitably, show that the displacement for the journey is 945 m .

MATHS CHECK

See Chapter 4 for various techniques for estimating the area under a non-linear graph, including the trapezoidal rule.

Using the square-counting method, the author obtained 0.65 m and 0.69 m respectively when using the large and small green squares.

Self-test

1.2.11

In Fig. 1.2.5, assuming that the acceleration is always downwards and 1.5 m s^{-2} , calculate the velocity of the ball when it hits the ground 20 s after **C**.

[Hint: calculate Δv using $\Delta v = a \Delta t$, then draw a triangle of vectors.]

(b) Motion in two dimensions

We'll see in section 1.2.4 how to study horizontal and vertical components of motion separately. Here we'll have a brief look at how to calculate acceleration when the direction of the motion changes. Consider the motion of the cricket ball in the interplanetary test match on the lunar Tycho base. The Jovian opening bat despatched a full toss to the (rather distant) boundary.

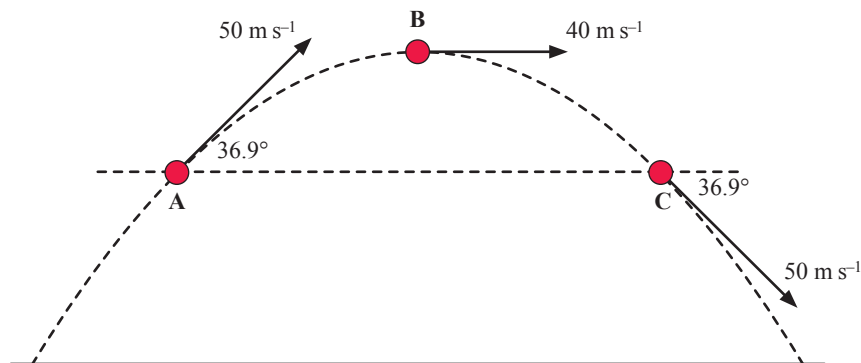


Fig. 1.2.5 Lunar cricket

The ball was at positions **A**, **B** and **C** at times of 20 s , 40 s and 60 s after it was struck. We'll determine the mean acceleration between positions **A** and **C**. Fig. 1.2.6 shows how to calculate Δv .

$$\Delta v = 2 \times 50 \sin 36.9^\circ = 60 \text{ m s}^{-1}$$

$\therefore$ Between **A** and **C**, $\langle a \rangle = \frac{\Delta v}{\Delta t} = \frac{60 \text{ m s}^{-1}}{40 \text{ s}} = 1.5 \text{ m s}^{-2}$ vertically downwards.

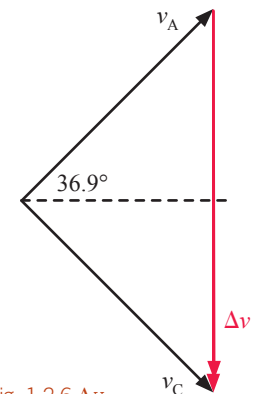


Fig. 1.2.6 Δv

Stretch & Challenge

A car takes a 90° bend at 15 m s^{-1} . It completes the bend in 7.0 s . See diagram. Calculate the mean acceleration.

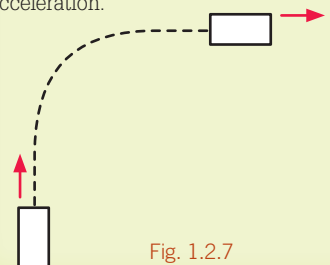
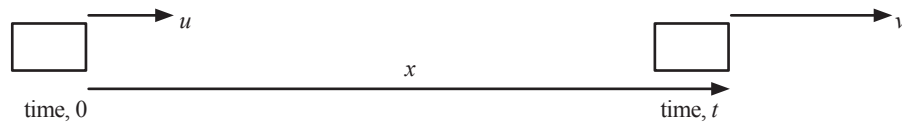


Fig. 1.2.7

1.2.3 Uniform acceleration equations

(a) Deriving the equations

Fig. 1.2.8 $xuvat$

In this section, we'll consider an object which is initially moving with a velocity, u , and accelerates with a constant acceleration, a , for a time, t . In this time, it attains a final velocity, v , and moves through a displacement, x . The motion is along a straight line. We'll derive some relationships between these quantities, x , u , v , a and t .

Terms & definitions

These equations are often referred to as $xuvat$.

- x = displacement
- u = initial velocity
- v = final velocity
- a = acceleration
- t = time

From the definition of acceleration, $a = \frac{v-u}{t}$, $\therefore$ (rearranging) $v = u + at$ [1]

The displacement, x , is the 'area under' the $v-t$ graph – see Fig. 1.2.9(a). The graph is a straight line because the acceleration is constant. For convenience we've assumed that $a > 0$ [so the gradient is positive] and $u > 0$. The equations we derive will still be valid for either a or u [or both] < 0 .

The displacement, x , is the area of the trapezium. From the formula for the area of a trapezium, $x = \frac{1}{2}(u+v)t$ [2]

If we break up the trapezium into a rectangle plus a triangle we obtain Fig. 1.2.9(c)

Adding the areas, $A_1 + A_2$, we get: $s = ut + \frac{1}{2}(v-u)t$

From equation [1] $v - u = at \therefore$ $x = ut + \frac{1}{2}at^2$ [3]

From equation [1], $t = \frac{v-u}{a}$. Substitute for t in [2] we get $x = \frac{1}{2}(u+v) \frac{(v-u)}{a}$,

which we can rearrange to give: $v^2 = u^2 + 2ax$ [4]

You should **learn equations [1] – [4]**. Make sure you can derive them.

A fifth equation to complete the set is: $x = vt - \frac{1}{2}at^2$ [5]

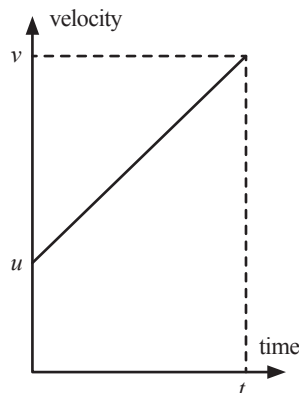
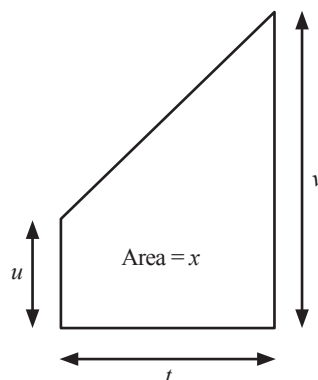
Fig. 1.2.9(a) $v-t$ graph

Fig. 1.2.9(b) Trapezium

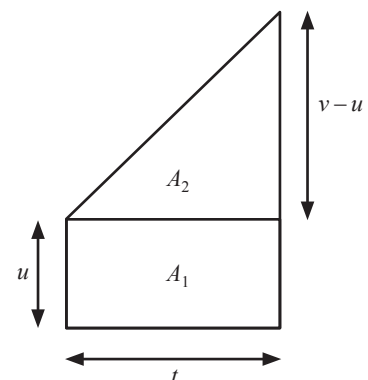


Fig. 1.2.9(c) rectangle plus triangle

(b) Applying the equations

It is important to be systematic when applying these equations.

- Identify and write down which of the quantities, x , a , etc., you already know and which you need to calculate.

Say you know the initial velocity (u), the acceleration (a) and the time (t) and are asked to calculate the displacement (x). The equation that contains these four quantities is $x = ut - \frac{1}{2}at^2$, so this is the equation to apply.

Example

A car, travelling at 26 m s^{-1} , decelerates at 1.2 m s^{-2} to a speed of 10 m s^{-1} . Calculate (a) the distance travelled and (b) the time taken in this process.

- (a) Writing down the quantities. $u = 26 \text{ m s}^{-1}$; $v = 10 \text{ m s}^{-1}$; $a = -1.2 \text{ m s}^{-2}$.

Unknown quantity = x . $\therefore$ Use the equation $v^2 = u^2 + 2ax$. From this we get $x = 240 \text{ m}$.

- (b) Now we know u , v , a and x and need to calculate t . So we can use any of the equations 1, 2 and 3. The easiest is $x = \frac{1}{2}(u + v)t$, which gives $t = 13.3 \text{ s}$.

Comments on the example:

1. It is possible to answer part (b) of the question before part (a).

With u , v and a known and t to be calculated, equation 1 is the obvious route.

2. If you use $x = ut - \frac{1}{2}at^2$ to calculate t , then you will usually obtain two possible solutions – see Study point. In this case, $t = 13.3 \text{ s}$ or 30.0 s . To see how this second solution arises, look at the sketch graph of x against t . A good reason for avoiding quadratics.

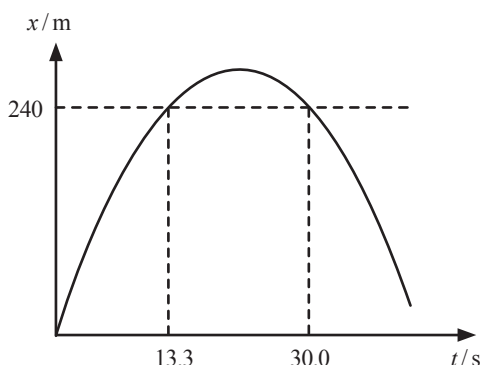


Fig. 1.2.10 x - t graph

(c) Vertical motion under gravity¹

The image in Fig. 1.2.11 is of a 'freely' falling golf ball (white) and table-tennis ball. They are lit by a strobe which is flashing at regular intervals. The distances between the positions of both spheres increases as they fall showing that they are accelerating. We can use the scale to investigate the acceleration.

The positions of the centre of the golf ball in the four images are approximately (in cm): 3.0, 10.5, 22.0 and 38.0. The distances fallen between the images are (approx.) 7.0, 11.5, 16.0 cm, showing a constant increase and hence a constant acceleration. We cannot measure the acceleration because we do not know the time interval between the flashes. Note that the golf ball is apparently overtaking the table-tennis ball, because the relative effect of air resistance is greater on the lower mass ball.

Around 1590, Galileo famously devised a thought experiment to justify by logic that all **freely** falling objects (i.e. in the absence of air resistance) accelerate to Earth with the same acceleration. We call this acceleration the *acceleration of free fall* or the *acceleration due to gravity*. The symbol for this acceleration is g . See Section 1.3.8 for some more discussion of this and also for the effect of air resistance on falling motion.

¹ see also Section 1.3.8

Self-test

1.2.12

- (a) Show that the answers given for the example are correct.
- (b) Use $v = u + at$ to solve part (b).

Stretch & Challenge

- (c) Use $x = ut + \frac{1}{2}at^2$ to solve part (b) in Self-test 1.2.12.

Exam tip

Although examiners apply ecf (*error carried forward*), it is a good idea not to rely on the answer to part (a) of a question when answering part (b).

Study point

If you have done Self-test 1.2.12(c) you will have obtained two solutions: $t = 13.3 \text{ s}$ and 30.0 s .

The appropriate answer is 13.3 s . The 30.0 s answer is commented on in the main text.

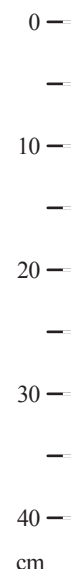


Fig. 1.2.11 Freely falling spheres

The acceleration due to gravity, g , close to the surface of the Earth is almost constant. The value of g at 39 km, the height from which Felix Baumgartner jumped in October 2012, is only 1.2% less than at ground level. Even at the altitude of the International Space Station (400 km) g is only 13% less than at ground level. Unless otherwise told, assume $g = 9.81 \text{ m s}^{-2}$. When making estimations, it is sensible to use the 10 m s^{-2} as the approximate value of g .

Example

A student drops a rock from a tall building. Estimate (a) its position and (b) its speed after 1 s, 2 s, 3 s and 4 s, assuming it hasn't hit the ground!

- (a) If the acceleration $\sim 10 \text{ m s}^{-2}$, this means that the speed increases by 10 m s^{-1} every second, so the speeds are $\sim 10, 20, 30$ and 40 m s^{-1} respectively.
- (b) Applying $x = ut - \frac{1}{2}at^2$ with $u = 0$ and $a = g \sim 10 \text{ m s}^{-2}$, we get $x = 5t^2$ leading to $x = 5 \text{ m}, 20 \text{ m}, 45 \text{ m}$ and 80 m .

Fig. 1.2.12 Galileo's thought experiment

1.2.13

Self-test

A passenger in a hot-air balloon at an altitude of 200 m throws a stone downwards at 10 m s^{-1} . Calculate the speed with which it hits the ground.

Study point

We could choose downwards as positive. If we do this, $a = g = +9.81 \text{ m s}^{-2}$.

In the example, u is -80 m s^{-1} and x is $+250 \text{ m}$ at the ground



Fig. 1.2.14

Study point

The equations in the example cannot distinguish between the given question and the situation in which an object is thrown upwards from the ground and achieves a velocity of 80 m s^{-1} at a height of 250 m at time 0. In this second situation, the time and velocity of projection would be -2.7 s and $+106 \text{ m s}^{-1}$ respectively.

Many problems involving motion under gravity involve upwards motion, which decreases and eventually becomes downwards motion. To solve such problems we have to decide which direction is positive.

If we choose *upwards as positive* then $a = -g = -9.81 \text{ m s}^{-2}$; a downward velocity will be negative and a position below the starting point will be negative. The following example illustrates these points.

Example

A toy rocket runs out of fuel at an altitude of 250 m and a vertical upwards speed of 80 m s^{-1} . Ignoring the effects of air resistance calculate the velocity with which it hits the ground.

Answer

$$u = 80 \text{ m s}^{-1}, a = -g = -9.81 \text{ m s}^{-2}.$$

The ground level is 250 m below the point at which our equations start to be applicable, i.e. $x = -250 \text{ m}$. We need to find v at this point.

$$\therefore v^2 = u^2 + 2ax. \therefore v^2 = 80^2 + 2(-9.81)(250) = 11305$$

$$\therefore v = \pm 106 \text{ m s}^{-1}$$

How do we distinguish between the two possible solutions? We could either say, 'Well, it's obvious that $+106 \text{ m s}^{-1}$ cannot be right, so the answer is -106 m s^{-1} , i.e. 106 m s^{-1} downwards.' This is certainly correct but we could look a little more deeply into it. See the Study point.

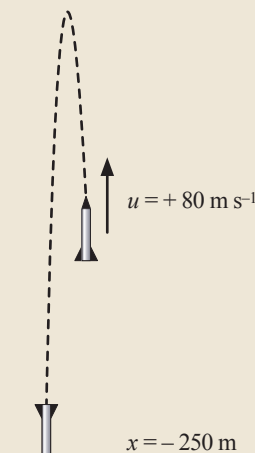


Fig. 1.2.13 Toy rocket

A difficult question

A more difficult question is to calculate the time until the rocket reaches the ground. Why is it more difficult? Because it involves the solution of a quadratic equation!

An easier solution

Some people find it easier to think of this problem in two parts:

1. Calculate the time and distance to the top of the flight (stage 1), and
2. Calculate the time to fall from the top to the ground (stage 2).

This involves the following sequence:

- For stage 1, $v = 0$. Use $v = u + at$ to calculate the time to the top.
- Use $x = \frac{1}{2}(u + v)t$ or $x = ut + \frac{1}{2}at^2$ to calculate the height gain in stage 1. Add the initial height (250 m) to give the total height.
- Use $x = ut + \frac{1}{2}at^2$ to calculate the time taken to fall 576 m from rest. This is a quadratic but the ut term is zero (because $u = 0$). [Hint: it is sensible to take *down* as positive for this part of the flight.]
- Add the two times together.

Now try Self-test 1.2.14.

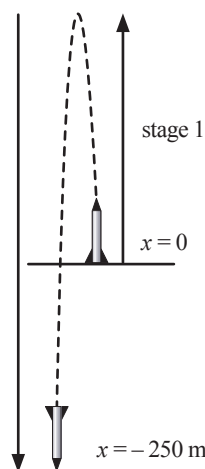


Fig. 1.2.15 Flight time

Self-test

1.2.14

Do the calculations for stages 1 and 2 and show that the total time is 19.0 s.

Why can't we find the total flight time of the rocket using the equations of motion 1–4?

Stretch & Challenge

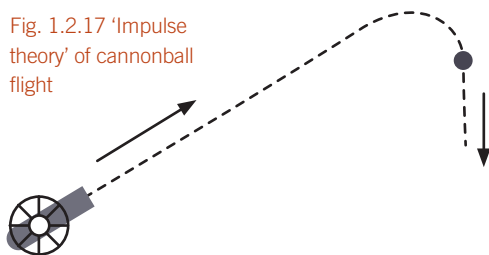
Use $x = ut + \frac{1}{2}at^2$ for the whole flight from 250 m, to find the time to impact with the ground.

1.2.4 Projectiles

A *projectile* is an object which is thrown/kicked/made to move obliquely upwards and carries on its path under the influence of gravity, e.g. a rugby ball during a conversion. The study of this kind of motion is called *ballistics* after the Roman siege weapon, the ballista.

Writings in military books in the middle ages suggested that the flight path of a cannon ball was as shown in Fig. 1.2.17.

Fig. 1.2.17 'Impulse theory' of cannonball flight



The idea was that the cannon ball had a certain 'impulse' which carried it along. When this ran out, gravity took over and caused it to plummet downwards. This is similar to the cartoon character's path when running off a cliff. The image of water jets shows the true parabolic path of projectiles Fig. 1.2.16.

Fig. 1.2.16 Parabolic jets of water

To see what is happening, we'll look at a strobe image of two spheres, one of which is moving vertically, the other starting off at the same moment horizontally Fig. 1.2.18.

The vertical white lines are equally spaced. From this image we see that:

1. The heights of the two spheres are the same at all instants, i.e. they are accelerating downwards at the same rate, which we know is g .
2. The white sphere moves horizontally at a constant velocity.

We therefore conclude that we can treat the horizontal and vertical motions of a projectile independently.

If we use x, y co-ordinates, with x being horizontal, y being vertically upwards and the projectile starting off from $(0,0)$ with an initial velocity u at θ to the horizontal, the equations of motion become:

Fig. 1.2.18 Independence of horizontal and vertical motion

1.2.15

Self-test

If $u = 30 \text{ m s}^{-1}$ and $\theta = 30^\circ$, calculate (a) the horizontal and vertical components of u , (b) the values of v_x and v_y after 5 seconds, (c) the velocity after 5 seconds and (d) the position after 5 seconds.

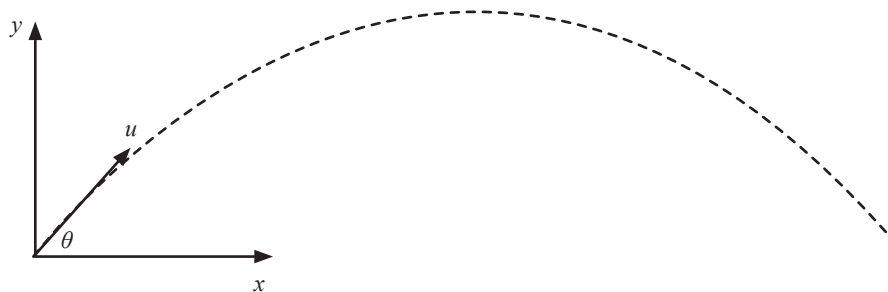


Fig. 1.2.19

Horizontal: $x = u_x t$ and $v_x = u_x$ [i.e. velocity is constant].

Vertical: $v_y = u_y - gt$; $y = \frac{1}{2}(u_y + v_y)t$; $y = u_y t - \frac{1}{2}gt^2$; $v_y^2 = u_y^2 - 2gy$

where $u_x = u \cos \theta$ and $u_y = u \sin \theta$ are the initial horizontal and vertical component of velocity and v_x and v_y the horizontal and vertical components of velocity at time, t .

Look at Self-test 1.2.15. This type of question can be tackled in a straightforward way by identifying the correct equation at each stage and substituting the data. It is strongly suggested that you do this question and check your answer before proceeding. Some problems, such as the example, involve a multi-stage approach. They often require a time to be calculated before the relevant distance.

Example

A trebuchet (a medieval siege engine; see Figure 1.2.20) hurls a rock at a speed of 40.0 m s^{-1} and an angle of 30° to the horizontal towards the vertical wall of a castle 100 m away. Calculate how high up the wall of the castle the rock will hit.

Plan Calculate the time it takes to reach the castle wall, using the horizontal motion and then calculate the height of the rock at this time using the vertical motion.

Fig. 1.2.20

1.2.16

Self-test

Use the **plan** in the trebuchet example to answer the question, then use the time to calculate the velocity with which the rock hits the castle wall.

1.2.5 Measurement of g by free fall

To measure the acceleration of free fall, all one needs to do is to measure the time, t , it takes an object to fall from a known height, h .

Then, using $x = ut + \frac{1}{2}at^2$, with $u = 0$, $x = h$ and $a = g$, the equation becomes:

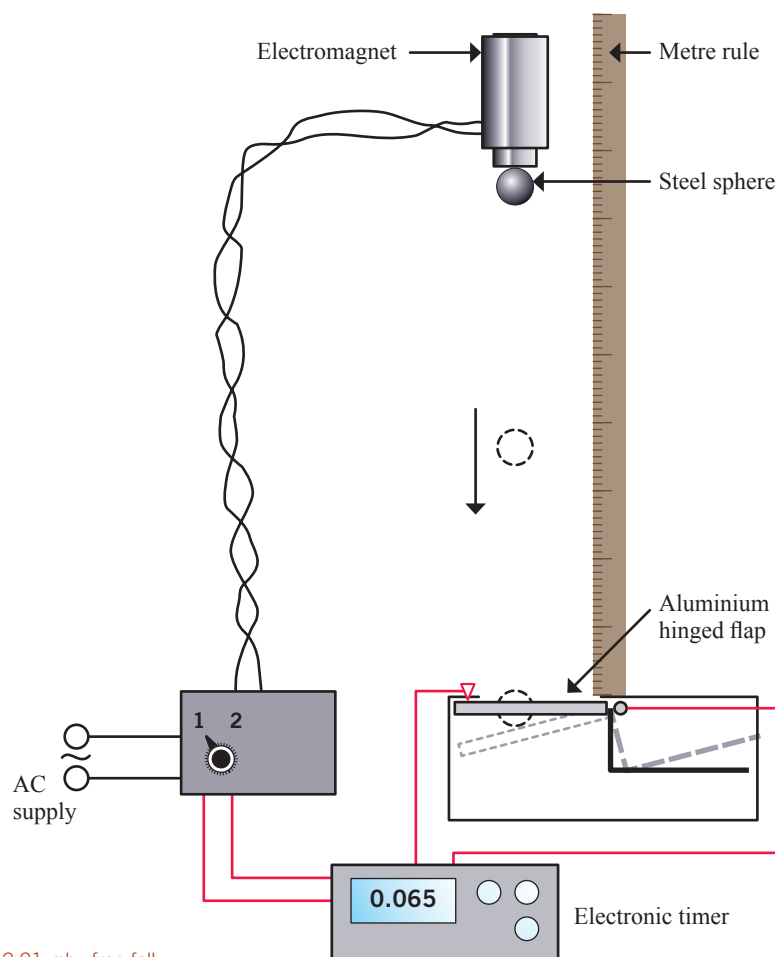
$$h = \frac{1}{2}gt^2 \quad [1]$$

$$\therefore g = \frac{2h}{t^2} \quad [2]$$

In principle, one could just drop an object from a high window and use a stopwatch to time its fall. The problem with this method is that the time taken to fall 20 m [quite a high window] is only $\sim 2 \text{ s}$, so the percentage uncertainty in t with a hand-operated stopwatch is quite high and then the equation for g involves t^2 which doubles the uncertainty; e.g. If you can measure time with an uncertainty of 0.1 s the percentage uncertainty, p , in 2 s is:

$$p = \frac{0.1}{2} \times 100 = 5\%$$

So the uncertainty in g will be 10% .

Fig. 1.2.21 g by free fall

In the apparatus in Fig. 1.2.21, the steel sphere is held in position by an electromagnet, the current for which is from the AC supply. Moving the switch from 1 to 2 simultaneously switches off the current to the electromagnet (releasing the sphere) and triggers the electronic timer to start. The sphere hits the aluminium flap, causing it to swing downwards breaking the red circuit, stopping the timer.

Measurements:

- Drop height, h , to $\pm 1 \text{ mm}$ using the metre rule. Typically h is up to 75 cm .
- Time, t , using the timer. The scale might be to 1 ms or 10 ms but the measurements of t typically vary with an uncertainty of 10 ms .

Analysis of results

Commonly t is measured for a range of values of h up to $\sim 75 \text{ cm}$, and a graph of h against t^2 plotted. From equation 1, the gradient is $\frac{1}{2}g$, so g is double the gradient.

Systematic error

A problem with this technique is that there is often a small delay before the sphere is released because it takes a short time for the magnetisation in the electromagnet and/or the steel sphere to decay. This adds an unknown time – so the true time for the fall is less by an unknown time, Δt . The effect of this Δt is to produce a curved graph for h against t^2 . See the margin for a technique for dealing with this.

Self-test

1.2.17

If $h \sim 50 \text{ cm}$,

- Estimate t using $g \sim 10 \text{ m s}^{-2}$.
- Use $\pm 1 \text{ mm}$ and $\pm 10 \text{ ms}$ to estimate the % uncertainty in g .

Systematic error

With the time delay of Δt the true relationship is:

$$h = \frac{1}{2}g(t - \Delta t)^2$$

Taking the square root gives:

$$\sqrt{h} = \sqrt{\frac{1}{2}gt} - \sqrt{\frac{1}{2}g\Delta t},$$

so if we plot a graph of

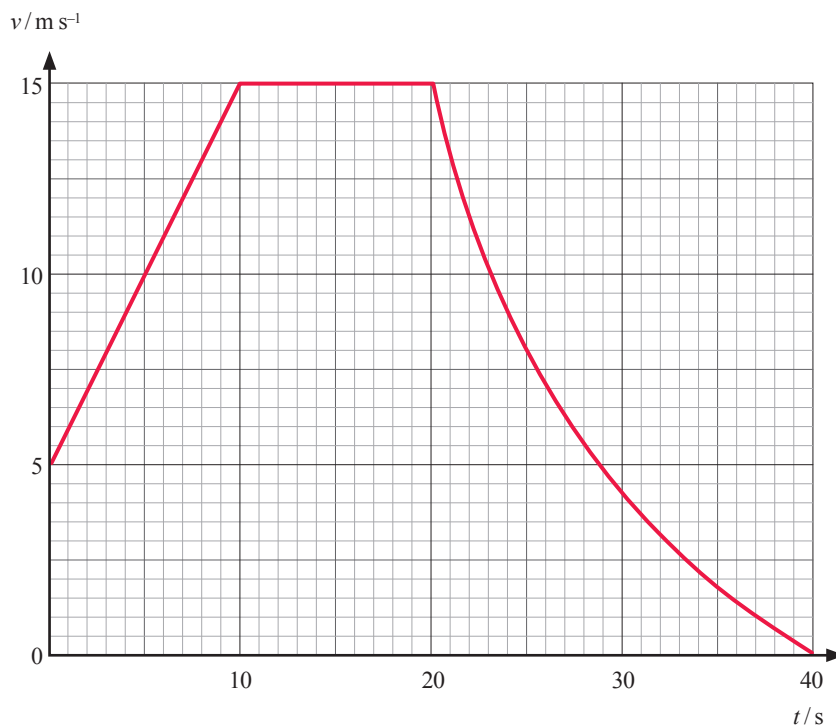
$\sqrt{h}$ against t we should get a straight line with gradient $\sqrt{\frac{1}{2}g}$.

Stretch & Challenge

- What is the intercept on the $\sqrt{h}$ axis?
- How can Δt be found more easily?

Exercise 1.2

1. A ball is dropped from a height of 5.0 m . It hits the ground at 10.0 m s^{-1} after 1 s . It rebounds at 8.9 m s^{-1} and reaches a maximum height of 4.0 m in a time of 0.90 s . Calculate:
 - (a) the mean speed in the descent,
 - (b) the mean speed in the ascent,
 - (c) the mean speed for the whole 1.9 s ,
 - (d) the mean velocity for the whole 1.9 s ,
 - (e) the change in velocity at the bounce.
2. Use relevant equations of motion to show that the acceleration in Q1 was assumed to be 10 m s^{-2} downwards.
3. A train accelerates from rest at 0.50 m s^{-2} for a period of 90 s . It travels at a steady velocity for 4.5 km before decelerating to rest in a further 1800 m . Calculate the mean velocity for the journey. [Hint: Calculate the total displacement and the total time taken.]
4. A car, travelling at a steady speed of 20 m s^{-1} , changes its direction from due North to due East in 5 seconds . Calculate the mean acceleration.



5. Use the $v-t$ graph to determine
 - (a) the acceleration between 0 and 10 seconds,
 - (b) the distance travelled in the first 20 s ,
 - (c) the acceleration at 30 s ,
 - (d) the mean velocity over the 40 s .
6. A stone is catapulted horizontally from the top of a cliff with a speed of 30 m s^{-1} . It hits the sea 5.0 s later. Calculate
 - (a) the height of the cliff,
 - (b) the vertical component of its velocity when it hits the water,
 - (c) the velocity with which it hits the water.

7. Taking the times between the positions of the spheres in Fig. 1.2.18 to be 0.050 s , estimate the time until the horizontal and vertical components of the velocity of the white sphere are equal and hence determine the distance between vertical white lines. For the purposes of this question you may take g to be 10 m s^{-2} .
8. An arrow is shot at 50 m s^{-1} at an angle of 30° to the horizontal. Ignoring air resistance and other aerodynamic effects, calculate
- the maximum height reached by the arrow,
 - the time to the maximum height,
 - the velocity at the maximum height and
 - the horizontal range of the arrow.
9. Car A starts from rest and accelerates at 1.5 m s^{-2} . Car B travels at a steady velocity of 30 m s^{-1} and passes car A as it starts off. How long does it take car A to overtake car B and how far have they both travelled by the time this happens?
10. An experiment is undertaken to determine a value for the acceleration due to gravity using the apparatus illustrated in Fig. 1.2.21. A centisecond timer was used. The measured times for the steel sphere to drop are given in the table:

Distance, x / cm	10.0	20.0	30.0	40.0	50.0	60.0	70.0
Time, t / ms	180	250	280	330	350	390	420

There is a small systematic error due to the magnet. Compare graphically the two methods of analysis given in the text and determine the delay time before the magnet drops the sphere.

Stretch & Challenge

The table gives the velocity of a bullet in a ballistics test from the instant it enters a tank of water.

Time / ms	0.0	0.125	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
Velocity / m s^{-1}	500	388	317	233	183	152	129	112	100	89	81	74

- Draw a $v-t$ graph for the motion of the bullet.
- By taking measurements from the $v-t$ graph, draw an $x-t$ graph.
- By taking measurements from the $v-t$ graph, draw an acceleration-time ($a-t$) graph.
- Use your graphs to investigate how the resistive force on the bullet depends upon its velocity through the water. Is this relationship likely to continue?

1.3 Dynamics

► Pointer

A car approaching a bend on an icy road could well fall victim to N1. Without the frictional grip of the tyres it will plough straight on!

Terms & definitions

An **isolated system** is one on which no external forces act and no particles enter or leave.

Terms & definitions

An alternative statement of Conservation of Momentum is: The vector sum of the momenta of the bodies in a system is constant provided there is no resultant force from outside the system.

UNIT

From the definition

$$[p] = [m][v].$$

So $[p] = \text{kg m s}^{-1}$. N s is also a correct unit. See Self-test 1.3.6

1.3.1

Self-test

Calculate p for:

- A 1000 kg car with a speed of 20 m s^{-1} ,
- The Earth in its orbit ($M_E = 6.0 \times 10^{24} \text{ kg}$, orbital speed = 30 km/s).

Section 1.2 was concerned with the mathematical language of motion – describing uniform and accelerated motion in terms of equations. This unit is concerned with the causes of motion and its changes. Before the 17th century, the natural state of motion of objects was considered to be at rest. Some agency was considered necessary for an object to be moving: it is difficult to force a log to move at all and the moment you stop dragging it, the log ceases motion; even a rolling ball stops in quite a short time; the Earth was ‘known’ to be at rest at the centre of the universe.

Different rules were held to apply to objects outside the Earth, i.e. the Moon and beyond. They were not made of normal stuff (earth, air, fire and water) but *quintessence*, a fifth substance whose natural state of motion was to circle the Earth. This unit presents the results of the revolution in the 16th and 17th centuries in which Isaac Newton built upon the work of such famous people as Copernicus, Kepler, Galileo and Descartes.¹

Newton’s three laws of motion may be stated as:

- A body’s velocity will be constant unless a force acts upon it.
- The rate of change of momentum of a body is directly proportional to the resultant force acting upon it.
- If a body **A** exerts a force on body **B**, then **B** exerts an equal and opposite force on **A**.

These laws are often referred to as N1, N2 and N3 for brevity.

The principle (law) of conservation of momentum states:

- The vector sum of the momenta of the bodies in an **isolated system** is constant.

In fact these four statements are not independent; the third law (N3) can be derived from N2 and conservation of momentum. Notice that these laws are framed in terms of velocity, momentum and force. We shall start with momentum.

1.3.1 Momentum

The momentum, p , of a body was defined by Newton by

$$p = mv,$$

where v is the velocity of the body and m is the mass. Mass is a scalar quantity which is a measure of the the body’s **inertia**. Inertial mass is assumed to be independent of velocity.

The principle of conservation of momentum deals with a system of particles which is free of outside influences, i.e. it does not gain or lose particles and is not subject to forces from the outside. We can also apply it in the case where there **are** external forces but they sum to zero, so there is no resultant force.

In the laboratory, we often investigate momentum changes using riders on air tracks. A rider sits on a cushion of air so that:

- There is zero resultant vertical force because the upward force on the rider due to air pressure is equal and opposite to the downward gravitational force and
- There is zero frictional force on the rider and the force due to air resistance is very small.

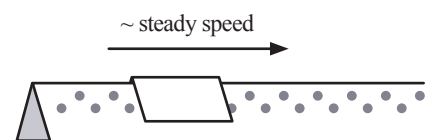


Fig. 1.3.1 Air track

¹ The effect of Newton on our understanding of the way that objects move cannot be overstated. In the words of Alexander Pope’s epitaph for him, ‘Nature and nature’s laws lay hid in night; God said “Let Newton be!” and all was light.’

Hence we can consider a collection of riders (usually two) on the air track to constitute an almost isolated system. A single rider moving at low speed is seen to travel at very close to constant velocity, in line with Newton's 1st law (N1).

a) When colliding objects stick together

Fig. 1.3.2 shows two collisions on the air track which are used by teachers in the classroom to illustrate the principle. These collisions, in which the objects combine on impact, are called **inelastic collisions**. For illustration purposes the velocities are deliberately idealised! The riders are identical, so their masses are the same – say 0.15 kg.

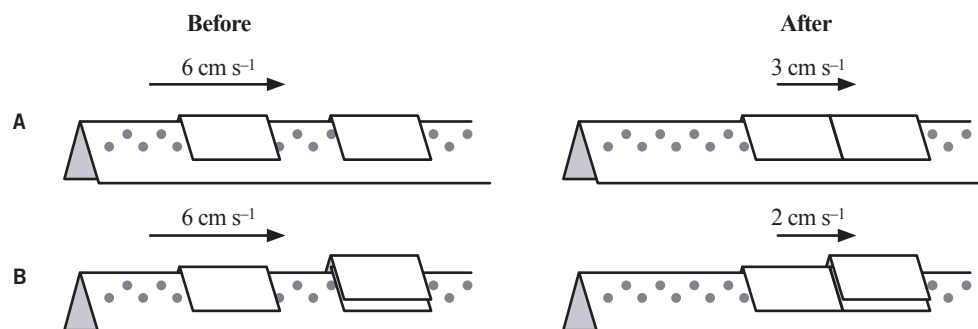


Fig. 1.3.2 Inelastic collisions

Assuming the riders each have a mass of 0.15 kg, the **momenta** before and after the collisions, p_1 and p_2 , are for collision **A**:

$$p_1 = 0.15 \text{ kg} \times 6 \text{ cm s}^{-1} + 0.15 \text{ kg} \times 0 = 0.90 \text{ kg cm s}^{-1}$$

$$p_2 = 0.30 \text{ kg} \times 3 \text{ cm s}^{-1} = 0.90 \text{ kg cm s}^{-1}.$$

The momenta before and after are the same. Of course, the actual mass of the riders is irrelevant as long as they are all the same.

b) When colliding objects bounce apart

If we mounted repelling magnets on the riders we might observe collision **C** in Fig. 1.3.3. There are two differences from **A** and **B**:

- There is movement in both directions.
- The riders move apart after the collision.

This could be performed, for example, by mounting repelling magnets on the riders so that no physical contact occurs between them.



Fig. 1.3.3 Elastic collision

Just as with motion under gravity (Section 1.2) we need to define a positive direction. Let's choose right as the positive direction.

$$\text{Then: } p_1 = 0.15 \text{ kg} \times 6 \text{ cm s}^{-1} + 0.15 \text{ kg} \times 0 = 0.90 \text{ kg cm s}^{-1}$$

$$p_2 = 0.15 \text{ kg} \times (-2) \text{ cm s}^{-1} + 0.30 \text{ kg} \times 4 \text{ cm s}^{-1} = 0.90 \text{ kg cm s}^{-1}.$$

The example shows how to apply the principle to calculate a final velocity.

Study point

It is important to realise that the Principle of Conservation of Momentum is an *experimental law*. In countless millions of interactions which physicists have investigated, it has never been known to be broken.

Self-test

1.3.2

Use the same method to show that momentum is conserved in collision **B**.

Terms & definitions

'**Momenta**' is the plural of momentum.

Terms & definitions

The **kinetic energy** of a body is its energy by virtue of its motion and is given by $\frac{1}{2}mv^2$.

Self-test

1.3.3

- Show that the 50% of the KE is lost in collision **A**.
- Calculate the percentage loss in KE in collision **B**.

Study point

Notice the minus sign (–) in the calculation of p_2 . The left-hand rider is moving to the left, so $v = -2 \text{ cm s}^{-1}$.

1.3.4

Self-test

Show the calculations for p_1 and p_2 in Fig. 1.3.4 with left as the positive direction.

Exam tip

In the example:

- (a) Remember that the momenta are vectors.
- (b) State the direction as well as the magnitude of the velocity.

Exam tip

Note that in calculating KE, only the magnitude of the velocity is important, not the direction. This is because $(-v)^2$ is the same as v^2 .

1.3.5

Self-test

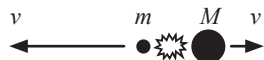
Show that the initial and final kinetic energies in collision **C** are both 2.7×10^{-4} J.

Terms & definitions

An **elastic** collision is one in which no kinetic energy is lost.

Study point

Conservation of momentum doesn't only apply to collisions. It also applies to situations where one object ejects another, such as a nucleus emitting an α particle or a gun firing a bullet. The momentum is zero before the event, so the total momentum is zero afterwards.



So: $mv = MV$. If we know the total energy, E then we can write $\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = E$ and solve the equations for v and V .

Example

An empty wagon of mass 1000 kg travelling at 6 m s^{-1} to the right collides with a full wagon of mass 4000 kg travelling at 2 m s^{-1} in the opposite direction. If they couple on impact, calculate their common velocity after the collision.

Step 1:

Diagram:



Fig. 1.3.4

Step 2: CoM equation: sum of initial momenta = sum of final momenta

Step 3: Taking right as positive: $1000 \times 6 + 4000 \times (-2) = 5000v$

Step 4: Solving this $\therefore v = -0.8 \text{ m s}^{-1}$

Answer

$\therefore$ The common velocity is 0.8 m s^{-1} to the left.

1.3.2 Elastic and inelastic collisions

Three of the collisions in Section 1.3.1 were **inelastic**: collision **A**, **B** and the example. These collisions result in a loss of *kinetic* energy. Note the word kinetic. Energy is always conserved but it can be transferred from one object to another or one form into another. You should be able to show that the collision in the example above results in a loss of 24.4 kJ of kinetic energy (from 26 kJ to 1.6 kJ).

If you've done Self-test 1.3.5 you'll have shown no kinetic energy is lost in collision **C** on page 37. This kind of collision is called an **elastic** (or sometimes a **perfectly elastic**) collision. Contact collisions between macroscopic objects, i.e. objects you can see, such as tennis balls and cars always involve some loss of kinetic energy – it is transferred to the vibrational energy of the molecules of the objects. Low energy collisions between subatomic particles or between molecules of monatomic gases at room temperature are usually elastic. Collisions between hard spheres such as snooker balls will typically preserve 90% of the kinetic energy.

Elastic collisions are more difficult to analyse than inelastic ones because there are two unknown velocities. We need to solve simultaneous equations.

Example

The collision in the diagram is elastic. Find v_1 and v_2 .

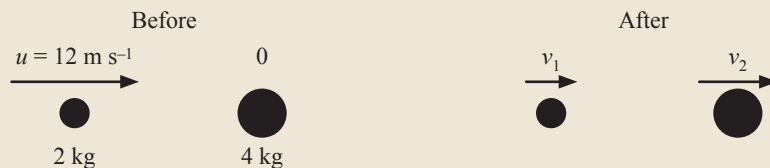


Fig. 1.3.5

Momentum before the collision = $2 \times 12 + 4 \times 0 = 24 \text{ N s}$

$\therefore$ Using the principle of conservation of momentum: $2v_1 + 4v_2 = 24$ [1]

Kinetic energy before the collision = $\frac{1}{2} \times 2 \times 12^2 + \frac{1}{2} \times 4 \times 0 = 144 \text{ J}$

$\therefore$ Using conservation of energy $\frac{1}{2} \times 2 \times v_1^2 + \frac{1}{2} \times 4 \times v_2^2 = 144$

$\therefore$ Simplifying $v_1^2 + 2v_2^2 = 144$ [2]

Solving equations 1 and 2 for v_2 , gives $v_2 = 0$ or 8 m s^{-1} . We can ignore the 0 because that clearly represents a 'no collision' [i.e. the 2 kg ball has missed!] so $v_2 = 8 \text{ m s}^{-1}$. Substituting in [1] gives $v_1 = -4 \text{ m s}^{-1}$, i.e. 4 m s^{-1} to the left: the lower mass ball has bounced back (as we might have expected).

Notice that in this collision the balls separate at the same rate (12 m s^{-1}) after the collision as they collided. This is typical of elastic collisions and we can use this fact to simplify the calculation of v_1 and v_2 . See **Stretch & challenge**.

Stretch & Challenge

If $v_2 - v_1 = 12 \text{ m s}^{-1}$ (see text underneath the example), we can use this as equation 2 instead. This makes the solution easier. Check this by solving:

$$2v_1 + 4v_2 = 24 \quad [1]$$

$$v_2 - v_1 = 12 \quad [2]$$

1.3.3 Force and momentum

The cricketer in Fig. 1.3.6 is playing a hook shot. The ball, which was originally moving rapidly towards him, is suddenly moving sideways even more rapidly. The change of momentum, Δp , is illustrated in Fig. 1.3.7.

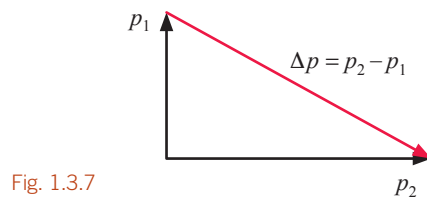


Fig. 1.3.7

We know (Newton's 1st law – N1) that an object's velocity and therefore its momentum stays constant in the absence of a force. So this change of momentum occurs because something, in this case the bat, applies a **force** to the ball.

Newton's 2nd law (N2), says that the resultant force applied is 'directly proportional to the rate of change of momentum'. In SI, we **define** the rate of change of momentum as equal to the force expressed in newtons, i.e.

$$F_{\text{res}} = \frac{\Delta p}{\Delta t} \text{ or } \Sigma F = \frac{\Delta p}{\Delta t}$$

where Δt is the duration of the force.

Example

A cricket ball has a mass of 0.16 kg . Its speed hitting the bat is 30 m s^{-1} , it turns through a right angle, leaves the bat at 40 m s^{-1} and the duration of the impact is 1.5 ms .

Calculate the magnitude of the mean force exerted by the bat on the ball.

Answer

Using Fig. 1.3.7 above: $p_1 = 0.16 \times 30 = 4.8 \text{ N s}$

$$p_2 = 0.16 \times 40 = 6.4 \text{ N s}$$

$$\therefore \Delta p = \sqrt{p_1^2 + p_2^2} = 8.0 \text{ N s}$$

$$\therefore \text{The mean force, } F = \frac{\Delta p}{\Delta t} = \frac{8.0}{1.5 \times 10^{-3}} = 5300 \text{ N (2 s.f.)}$$

Just as the gradient of a v - t graph is the acceleration of an object, the gradient of the momentum-time graph is the resultant force upon it.

This is because $F = \frac{\Delta p}{\Delta t}$ and $\frac{\Delta p}{\Delta t}$ is the gradient of the p - t graph.

Fig. 1.3.6

Study point

If several forces act on an object, it is the resultant force, $F_{\text{res}} = \Sigma F$ which is equal to $\frac{\Delta p}{\Delta t}$.

Study point

The first law is just a special case of the second law:

If $\frac{\Delta p}{\Delta t} = 0$ then $F_{\text{res}} = 0$,

so the momentum is constant.

Self-test

1.3.6

Use the equation $F = \frac{\Delta p}{\Delta t}$ to show that 'N s' is a unit of momentum.

1.3.7

Self-test

The sloping straight line in Fig. 1.3.8 is the tangent at 15 s. Use the graph to find:

- The (constant) resultant force on the car from 0 – 10 s.
- The resultant force at 15 s.
- The maximum braking force.

Study the p - t graph for a car moving east along a straight road and braking to rest (Fig. 1.3.8). Now answer Self-test 1.3.7.

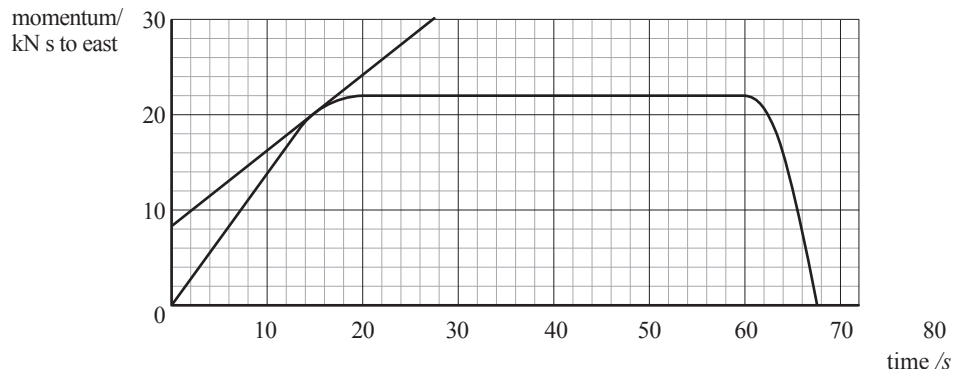


Fig. 1.3.8 Momentum-time graph for Self-test 1.3.7

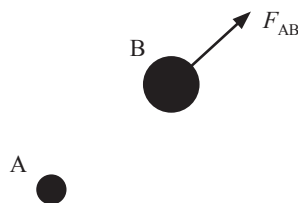


Fig. 1.3.9 Body A exerts a force on B

Study point

N3 is a bit tricky so it's important to understand what it is saying:

All forces arise from interactions between two bodies. In the interaction, both bodies experience a force. These two forces are equal and opposite.

1.3.4 Momentum and Newton's 3rd law of motion

As promised in the introduction we are going to show that N3 is a consequence of the principle of conservation of momentum and the definition of a force from N2. Fig. 1.3.9 shows two isolated bodies, **A** and **B**. **A** exerts a force on **B**, which we'll call F_{AB} . This is drawn as a repulsive force but it could act in any direction.

Consider a small time interval Δt . From N2, body **B** suffers a change in momentum, Δp_B given by:

$$\Delta p_B = F_{AB} \Delta t.$$

But, if the bodies are isolated, their total momentum, $p_A + p_B$ is constant so body **A** must suffer an equal and opposite momentum, Δp_A , i.e.

$$\Delta p_A = -F_{AB} \Delta t.$$

Because the momentum of **A** has changed it must have a force on it, which can only be exerted by **B** (because the bodies are isolated). Then F_{BA} , the force exerted by **B** on **A**, is given by:

$$F_{BA} = \frac{\Delta p_A}{\Delta t} = -F_{AB} \text{ (from above).}$$

In other words, the force exerted by body **B** upon body **A** is equal and oppositely directed to the force exerted by body **A** upon body **B**. Let's consider the common examiner's standby of a falling skydiver. The forces acting on the skydiver at a particular instant are shown in Fig. 1.3.10.

We analyse this in terms of N3 and look for the equal and opposite force to each force in the diagram.

- The **800 N weight**: This is the gravitational force that the Earth exerts on the skydiver. So the skydiver exerts an equal and opposite gravitational force on the Earth. The Earth is pulled (upwards) by an **800 N force**! [The mass of the Earth is 6×10^{24} kg so its acceleration is rather small!]
- The **600 N air resistance**: This is the drag of the air molecules on the clothing as the skydiver plummets down. So (the clothing of) the skydiver exerts an equal and opposite drag force (i.e. **600 N downwards**) on the molecules of air, which doubtless causes some air turbulence.



Fig. 1.3.10 The skydiver

The N3 trap: not all equal and opposite forces arising are N3 pairs!

Fig. 1.3.11 shows a sea lion holding up a ball. The ball is in equilibrium under the action of the two labelled equal and opposite forces.

Why aren't they an N3 pair? There are two reasons:

1. **They act on the same body** (the ball). In an N3 pair, there are two bodies: one force acts on one body; the other force acts on ... the other!
2. **The forces are not of the same kind.*** The force exerted by the Earth on the ball is gravitational, so the other member of its N3 partner must also be gravitational (it is, as with the skydiver, the gravitation force of the ball on the Earth).
3. [Additional reason] N3 is a universal law. There must always be an equal and opposite force. Imagine the sea lion quickly whipping its head out from under the ball: the downward force is still there but the upward force has disappeared – so they can't have been an N3 pair!

*Think about reason 2. What 'kinds' of forces are there? Physicists recognise four fundamental forces: the strong nuclear force; the weak force; the electromagnetic force and the gravitational force. The first two are only of significance for subatomic particle interactions so all other forces are gravity and electromagnetic. The forces between the molecules of the ball and sea lion nose are electromagnetic – they are caused by repulsion of the outer electrons in the molecules in the ball and nose which are in close proximity.

Don't worry about this last point – any legitimate way of describing a force must also apply to its N3 partner, e.g. the sea lion exerts an inter-atomic force on the ball, so the ball exerts an inter-atomic force on the sea lion.

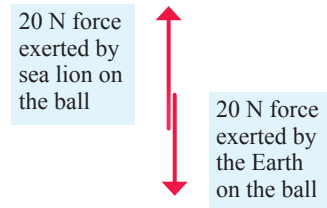


Fig. 1.3.11 Equal and opposite forces but not N3!

Study point

What does reason 2 mean? The upward force on the ball arises from the repulsion between the molecules of the ball and the sea lion's nose. It is definitely not gravitational; in fact it is electromagnetic – caused by the repulsion of the outer electrons in the molecules of the ball and nose which are in close proximity.

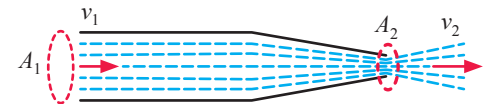


Fig. 1.3.12

Example – A difficult example of N2 and N3

Most exam questions are about forces on single objects. We can also apply the equation

$$F = \frac{\Delta p}{\Delta t} \text{ to streams of objects, e.g. water in a pipe.}$$

Water flows at a rate of V [in $\text{m}^3 \text{s}^{-1}$] through the nozzle shown in Fig. 1.3.12. Calculate the force that the water exerts on the nozzle.

Answer

The mass flow rate, $m = \rho V$, where ρ is the density of the water.

The initial velocity, $v_1 = \frac{V}{A_1}$; the final velocity, $v_2 = \frac{V}{A_2}$

$$\therefore \frac{\Delta p}{\Delta t} \text{ of water} = m(v_2 - v_1) = \rho V \left(\frac{V}{A_2} - \frac{V}{A_1} \right) = \rho V^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right).$$

As $A_2 < A_1$, $v_2 > v_1$ so this change of momentum is to the right.

So, by N2, the force exerted on the water by the nozzle is $\rho V^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$ to the right.

So, by N3, the force exerted on the nozzle by the water is $\rho V^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$ to the left.

1.3.8

Self-test

In the hose nozzle example, if the water flows at $20 \text{ dm}^3 \text{s}^{-1}$, $A_1 = 20 \text{ cm}^2$ and $A_2 = 10 \text{ cm}^2$, could you hold the nozzle?

[$1 \text{ dm}^3 = 10^{-3} \text{ m}^3$

$\rho = 1000 \text{ kg m}^{-3}$.]

1.3.9

Self-test

With the above data, what is the increase in the KE of the water per second? How is this supplied?

1.3.5 Forces between materials in contact

Study point

Avoid referring to the normal force as the normal reaction. This sounds too much like part of a poor statement of Newton's 3rd law!

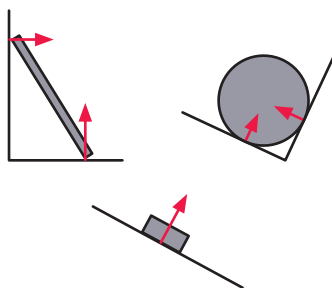


Fig. 1.3.13 Normal contact force examples

Study point

Static and dynamic friction oppose relative motion between surfaces. If a surface would slide to the right, the frictional force on it is to the left.

1.3.10

Self-test

Sand is fed vertically onto a horizontal conveyor belt which is moving towards the right. In which direction does the friction act on (a) the sand and (b) the belt? Explain your answers.

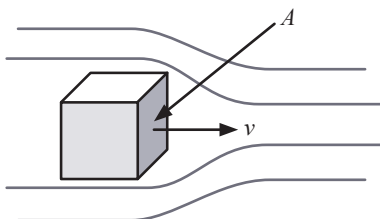


Fig. 1.3.15 Viscous drag

1.3.11

Self test

Show that c_d has no units (i.e. it is dimensionless).

Forces arise between objects due to molecular interactions. We shall consider some of these forces: the normal force, friction and air resistance. This section links to Section 1.4.5.

(a) The normal force, F_N

If an object rests against a surface, the surface exerts a force on the object. This force arises because the molecules in the two bodies are placed in close contact. If molecules are close together the electrons in the outer shells repel one another and so, in this case, the bodies experience a force at right angles to the surface. The word '*normal*' is used to mean 'at 90° '.

(b) Friction

The box on the slope in Fig. 1.3.13 will remain stationary on the slope as long as the gradient is not too great. This can only be because a force acts on the box up the slope which counteracts the component of the weight, $W \cos \theta$, down the slope, see Fig. 1.3.14.

This force is called *static friction*, F_R , or *grip*.

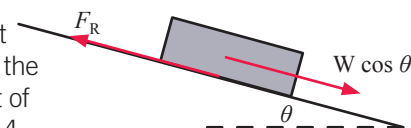


Fig. 1.3.14 Static friction

This force acts to stop the two surfaces sliding over each other, i.e. it opposes relative motion. For the stationary box, $F_R = W \cos \theta$, i.e. the friction is just big enough to stop motion. F_R has a maximum value, often called *limiting friction*. In the case of the box on the slope, if the slope is gradually made steeper, the value of F_R will increase up to this limiting value; at greater angles, $W \cos \theta > F_R$ and the box will start to accelerate down the slope. Grip arises from bonds between molecules of the two surfaces in contact.

The force which opposes the relative motion when one surface slides over another is also called friction – in this case *dynamic friction*. It arises from temporary bonds which form as molecules in the surfaces move past one another. When they stretch and break, the stored energy in the bonds is converted to vibrational energy of the molecules, i.e. the temperatures of the bodies increase. The value of dynamic friction is typically smaller than the limiting value of grip. This means that once an object starts to slide, it usually accelerates rather than just moving very slowly.

(c) Air resistance

Air resistance, is one example of *viscous drag*, or just *drag*. As with friction it opposes relative motion between the object and the fluid [= liquid or gas] through which the object is moving. It also exists when the fluid flows past a stationary object, e.g. the wind on a building. The mechanism of viscous drag is complicated but a simplification is that the molecules of a fluid bounce off a moving object slightly faster than they hit it: they gain velocity in the direction that the object is moving. So there is a momentum transfer to the fluid – and momentum transfer means a force on the fluid in the direction of the object's motion. By N3 the fluid exerts an equal and opposite force on the body.

The drag force, F_d , is given by the following equation:¹ $F_d = \frac{1}{2} \rho v^2 c_d A$, where ρ is the density of the fluid and c_d is a dimensionless quantity called the drag coefficient, which depends upon the shape of the object. You do not need to know this equation but you should know that F_d increases with the area of the object, the velocity and the density of the fluid.

¹ Stokes' formula on page 11 is applicable only at low speeds.

1.3.6 Free-body diagrams

Drawing a free-body diagram is useful in identifying the forces which are on interacting objects. Let's go back to the sea lion and her ball. See Fig. 1.3.16(a).

N3 pairs are drawn in matching colours but identifying them is still quite difficult, not least because lots of them overlap. To make the forces easier to identify, we take each object in the diagram and separate it from the others. We'll isolate the sea lion and the forces acting on it – see Fig. 1.3.16(b).

Now the forces are much easier to identify:

1. Contact force of ball on sea lion.
2. Gravitational force of Earth on sea lion.
3. Contact force of ground on back flipper of sea lion – at right angles to the slope.
4. Contact force of ground on front flipper of sea lion – at right angles to the slope.
5. Frictional force of ground on back flipper of sea lion – up the slope.
6. Frictional force of ground on front flipper of sea lion – up the slope.

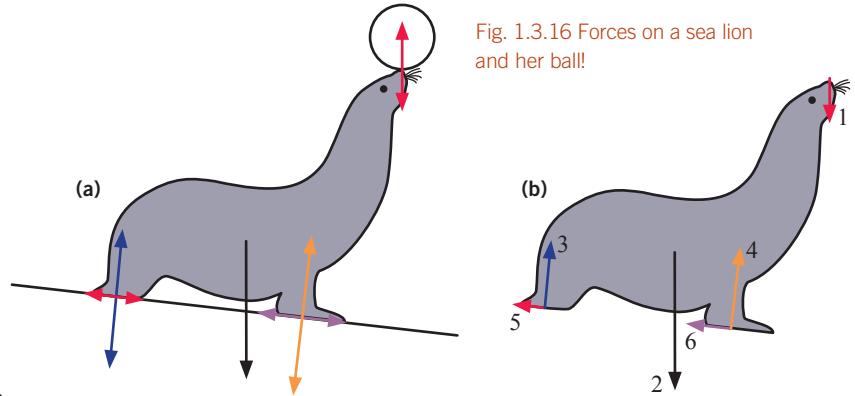


Fig. 1.3.16 Forces on a sea lion and her ball!

1.3.7 Force and acceleration

When Newton's second law of motion is applied to a single body with constant mass, acted on by a number of forces, the equation is often written in the form:

$$F = ma.$$

We can derive this from $F = \frac{\Delta p}{\Delta t}$ as follows:

By definition, $p = mv$, $\therefore \Delta p = \Delta(mv) = m\Delta v$ because m is constant.

$$\therefore F = \frac{\Delta p}{\Delta t} = m \frac{\Delta v}{\Delta t}. \text{ But } \frac{\Delta v}{\Delta t} = a, \therefore F = ma$$

We can re-arrange this as $a = \frac{\Sigma F}{m}$.

So, in words, the acceleration of an object is the resultant force acting on it divided by its inertial mass.

Example

Calculate the acceleration of the body in Fig. 1.3.17

Step 1: Calculate the resultant force:

$$\text{From the force diagram } F_{\text{res}} = \sqrt{8^2 + 12^2} = 14.4 \text{ N}$$

$$\text{and } \theta = \tan^{-1} \frac{8}{12} = 33.7^\circ$$

Step 2: Calculate the acceleration: $a = \frac{F_{\text{res}}}{m} = \frac{14.4}{2.5} = 5.76 \text{ m s}^{-2}$.

Answer: The acceleration is 5.76 m s^{-2} at 33.7° to the 12 N force (see Exam tip).

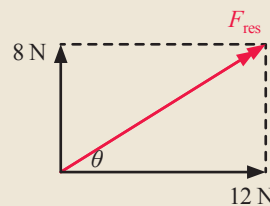


Fig. 1.3.18

Study point

An effect of air resistance is to transfer energy from the moving body to the molecules of air, as kinetic energy.

Self-test

1.3.12

Identify the N3 partner to each force, 1 – 6, on the sea lion in Fig. 1.3.16.

Self-test

1.3.13

Draw a free-body diagram for the ball in Fig. 1.3.16(a) and identify the forces.

Study point

Note that a and ΣF are vectors and m is a scalar (> 0) so the direction of the acceleration is the same as that of the resultant force.

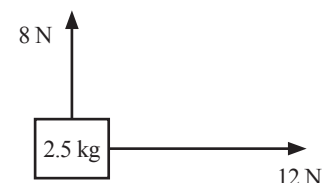


Fig. 1.3.17 $\Sigma F = ma$

Exam tip

Acceleration is a vector so, if asked for an acceleration, you should normally give a direction as well as a magnitude.

1.3.8 Gravitational force¹

It is an experimental fact that if an object falls freely, i.e. in the absence of air resistance, its acceleration is independent of its mass, density or shape. The classic ‘guinea and feather’ illustration of this is shown in Fig. 1.13.19. This is a modern version of Galileo’s famous thought experiment, in which he imagined dropping two cannon balls of different masses from the top of the Leaning Tower of Pisa. US Apollo 15 astronaut, David Scott, repeated this experiment live on the Moon in 1971 using a hammer and a feather; more recently Brian Cox used a NASA facility to show the same experiment – you can find videos of these using a search engine.

The acceleration of free fall is given the symbol g for (gravity) and is also known as the ‘acceleration due to gravity’. Its value on the surface of the Earth is approximately 9.81 m s^{-2} , though this depends on the location because of the shape of the Earth and height of the terrain.

Consider a falling object of mass m . Its acceleration is g , so N2 tells us that the gravitational force on it, which we call its *weight*, W , is given by

$$W = mg.$$

The following example illustrates the use of $W = mg$ in conjunction with other concepts, which we have already met.

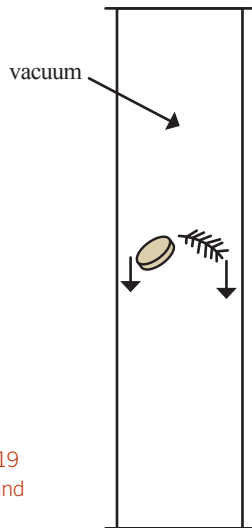


Fig. 1.3.19
Guinea and
feather

Study point

We can rearrange the equation

$$W = mg, \text{ as } g = \frac{W}{m}, \text{ so } \text{N kg}^{-1}$$

can be used as the unit of g . If N kg^{-1} is used, g is referred to as the *gravitational field strength* – it gives the force per unit mass on a body placed in the gravitational field.

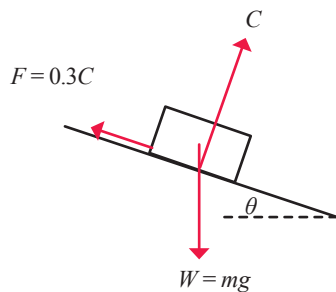


Fig. 1.3.20 Accelerating box on a slope

Example

Fig. 1.3.20 shows a box accelerating down a slope. F , the frictional force has a value which is $0.3C$, where C is the normal contact force. Calculate:

- the minimum value of θ for the box to accelerate down the slope,
- the acceleration of the box if $\theta = 20^\circ$. [Take $g = 9.81 \text{ m s}^{-2}$]

- There is no motion perpendicular to the slope so the resultant force in this direction is 0.

$$\therefore \text{(Resolving perpendicular to the slope): } C = mg \cos \theta$$

$$\text{But } F = 0.3C, \therefore$$

$$F = 0.3 mg \cos \theta$$

$$\text{The component of } W \text{ down the slope} = W \sin \theta = mg \sin \theta$$

$$\therefore \text{The resultant force, } \Sigma F, \text{ down the slope} = mg \sin \theta - 0.3 mg \cos \theta \\ = mg (\sin \theta - 0.3 \cos \theta)$$

$$\text{The box can only accelerate if } \Sigma F > 0. \therefore \sin \theta > 0.3 \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta \therefore \tan \theta > 0.3 \therefore \theta \text{ must be at least } 16.7^\circ \text{ (0.29 rad)}$$

- At 20° , ΣF down the slope $= mg (\sin 20^\circ - 0.3 \cos 20^\circ) = 0.590m$.

$$\therefore \text{Using } \frac{\Sigma F}{m}, \text{ the acceleration is } 0.590 \text{ m s}^{-2}.$$

1.3.14

Self-test

Calculate the acceleration of a cyclist who is freewheeling down a 10° slope. The mass of the cyclist and bike is 75 kg ; the resistance force is 50 N .

¹ This section links to Section 1.2.3(c)

Your *sensation* of weight is not the same as the force of gravity on you. It arises from the compression of your body between the force of gravity, which is distributed through your body, and the upward contact force on you from the ground. Hence an astronaut in the *International Space Station* has no sensation of weight. When you are in a lift, your apparent weight depends upon the motion of the lift. Our final example illustrates this effect.

Example

A man of mass 85 kg is standing on a bathroom scales whilst in a lift, which is accelerating upwards at 1.5 m s^{-2} , as shown in Fig. 1.3.21.

- Calculate the upward force, U , exerted by the scales on the man.
- The scales measure this upward force but their reading, R , is in kg , which relates to U by:

$$U = Rg.$$

What is the reading on the scales?

Answer

$$(a) \quad W = mg = 85 \times 9.81 = 833.9 \text{ N}.$$

$$\text{From N2: } \Sigma F = ma. \therefore U - 833.9 = 85 \times 1.5 = 127.5 \text{ N}$$

$$\therefore U = 127.5 + 833.9 = 961.4 \text{ N [= 961 N to 3 s.f.]}$$

$$(b) \quad R = \frac{U}{g} = \frac{961.4}{9.81} = 98.0 \text{ kg}$$

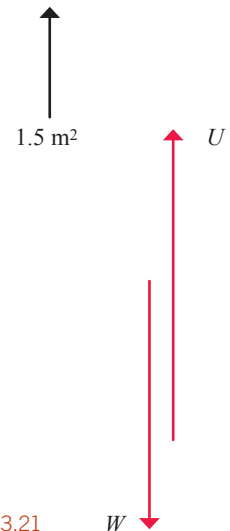


Fig. 1.3.21

Self-test

1.3.15

What would be the reading on the scale if the lift were accelerating downwards at 1.5 m s^{-2} ?

What would be the result on the Moon ($g = 1.5 \text{ m s}^{-2}$)?

Motion under gravity and air resistance

If the 'guinea and feather' in Fig. 1.3.19 are dropped in air, the former reaches the ground first. Whereas the feather appears to drift downwards at a constant speed, the coin accelerates all the way down.

The acceleration of falling objects, such as the guinea, the feather or a skydiver is determined by the resultant force on them. The two significant forces are:

- The weight, W , which is a constant for objects close to the Earth.
- Air resistance, F_d , which varies according to $F_d = \frac{1}{2}\rho v^2 c_d A$ – see Section 1.3.5(c).

Fig. 1.3.23 shows the effect of this combination of forces for a skydiver.

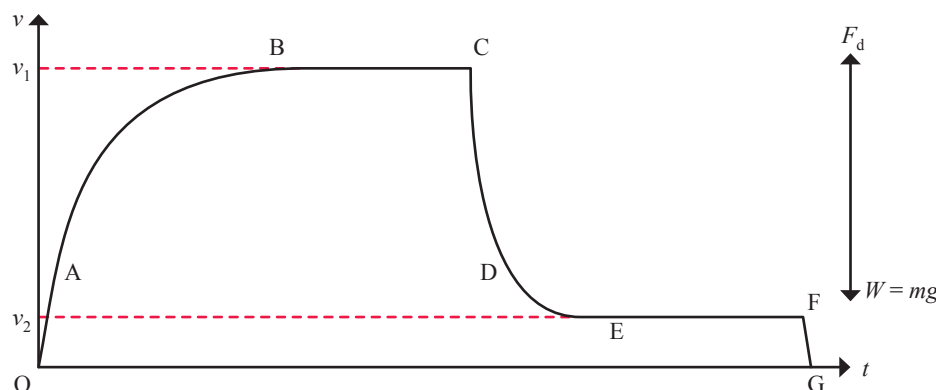


Fig. 1.3.23 Air resistance on a skydiver

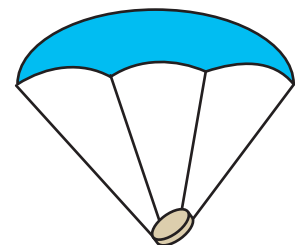


Fig. 1.3.22 Guinea and parachute

Study point

A guinea tied to a toy parachute (Fig. 1.3.22) would drift downwards after an initial acceleration.

1.3.16

Self-test

Sketch a v - t graph for the skydiver on the assumption that the air density increased as the skydiver descended. Justify your answer.

Hint: Concentrate on section BC.

1.3.17

Self-test

Assuming that $c_d = 1$, estimate the terminal velocity, v_1 , of a skydiver. Take $\rho_{\text{air}} = 1 \text{ kg m}^{-3}$. State your assumptions for other quantities.

If we assume, to a first approximation, that ρ , the air density, is constant, the only significant variables which affect F_d are v and A , so the resultant force (F_{res} or ΣF) is given by:

$$F_{\text{res}} = mg - \frac{1}{2}\rho v^2 c_d A$$

Acceleration, $a = \frac{F_{\text{res}}}{m} \therefore a = g - kAv^2$ (where $k = \frac{c_d}{2m}$, a constant)

So, to take the descent in stages:

- OB** At **O**, $v = 0$ so the acceleration, $a = g$. As the skydiver speeds up, F_d increases so F_{res} and a decrease. At **B**, $F_d = mg$, the two forces balance, so $F_{\text{res}} = 0$ and $a = 0$. The velocity is constant $= v_1$ in Fig. 1.3.23. This is called a *terminal velocity*.
- BC** Assuming the density of the air stays constant, F_d is constant so the skydiver continues to fall at v_1 .
- C** The parachute opens: A increases (massively); F_d increases so $F_{\text{res}} \ll 0$ and so $a \ll 0$, i.e. the skydiver very rapidly decelerates.
- CD** As the speed decreases, F_d decreases ($\propto v^2$) until once again $F_{\text{res}} = 0$ and so $a = 0$. This is the second terminal velocity, v_2 .
- DE** As BC (at velocity v_2 this time).
- EF** Contact with ground. The ground exerts a large upward force on the skydiver. F_{res} is now large and upwards. It quickly reduces the speed to 0.

Detail

The rider on an air track consists of two flanges at 90° to each other on either side of the track. A blower pumps air into the track. This escapes through holes and the rider sits on this cushion of air.

50 g masses can be added to spigots on each flange.

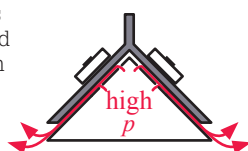


Fig. 1.3.24

Technique 1

– increasing the accuracy

If a broader light shield is used, the time for the shield to pass through the light gate is increased. The timer is usually a millisecond timer. The greater the time, t , the lower the uncertainty in v .

1.3.9 Investigating Newton's 2nd law of motion

In A Level Physics, this normally involves demonstrating that experimental results are consistent with $F = ma$, which means that

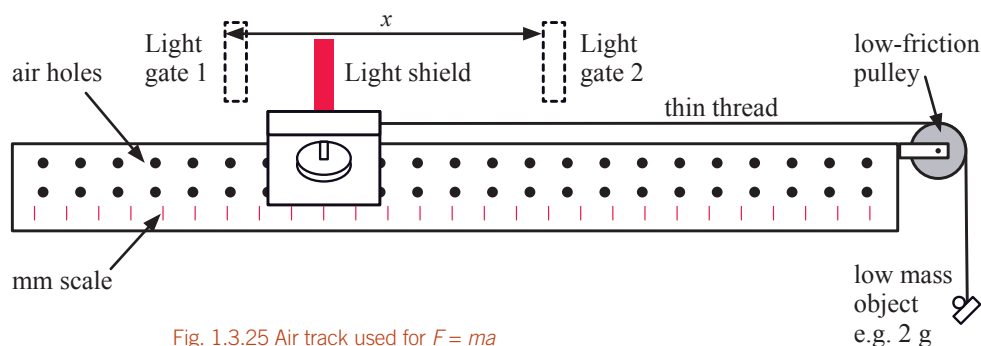
- For constant mass: $a \propto F$
- For constant force: $a \propto \frac{1}{m}$

In order to investigate these relationships, a preliminary experiment is useful, to show that a constant force on an object produces a constant acceleration. This lets us subsequently measure accelerations with just three variable measurements, with the help of the equations of motion:

$$x = ut + \frac{1}{2}at^2 \text{ and } v^2 = u^2 + 2ax$$

It also establishes the general technique.

The apparatus normally used in school is the air track, a hollow, triangular-section tube with air holes. Air is pumped in and metal riders are held above the track by a cushion of air which escapes from the air holes (see Detail). Fig. 1.3.25 shows the setup.


 Fig. 1.3.25 Air track used for $F = ma$

The accelerating force is provided by the gravitational force on the 'low mass object' and transmitted along the thin thread to the rider.

Experiment 1: showing that a constant force gives a constant acceleration

Technique 1: Using only one light gate, light gate 2 (LG2)

- Release the rider from rest and use the light gate (LG2) and electronic timer to measure the time, Δt , taken for the light shield of width Δx , to cut off the light.
- Use the equation $v = \frac{\Delta x}{\Delta t}$ to calculate the velocity, v , acquired by the rider after travelling a distance, x , from rest.
- Repeat for a series of distances.
- Draw a graph of v^2 against x . If the acceleration is constant, v and x are related by $v^2 = u^2 + 2ax$, so the graph should be a straight line through the origin ($u = 0$). The gradient is $2a$, allowing a to be calculated.

Measurements: x , the distance between the position of the midpoint of the light shield at the start and at the light gate; Δt ; Δx .

Technique 2: Using two light gates: Release the rider to the left of LG1; set the timer to record the time, t , to travel the distance, x , between LG1 and LG2. Vary x and repeat.

$x = ut + \frac{1}{2}at^2$, $\therefore \frac{x}{t} = u + \frac{1}{2}at$. A graph of x against t should be a straight line of gradient $\frac{1}{2}a$ if a is constant.

Experiment 2: Showing that acceleration is proportional to force

We vary the accelerating force by suspending a series of low mass objects from the end of the thread, e.g. 2 g, 5 g, 7 g, 10 g, 12 g, 15 g. Alternatively, we can just use numbers of paper clips or other objects of uniform mass. We measure the acceleration for each of these accelerating masses using the method of technique 1 above with one light gate (LG2) and a suitable value of x (the larger the better).

Problem: How do we keep the mass of the system constant? This is not quite as easy as it sounds because the small mass on the end of the thread forms part of the mass which is accelerating.

Solution: We take all the masses we intend to use as the weight on the thread, and load them onto the rider. If we put any of these, e.g. a 5 g mass, onto the end of the thread, the total mass stays constant but the accelerating force will be determined by mg hanging on the thread.

Experiment 3: Showing that acceleration is inversely proportional to mass

We vary the mass, M , of the system by adding or subtracting pairs of 50 g masses to the rider. The rider itself typically has a mass of ~ 150 g and will take up to an additional 300 g.

The force is kept constant by keeping the accelerating weight on the thread the same. We must remember to include its mass in the total mass of the system.

Again we use technique 1 to measure the acceleration over a suitably large x .

Data analysis

Results: $x = 2.5$ cm.

x / cm	Δt / ms
20	91
40	62
60	51
80	45
100	40

Show that the acceleration is constant and find its value.

Data analysis: technique 2

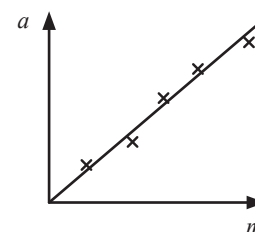
Results:

x / cm	Δt / ms
10	148
25	268
40	341
70	475
100	575
140	684

Show that the acceleration is constant and find its value.

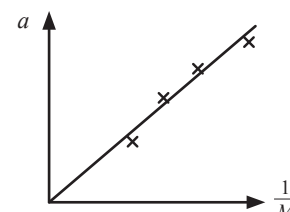
Experiment 2 analysis

Calculate the acceleration for each of the masses, m , on the thread and plot a graph of a against m . It should be as follows:



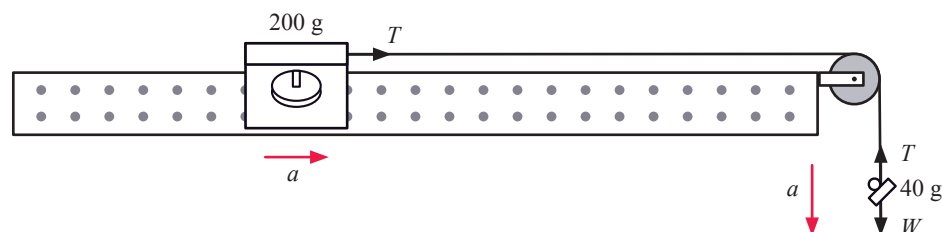
Experiment 3 analysis

Calculate the acceleration for each value of the total mass, M , remembering to include the mass on the thread on the thread. Plot a graph of a against $\frac{1}{M}$. It should be as follows:



Exercise 1.3

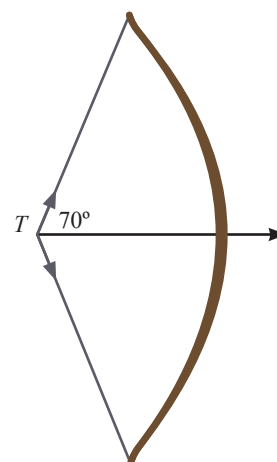
1. Two objects, A and B, have masses, 20 kg and 30 kg, and speeds, 25 m s⁻¹ and 10 m s⁻¹ respectively. Calculate the total momentum of the objects if:
 - (a) A and B are both travelling to the right,
 - (b) A is travelling to the right and B is travelling to the left,
 - (c) A is travelling due North and B is travelling due East.
2. Calculate the total kinetic energy for the bodies in question 1.
3. On an air track, two riders together travelling at 6 cm s⁻¹ collide inelastically with a third identical rider at rest. The common velocity after the collision is 4 cm s⁻¹. Show that this is as predicted by the principle of conservation of momentum.
4. Calculate the fraction of the initial kinetic energy which is lost in the collision in question 3.
5. A car of mass 1300 kg travelling at 40 m s⁻¹ collides head on with a second car of mass 1200 kg travelling at 20 m s⁻¹ and they stick together. Calculate
 - (a) the total momentum before the collision and
 - (b) their velocity immediately after the collision. What have you assumed?
6. A neutron travelling at 1200 m s⁻¹ collides with a stationary U-238 nucleus and is absorbed. Estimate the speed of the resulting U-239 nucleus. You may assume that protons and neutrons have the same mass.
7. Samarium-147 undergoes α decay. The α particles of mass 6.68×10^{-27} kg are ejected at 1.00×10^7 m s⁻¹, leaving $^{143}_{60}\text{Nd}$ nuclei of mass 2.39×10^{-25} kg. Calculate
 - (a) the recoil velocity of the nuclei and
 - (b) the kinetic energy released in the decay.
8. Find the resulting velocities if the objects in question 1 collide elastically head on.
9. A football of mass 450 g and is kicked at 30 m s⁻¹ against a wall at right angles. It rebounds at 25 m s⁻¹. If the duration of the collision is 0.04 s calculate the mean force which the ball exerts on the wall. Explain your answer clearly in terms of N2 and N3.
10. Two forces of magnitude 5 N and 12 N are simultaneously applied to a body of mass 1.55 kg. The directions at which the forces act can be varied.
 - (a) Calculate the magnitudes of the maximum and minimum accelerations.
 - (b) Calculate the acceleration of the object if the two forces act at right angles to each other.
11. The diagram shows an air track set up. The chord joining the 40 g mass and the rider has a tension, T .



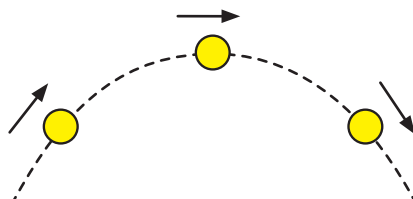
- (a) Calculate the weight, W , of the small mass.
- (b) Write equations in terms of T for the acceleration of
 - (i) the 40 g mass and
 - (ii) the 200 g rider.
- (c) Solve the two equations and hence determine the acceleration, a .

12. The tension, T , in the longbow string is 4700 N. The mass of the arrow is 0.065 kg.

- (a) Calculate the acceleration of the arrow.
 (b) The arrow leaves contact with the string after 70 cm. Estimate the speed with which it leaves the bow. State your assumption.



13. The diagram shows the flight of a thrown ball.



Draw free body diagrams for the ball in each of the 3 positions

- (a) without air resistance,
 (b) with air resistance.

14. A student obtained a set of results to demonstrate that a constant force produces a constant acceleration, using technique 2 (Section 1.3.9).

x/cm	10.0	20.0	30.0	40.0	50.0	60.0
t/s	0.45	0.74	1.00	1.25	1.38	1.55

The suspended mass, m , was 5.0 g. By drawing a suitable graph, use the results

- (a) to show that the acceleration is constant,
 (b) to calculate the acceleration, a , and
 (c) to determine the mass, M , of the rider.

15. The uncertainty in each of the times in question 14 is estimated to be ± 0.02 s. The uncertainties in x and m are negligible. Add error bars to your graph and estimate the uncertainties in a and M .

Stretch & Challenge

This question involves only physics concepts from the AS physics specification but uses techniques from A level mathematics. It is suitable for students of maths and physics towards the end of their A level courses.

The resistive force on the bullet in the Stretch & Challenge question on page 35 is assumed to be directly proportional to the square of the velocity, i.e. $ma = -kv^2$, where m is the mass of the bullet and k is a constant.

- (a) Suggest a unit for k .

- (b) The acceleration, a , can be written in calculus terms as $a = \frac{dv}{dt}$ or $a = v \frac{dv}{dx}$.

- (i) Using the $a = v \frac{dv}{dx}$, we can write: $mv \frac{dv}{dx} = kv^2$. Solve this equation to find v as a function of x , with v_0 as the value of v when $x = 0$. Then, writing v as $\frac{dx}{dt}$, integrate to find $x(t)$.
 (ii) Find $v(t)$ and $x(t)$ from $m \frac{dv}{dt} = -kv^2$ and confirm that the solutions are the same as in (i).
 (iii) Using the values $v_0 = 500 \text{ m s}^{-1}$, $m = 0.050 \text{ kg}$ and $k = 0.23$ units, compare the results with the data given in the table on page 35.

1.4 Energy concepts

Terms & definitions

Work is done when a force moves its point of application.

Terms & definitions

Energy is the ability to do work.

The unit of work is the **joule** (J).

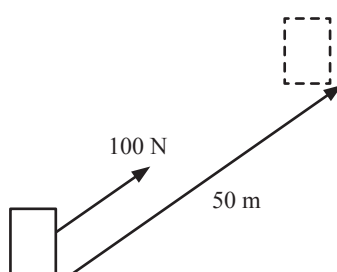


Fig. 1.4.1 Work done

Terms & definitions

The **internal energy** of an object is total energy (KE + PE) possessed by its particles. This should not be called heat.

Study point

Work done = Energy transfer

So, if a force does 5 kJ of work, 5 kJ of energy is transferred.

The concept of energy is one which physicists developed over a long period, between the late 17th and the early 20th centuries. Unlike momentum, which is a vector quantity and only conserved if there are no external forces, energy is a scalar quantity. The first type of energy to be identified was *kinetic energy* – the energy due to motion. An object that possesses energy can cause events to happen, which means that it can make things move. For example, an asteroid hitting the Earth can cause a large crater to be formed – moving huge quantities of material (and finishing off the dinosaurs in the process). We shall refine these ideas in this section of the book and start by considering the work done by a force.

1.4.1 Work and energy

If a force moves something, we say that the force *does work*. For example, the following forces are doing work:

- The force exerted by an electric winch which pulls a car out of a ditch.
- The force exerted by the wind which turns a turbine connected to a dynamo.
- The force exerted by an archer's muscles when he draws a bow.
- The force exerted by the archer's bow when it straightens up and shoots the arrow.

You'll recognise that all these examples need something to drive them, unlike, for example, the force exerted by a table in holding up a book or the force exerted by a nail which holds up a shelf. In these last two examples there is no motion, so no work is done – and there is no need of any 'input' to keep the objects in place. We use the concept of work to define energy.

The work done by a force is defined as follows:

$$\begin{array}{ccccc} \text{Work done} & = & \text{Force} & \times & \text{Distance moved in the} \\ \text{(J)} & & \text{(N)} & & \text{direction of the force} \\ & & & & \text{(m)} \end{array}$$

or, in symbols: $W = Fx$.

Applying this to Fig. 1.4.1:

The work done by the 100 N force, $W = 100 \text{ N} \times 50 \text{ m} = 5\,000 \text{ J}$ (or 5 kJ)

We define **energy** in such a way that the quantity of energy transferred is equal to the work done by the force. How is this energy transferred? This depends upon the details. Various possibilities are shown in Figs 1.4.2 to 1.4.4.

Fig. 1.4.2 Kinetic energy

Fig. 1.4.3 Internal energy

Fig. 1.4.4 Gravitational potential energy

The labels in Figs 1.4.2 to 1.4.4 give the form the energy takes following the transfer. The crossbow stored energy as elastic potential energy before it is transferred to the quarrel; in the other two diagrams the energy was previously stored in the muscles by the compound ATP.

Example

A 20 kg load is raised by 5.0 m using a winch as shown in Fig. 1.4.5. The handle of the winch is 0.60 m long and the radius of the drum on which the rope is wound is 0.15 m. A force of 50 N is required to turn the handle. Calculate:

- (a) the work done on the 20 kg block and
- (b) the work done by the 50 N force

Answer

- (a) The force needed to raise the load is $mg = 20 \times 9.81 = 196.2 \text{ N}$

The work done, W , on the 20 kg block = $Fx \cos \theta$

$$\therefore W = 196.2 \times 5 \cos 0^\circ = 980 \text{ J (2. s.f.)}$$

- (b) We need to calculate the distance moved by the 55 N force. The length of the handle is 4x the radius of the drum, so the distance moved is $4 \times 5 \text{ m} = 20 \text{ m}$.

$$\therefore \text{The work done by the 55 N force} = 55 \times 20 = 1100 \text{ J}$$

Note that the work done by the person winding the crank is more than the work done raising the load. We'll return to this in Section 1.4.5.

1.4.2 Directions of the force and the displacement

In the examples in the last section the force and displacement were in the same direction. What if this is not the case?

The tug in Fig. 1.4.6 is pulling at an angle θ to the direction of motion of the ship.

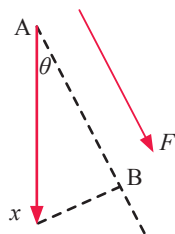


Fig. 1.4.7

The vector diagram shows this more clearly. The distance moved in the direction of the force is $AB = Fx \cos \theta$.

$$\therefore W = Fx \cos \theta.$$

The expression $x \cos \theta$ is the component of x in the direction of F . Also $x \cos \theta$ is the component of the force in the direction of the displacement.

So the definition of work done can be written:

Either: Work done = Force $\times$ Component of displacement in the direction of the force

or: Work done = Component of force in the direction of displacement $\times$ Displacement

We can see that these two ways of writing the work equation give the same answer if we consider the cyclist on a slope in Fig. 1.4.8.

Study point

Work is not the only way of transferring energy. Internal energy can be transferred by heat, i.e. conduction, convection or radiation.

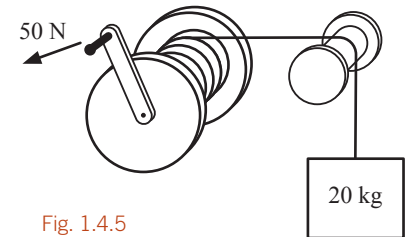


Fig. 1.4.5
Work done using a winch

Self-test

1.4.1

What is the excess energy in the example transferred to? [Do not say *heat* or *sound*!]

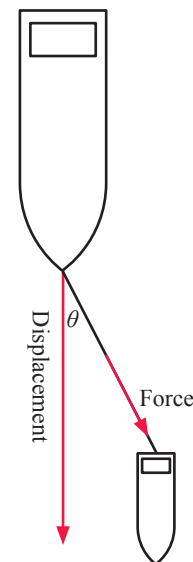


Fig. 1.4.6
Motion at an angle to the force

Self-test

1.4.2

Calculate W given the following data:

$$F = 500 \text{ kN:}$$

$$x = 1.5 \text{ mm: } \theta = 35^\circ$$

Study point

Notice that, in the case of the cyclist on the slope, the cyclist loses gravitational potential energy and either picks up a lot of speed (i.e. gains kinetic energy) or the brakes get very hot – or both!

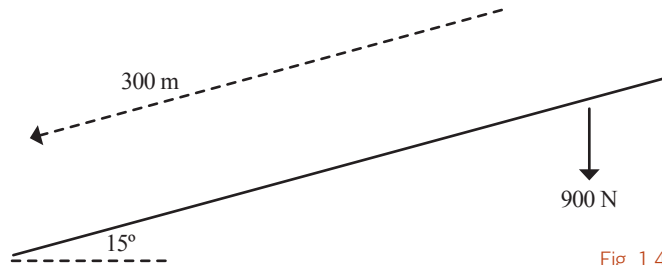


Fig. 1.4.8 Cyclist freewheeling downhill

Using the data in the diagram:

The vertical component of the displacement is $300 \sin 15^\circ$; the component of the weight down the slope is $900 \sin 15^\circ$. So applying the either/or from above:

Either: $W = (900 \text{ N}) \times (300 \sin 15^\circ \text{ m}) = 270\,000 \sin 15^\circ \text{ J} = 70 \text{ kJ}$ (2 s.f.)

Or: $W = (900 \sin 15^\circ \text{ N}) \times (300 \text{ m}) = 270\,000 \sin 15^\circ \text{ J} = 70 \text{ kJ}$

The angle between the force and the displacement in Fig. 1.4.8 was 75° . Here are some examples where the angle is 90° or more:

1. A satellite in a circular orbit. The gravitational force is at right angles to the movement if the satellite is in its orbit. Hence, $\theta = 90^\circ$, so $W = 0$. The gravitational force does no work! This fits in with the fact that the energy of the satellite is constant – there is no change in either the potential or kinetic energies. **Conclusion:** If $\theta = 90^\circ$, no work is done; no energy is transferred.
2. A bullet fired into a tree. The diagram shows the frictional force exerted on the bullet by the tree. The angle between the motion and the force is 180° . But $\cos 180^\circ = -1$, so the work done by the force is negative. This negative work means that a negative energy is transferred to the bullet – hence its kinetic energy decreases, i.e. it slows down. **Conclusion:** If $\theta > 90^\circ$, the work is negative and energy is transferred **from** the object.

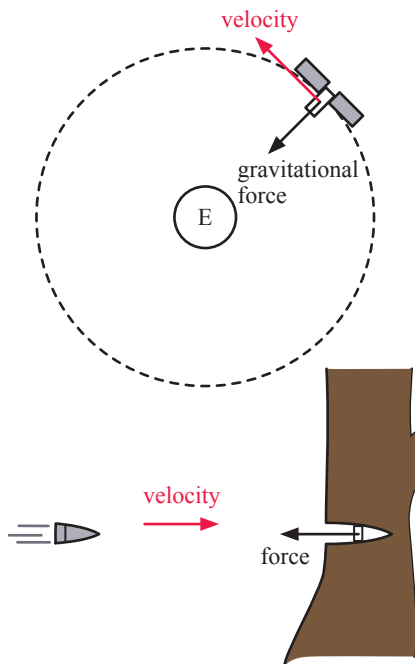


Fig. 1.4.9 & 1.4.10 Large values of θ

1.4.3 Energy conservation: kinetic and potential energy

(a) The principle of conservation of energy

Consider the box sliding to a halt on a surface in Fig. 1.4.11. Free body diagrams for these two objects are shown in Fig. 1.4.12:

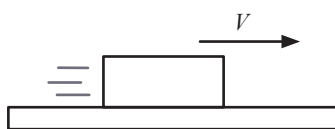
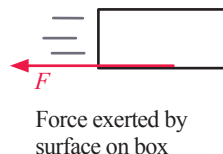
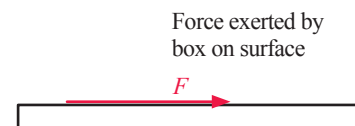


Fig. 1.4.11 Interacting objects



Force exerted by
surface on box



Force exerted by
box on surface

Fig. 1.4.12 Free body diagram

Terms & definitions

The principle of conservation of energy: the total energy of an isolated system is constant though it can be transferred within the system.

From N3, we know that the two objects exert equal and opposite forces, F , on each other. In a short time Δt , the box slides $v\Delta t$ to the right. In this time, the surface does an amount of work $-Fv\Delta t$ on the box, reducing its kinetic energy by $Fv\Delta t$. At the same time the box does an amount of work $+Fv\Delta t$ transferring this quantity of energy (as internal energy, shared between the surface and the box).

Hence the total quantity of energy in the system is constant; i.e.

$$\Delta E = Fv\Delta t - Fv\Delta t = 0$$

This illustrates the **principle of conservation of energy**.

We shall now use the principle that:

$$\text{Work done} = \text{energy transfer}$$

to derive expressions for kinetic and potential energy.

(b) Kinetic energy

A moving object can do work on other objects in coming to rest. Thus it possesses energy because of its motion. We call this **kinetic energy**. Its symbol is E_k , but it is often just abbreviated to KE.

Consider an object of mass m moving with velocity u subject to a force F over a displacement x as shown in Fig. 1.4.13:

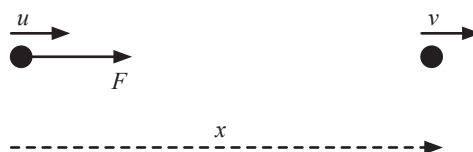


Fig. 1.4.13 Deriving the formula for KE

F and x are in the same direction, $\therefore W = Fx$.

But $F = ma$ $\therefore W = max$

The 4th kinematic equation for constant acceleration is $v^2 = u^2 + 2ax$.

$\therefore$ Rearranging, $ax = \frac{1}{2}v^2 - \frac{1}{2}u^2$

So substituting for ax above $\therefore W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

From this equation we see that the work done is the change in the value of the quantity $\frac{1}{2} \text{ mass} \times \text{velocity}^2$. So, because work done is the same thing as energy transfer, we conclude that $\frac{1}{2}mv^2$ is the kinetic energy of a body of mass m travelling with velocity v .

Example

A car of mass 800 kg, travelling at 15 m s⁻¹ is accelerated by a force of 1200 N over a distance of 250 m. Calculate the final velocity.

Answer

Work done = change in kinetic energy

$$\therefore 1200 \times 250 = \frac{1}{2} \times 800(v^2 - 15^2) \therefore 300\,000 = 400(v^2 - 225)$$

$$\therefore \text{Dividing by 400 and rearranging: } v^2 = 750 + 225 = 975 \rightarrow v = 31.2 \text{ m s}^{-1}.$$

Terms & definitions

Kinetic energy is the energy possessed by a body by virtue of its speed.

Terms & definitions

Gravitational potential energy is the energy possessed by a system of gravitating bodies by virtue of their separation.

Terms & definitions

Kinetic and potential energies are often grouped together as **mechanical energy**.

Exam tip

The change in kinetic energy is

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2,$$

which you can write as $\frac{1}{2}m(v^2 - u^2)$

This is **not** the same as

$$\frac{1}{2}m(v - u)^2!$$

Self-test

1.4.3

The example can be solved by using $F = ma$ to find a and then using $v^2 = u^2 + 2ax$ to find v .

Do this and compare the methods.

(c) Gravitational potential energy

An object in a raised position can do work on other objects as it descends. Thus it possesses energy because of its height. We call this **gravitational potential energy**. The symbol is E_p or U , but it is often just abbreviated to GPE or PE. Strictly speaking, the energy is possessed not by the body on its own but by the Earth-body system: the GPE depends upon their separation.

As with the kinetic energy, we'll imagine doing work on an object of mass m in such a way as to increase its GPE by raising it a distance Δh .

This is shown in Fig. 1.4.14. This will be carried out at constant speed, so that there is no change in kinetic energy.

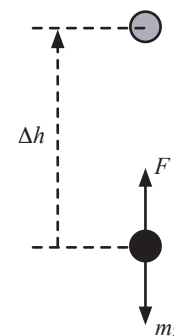


Fig. 1.4.14 Deriving the formula for GPE

Study point

Unlike kinetic energy, gravitational potential energy has no obvious place where $E_p = 0$.

Stretch & Challenge

For larger changes in height we use the formula

$E_p = -\frac{GM_E m}{r}$, which you will not need in AS! Look up the symbols in this formula and their units.

Terms & definitions

Elastic potential energy is the energy stored in a body by virtue of its deformation.

UNIT

Particle physicists express energy in *electron volts* (eV). 1 eV is the energy transfer when an electron moves through a potential difference of 1 V:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Study point

A stationary object can only be stretched or squashed if **two** forces are applied in opposite directions.

Stretch & Challenge

The work done, W , in stretching the spring is given by:

$W = \int F(x) dx$. Given that

$F = kx$, show that $W = \frac{1}{2} kx^2$.

NB Don't just ignore the constant of integration!

If the body is not accelerating, $\Sigma F = 0$ [Newton's 1st law]

$$\therefore F = mg$$

Because only potential energy is changing, no other kinds of energy are involved.

$\therefore$ By definition, $\Delta E_p = \text{work done by } F = F\Delta h$

$$\therefore \Delta E_p = mg\Delta h$$

Warning. This equation for ΔE_p is only valid for small changes in height. Small? Small compared with the distance from the centre of the Earth. Because the radius of the Earth is approximately **6400 km** this restriction is not a problem for changes in height within the atmosphere, even up to Felix Baumgartner's free fall jump from **39 km**!

(d) Elastic potential energy

Elastic objects (e.g. rubber bands, springs, rulers) which are deformed (stretched, compressed or bent) are able to do work on other objects when they return to their normal shape. Thus they possess energy because of their deformation. We call this **elastic potential energy**. The symbols used are the same as for gravitational potential energy.

When an object is stretched, squashed or bent, the extent of the deformation depends upon the applied force. Many objects obey Hooke's law, at least for small deformations. (See Section 1.5.1.)

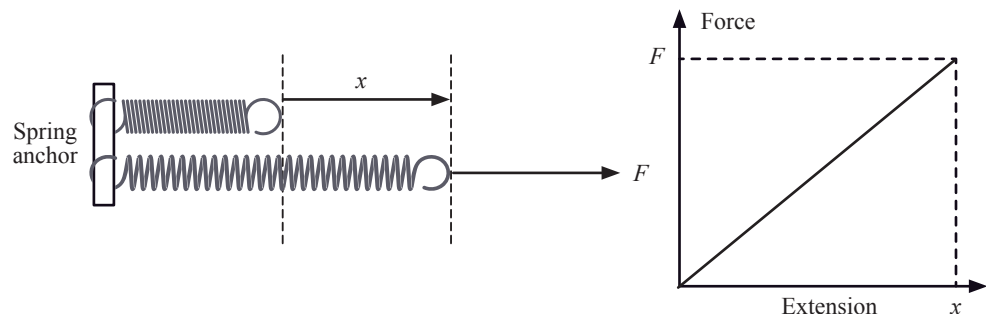


Fig. 1.4.15 Elastic potential energy

The spring constant, k , is defined by $F = kx$, where F is applied force and x the deformation (e.g. stretch) as shown in Fig. 1.4.15. Calculating the work done by the varying force is analogous to calculating the displacement from a velocity–time graph: the work done is the area under the graph.

$\therefore$ The work done in stretching the spring, $W = \frac{1}{2}Fx$.

$\therefore$ Substituting for F from $F = kx$, $W = \frac{1}{2}kx^2$,

If the spring is allowed to relax, it can do work as it does so equal to the work done in stretching it, so the elastic potential energy is given by $E_p = \frac{1}{2}kx^2$.

Example

A 200 g mass is hung on a spring of constant 15 N m^{-1} , as in Fig. 1.4.16, and dropped. Calculate:

- The distance the mass falls before it comes to rest.
- The speed of the mass when it has fallen half this distance.

Answer

- Using conservation of energy: Loss in GPE = gain in elastic PE.

Let x be the distance fallen

$$\text{Then: } mgx = \frac{1}{2}kx^2, \therefore x = \frac{2mg}{k} = \frac{2 \times 0.2 \times 9.81}{15} = 0.262 \text{ m}$$

- When $x = 0.131 \text{ m}$, Kinetic energy = Loss in gravitational PE – gain in elastic PE

$$\therefore \frac{1}{2}mv^2 = mgx - \frac{1}{2}kx^2$$

Putting in the values and rearranging leads to $v^2 = 1.28$, $\therefore v = 1.13 \text{ m s}^{-1}$



Fig. 1.4.16 Mass on spring

Terms & definitions

Power is the rate of transfer of energy. Its SI unit is the watt (W) which is equivalent to J s^{-1} .

1.4.4 Energy and power

Power, P , is calculated using $\text{Power} = \frac{\text{energy transfer}}{\text{time}}$ or, in symbols, $P = \frac{E}{t}$.

It is not restricted to energy transfer in the context of mechanical work. For example, a 15 W light bulb transfers 15 J into electromagnetic radiation and internal energy every second. For many purposes, the watt is a rather small unit, especially in the context of heating. A domestic electric kettle typically has a power of 2–3 kW, a small windfarm might have an installed power of ~10 MW and a typical thermal power station has a power of 1–2 GW. A similar approach could be taken with energy (using kJ, MJ, GJ, etc.) but often a different approach is taken:

Rearranging $P = \frac{E}{t}$, Energy transfer = Power $\times$ time

If we express power in kW and the time in hours, the unit of energy transfer is the kilowatt-hour (kW h).

Example

A nuclear power reactor has an electrical power output of 1.2 GW. Estimate electrical energy output in 1 year.

No. of hours in 1 year = 365.25 days $\times$ 24 hours / day = 8766.

$\therefore$ Electrical energy output = 1.2 GW $\times$ 8766 h = 11 000 GW h = 11 TW h (2 s.f.)

If the energy transfer arises from mechanical work, then we can re-write the power equation as:

$$\text{Power} = \frac{\text{Work done}}{\text{time}}, \text{ i.e. } P = \frac{W}{t}$$

Consider a force F applied to an object moving at velocity v at an angle θ to F (see Figure 1.4.17). In time Δt , the work done is given by: $W = Fv\Delta t \cos \theta$

$\therefore$ Dividing by Δt $P = \frac{W}{\Delta t} = Fv \cos \theta$

If F and v are in the same direction, i.e. $\theta = 0$, then $P = Fv$

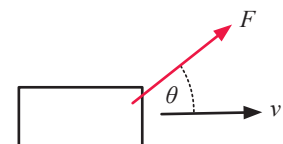


Fig. 1.4.17 Power and velocity

Self-test

1.4.4

By considering a power of 1 kW and a time of 1 hour, find the number of joules in 1 kW h.

Self-test

1.4.5

Calculate the energy transferred by a 2.5 kW electric kettle in 3 minutes.

Study point

The annual electrical output of the British national grid is sensibly expressed in TW h.

Self-test

1.4.6

Express the electrical energy output of the nuclear reactor in J.

Study point

The formula $P = VI$ is equivalent to $P = Fv$. The pd, V , drives the current, I , similarly to the way F drives v .

1.4.7

Self-test

The drag, F_d , on a car is given by

$$F_d = 0.3\rho_{\text{air}}v^2.$$

(a) Show that the unit of the 0.3 term is m^2 .

(b) Calculate the power developed by the engine at 25 m s^{-1} .

$$[\rho_{\text{air}} = 1.3 \text{ kg m}^{-3}]$$

Fig. 1.4.18 Energy dissipation

Study point

The symbol η (eta) is often used for efficiency.

Study point

What constitutes *useful* transfer depends on the context. The rise in temperature produced by rubbing your hands together could be desired output! The 'waste heat' from a car engine can be used to warm the passengers.

Exam tip

We can write the efficiency equation in terms of power instead of energy, i.e.

$$\eta = \frac{\text{Useful power output}}{\text{Total power input}} \times 100\%$$

1.4.5 Dissipative forces and energy¹

In Section 1.4.3 we derived the formulae for various forms of energy by considering the work done against external forces:

- The increase in gravitational potential energy is equal to the work done against the force of gravity.
- The increase in elastic potential energy is equal to the work done against the tension within the object when we stretch it.

The reason that we can do this is that these processes are *reversible*. If we release the systems, they will naturally return to their previous states – the energy can transfer in the opposite direction.

However, if a force does work by moving an object against a frictional force or aerodynamic drag, the energy that is transferred cannot be recovered in the same way. This is because the energy is then possessed in an increase in the disordered motion of the molecules of the system (the internal energy) and generally results in a rise in temperature. Reversing the process and turning some of the energy of random motion once more into ordered motion is possible but the efficiency is limited by the 2nd law of thermodynamics, which is beyond the scope of this course.

The familiar pattern in Fig. 1.4.18 is of the behaviour of water after an object, e.g. a pebble, is dropped in. A very small fraction of the energy is retained as ordered motion in the rising of the small water droplets and the spreading ripples. This energy is soon converted into random motion of water molecules by the action of viscous drag forces.

Looking back to the example in Section 1.4.1 and to Fig. 1.4.5, the work input into the system is 1100 J but the increase in GPE is only 980 J. This is the *useful* energy transfer. The remaining transfer, 120 J, represents a loss of useful energy. We define the efficiency of the system as the fraction of the energy input which is transferred usefully by the system. This is often expressed as a percentage, i.e.

$$\text{Efficiency} = \frac{\text{useful energy transfer}}{\text{total energy input}} \times 100\%$$

In the example this becomes: $\eta = \frac{980}{1100} \times 100\% = 89\%$

In calculations, efficiency is most conveniently expressed as a number between 0 and 1 (i.e. 0.89 for the winch), with the % figure being reserved for a final communication. In a chain of energy transfers, the total efficiency is the product of the efficiencies at each stage.

Example

A gas turbine power station, with an electrical power output of 1.0 GW has an efficiency of 60%. It is connected to consumers via a step up transformer (98% efficient), the national grid (97%) and local distribution network (95%).

Calculate the overall efficiency achieved.

$\eta = \frac{\text{Useful power output}}{\text{Total power input}}$; The power input to the power station is given by:

$$\therefore \text{Total power input} = \frac{1.0 \text{ GW}}{0.6} = 1.67 \text{ GW}$$

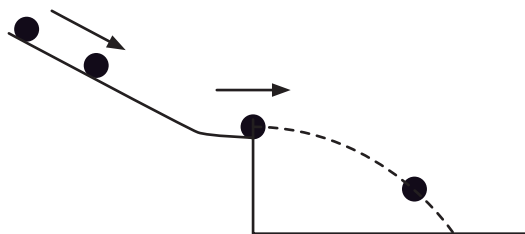
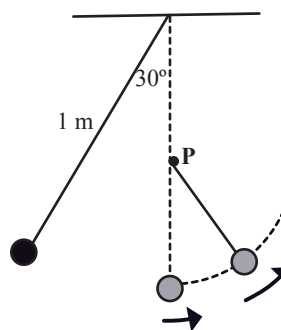
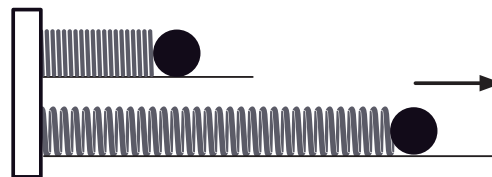
$$\text{Power delivered to consumers} = 1.0 \text{ GW} \times 0.98 \times 0.97 \times 0.95 = 0.90 \text{ GW}$$

$$\therefore \text{The overall efficiency} = \frac{0.9 \text{ GW}}{1.67 \text{ GW}} = 0.54, \text{ i.e. } 54\%$$

¹ See also Section 1.3.5(b) and (c)

Exercise 1.4

- Calculate the kinetic energy of
 - a car of mass **1200 kg** travelling at **30 m s⁻¹**,
 - a bullet of mass **0.04 kg** travelling at **500 m s⁻¹**.
- A car of mass **1600 kg**, travelling at a speed of **25 m s⁻¹**, brakes uniformly to rest in a distance of **100 m**. Calculate
 - the initial kinetic energy of the car and
 - the braking force.
- A cyclist and cycle, of joint mass of **85 kg**, freewheel from rest down a sloping road of height **20 m** and length (measured along the slope) of **200 m**.
 - Calculate the loss in gravitational potential energy.
 - Calculate the speed the cyclist would attain in the absence of resistive forces.
 - The cyclist attains a speed of **10 m s⁻¹**. Calculate the mean resistive force acting.
- A steel sphere of mass **4 g** is pushed against a spring with a force of **5 N**, compressing the spring by **20 cm**. Calculate:
 - The elastic potential energy when the spring is compressed.
 - The speed the sphere attains when the spring is released.
 State your assumptions.
- A small heavy sphere is suspended from the ceiling by a **1.00 m** long thread. A horizontal pin, **P**, is placed **50 cm** below the suspension point. The pendulum is pulled aside to the left by **30°** and released. Calculate:
 - The speed of the sphere at the lowest point.
 - The greatest angle attained by the pendulum thread on the right.
 Show your reasoning and your working clearly.
- A rolling ball possesses kinetic energy in its rotation as well as its translation (forward movement). The rotational kinetic energy is **28.6%** of the total. A ball rolls down a slope of height **1.00 m** which becomes horizontal before ending in a step of height **1.00 m**. Calculate the horizontal distance from the step to the point where the ball hits the ground.



[Note: You can take $g = 9.81 \text{ m s}^{-2}$ but, in fact, the answer is independent of the value of g ! You should check this algebraically.]

Stretch & Challenge

The formulae for momentum, p , and kinetic energy, E_k , are:

$$p = mv \text{ and } E_k = \frac{1}{2}mv^2$$

- Show that $E_k = \frac{p^2}{2m}$.
- A positively charged particle has a kinetic energy of **5.0 MeV** and a momentum of $1.03 \times 10^{-19} \text{ N s}$. Identify the particle by calculating its mass.

1.5 Solids under stress

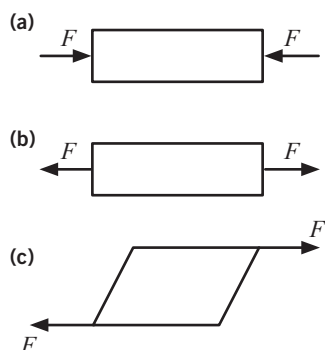


Fig. 1.5.3 Different types of stress

Study point

Transverse waves cannot propagate through gases and liquids because of their lack of ability to withstand shear forces – hence the inability of S-waves to propagate through the Earth's outer core.

Terms & definitions

Tension is the force which an object exerts on external objects because it is being stretched.

Terms & definitions

Hooke's law: the tension is directly proportional to the extension provided that the extension is not too great.

Terms & definitions

A material is **elastic** if it returns to its original form when stress is removed.

1.5.1

Self-test

A spring stretches by 12.5 cm when a load of mass 300 g is suspended from it. Calculate the spring constant k ($F = k \Delta l$).

If equal and opposite forces are applied to the opposite ends of an object, its particles (molecules / atoms / ions) will be forced into new equilibrium positions with respect to one another. The forces can be: (a) **compressive**, (b) **tensile** or (c) **shear** (see Fig. 1.5.3). The objects are said to be under **stress**. The forces shown are those applied externally to the object; by Newton's 3rd law the object exerts equal and opposite forces on the external objects.

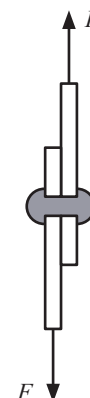


Fig. 1.5.1 & 1.5.2 Solids under stress

The material of Nelson's column is clearly under compression from the weight of the material above, the bungee cord is subject to tensile forces and the rivet is under shear. The response of different materials to stress is very variable: gases cannot be put under tension because the molecules are not bound together; liquids and gases cannot withstand a shear because they have no rigidity to their shape; many engineering materials, e.g. masonry, fracture easily under tension whereas steel is very strong. This section of the book deals with how materials behave under (mainly) tensile forces.

1.5.1 Hooke's law

When an object is subject to a tensile force it stretches. For most objects the degree of stretch is directly proportional to the tension – at least for small extensions. This is **Hooke's law** (after Robert Hooke, the 17th-century physicist). Where Hooke's law applies, an object also exhibits **elastic** behaviour. If an object is subject to too great a stress it will fracture but, often before that happens, the tension–extension graph ceases to be linear. Some objects, particularly made of ductile metals, enter a **plastic** region, in which they are permanently deformed by the tension.

The usual graphs of tensile behaviour are of tension against extension (rather than the other way round). There are several reasons for this:

- Tensile testing machines are usually designed to apply a specific extension and measure the tension produced.
- The gradient of the graph gives the **stiffness** of the object, which is called the **spring constant** in the case of a spring.
- The area under the graph is the work done in stretching the object, which for elastic materials is also the **elastic potential energy** stored.

You should refer to Section 1.4.3 for further treatment of these concepts.

1.5.2 Stress, strain and the Young modulus

The tension, F , extension, Δl and spring constant, k , relate to an object: a particular spring, piece of wire, block of concrete, etc. It is more useful for engineers to have quantities which relate to materials and which can be used to predict the properties of many different objects.

The bar in Fig. 1.5.4 of length l and cross-sectional area (c.s.a.) A , is stretched by Δl , which requires a force F . If we imagine two such bars side by side, making a total c.s.a. of $2A$, then each will require F to stretch it by Δl , so the total tension is $2F$.

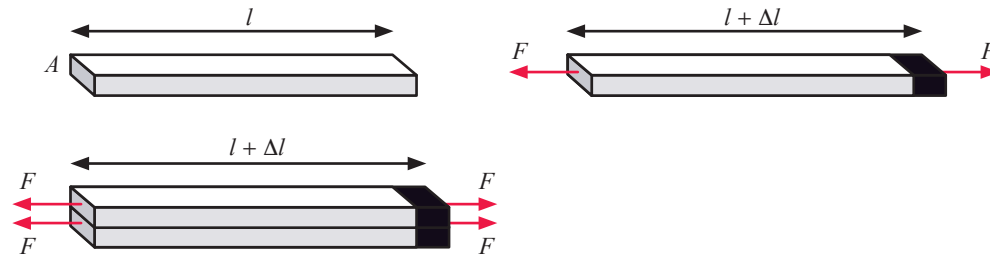


Fig. 1.5.4 Tension $\propto$ c.s.a

Conclusion: two bars of the same composition, with the same length, will be stretched by the same amount if the ratio $\frac{F}{A}$ is the same. This quantity is called the (tensile) **stress**, σ .

We now imagine the two bars being welded end to end, making a total length $2l$, and the same stretching force applied. The tension has the same value, F , in each half of the composite bar, so each half extends by Δl making the total extension $2\Delta l$, as in Fig. 1.5.5.

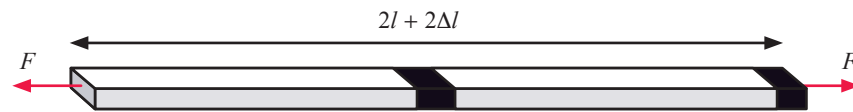


Fig. 1.5.5 Extension $\propto$ original length

Conclusion: for two bars of the same composition, with the same c.s.a. and tension, the ratio $\frac{\Delta l}{l}$ is the same. This quantity is called the (tensile) **strain**, ϵ .

If the bar obeys Hooke's law, then $F \propto \Delta l$, so from the definition of σ and ϵ , it must be true that $\sigma \propto \epsilon$ and we define the Young modulus, E , of the material by:

$$E = \frac{\sigma}{\epsilon}, \text{ which we can rewrite as } E = \frac{Fl}{A\Delta l}, \text{ from the definitions of } \sigma \text{ and } \epsilon.$$

Typical values of E , σ and ϵ

The newton is, in engineering terms, a rather small unit of force. The m^2 on the other hand is quite a large unit of area. Engineering materials, such as steel and concrete, are very resistant to deformation. Taking these points together it is not surprising that values of stress are very large (in the 100 MPa range) and strains are typically very small: a strain of 0.001 or less for a **Hookean material**. Hence, the Young moduli tend to have values in the 100 GPa range. Table 1.5.1 gives E for some common materials.

Terms & definitions

Tensile stress, $\sigma = \frac{F}{A}$ and **tensile strain**, $\epsilon = \frac{\Delta l}{l}$, where F is the tension, A the c.s.a., l the original length and Δl the increase in length.

Terms & definitions

For a material that obeys Hooke's law, the **Young modulus**, $E = \frac{\sigma}{\epsilon}$, where σ is the stress and ϵ the strain.

Study point

The **compressive stress** and **strain** are defined in exactly the same way as tensile stress and strain. The ratio of compressive stress to strain is called the **compression modulus** and, for small compressions can be taken as having the same value as the Young modulus.

UNIT

The unit of stress is the pascal, Pa. This is equivalent to N m^{-2} .

Strain has no unit because it is the ratio of one length to another.

Young modulus is the ratio of stress to strain so its unit is Pa.

Exam tip

E has a very large value (typically 10^{10} – 10^{11} Pa). In the formula,

$$E = \frac{Fl}{A\Delta l}, \text{ the large quantities}$$

are on the top and the small ones on the bottom of the fraction.

Terms & definitions

A **Hookean object** (or material) is one which obeys Hooke's law.

Table 1.5.1 Miscellaneous values of the Young modulus

Material	E / GPa
Mild steel	210
Copper	117
Aluminium	69
Human long bone	14
Concrete	14–30
Oak (along grain)	11
Glass	50–90
Rubber (small strain)	~0.1
Diamond	1220

1.5.2

Self-test

A 1 cm^2 c.s.a. bar of length 2.0 m is subject to a tension of 1 kN . The Young modulus is 100 GPa .

Calculate the extension and give your answer in μm .

Study point

Sometimes the symbol x is used for extension and sometimes Δl , so the formula for the work done in stretching can be written:

$$W = \frac{1}{2}Fx \text{ or } W = \frac{1}{2}F\Delta l$$

Because $F = kx$ [or $k\Delta l$] we can also write:

$$W = \frac{1}{2}kx^2 \text{ or } W = \frac{1}{2}k(\Delta l)^2 \text{ and}$$

$$W = \frac{1}{2}\frac{F^2}{k} \text{ or } W = \frac{1}{2}\frac{F^2}{\Delta l}$$

Study point

For Hookean materials, the product

$\frac{1}{2}\sigma\epsilon$ is the **strain energy per unit volume**. To show this:

Energy stored, $W = \frac{1}{2}F\Delta l$

$$\therefore \frac{W}{V} = \frac{1}{2}\frac{F\Delta l}{Al} = W = \frac{1}{2}\frac{F}{A}\frac{\Delta l}{l} = \frac{1}{2}\sigma\epsilon$$

1.5.3

Self-test

Calculate the strain energy in the bar in Self-test 1.5.2.

Working with E , σ and ϵ requires care with the use of SI multipliers and standard form. Study the following example.

Example

Calculate the extension of a 100 m long steel wire of diameter 1 mm if it is placed under tension by suspending a 10.0 kg mass from it. [$E_{\text{steel}} = 210 \text{ GPa}$.]

Solution

$$\text{Tension} = mg = 10.0 \times 9.81 = 78.5 \text{ N}; \text{ csa} = \pi(0.5 \times 10^{-3})^2 = 7.85 \times 10^{-7} \text{ m}^2.$$

$$\therefore \text{stress, } \sigma = \frac{F}{A} = \frac{78.5 \text{ N}}{7.85 \times 10^{-7} \text{ m}^2} = 1.249 \times 10^8 \text{ Pa [125 MPa]}$$

$$E = \frac{\sigma}{\epsilon} \therefore \epsilon = \frac{\sigma}{E} = \frac{1.250 \times 10^8 \text{ Pa}}{210 \times 10^9 \text{ Pa}} = 0.000 595$$

$$\therefore \Delta l = \epsilon l = 0.000 595 \times 100 \text{ m} = 0.059 5 \text{ m} = 6.0 \text{ cm (2 s.f.)}$$

Alternatively, we could just start from $E = \frac{Fl}{A\Delta l}$ and substitute for F and A .

The approach to take is a matter of choice.

1.5.3 Work of deformation and strain energy

As we showed in Section 1.4.4(d), the work done in stretching a Hookean object is given by $\frac{1}{2}F\Delta l$, this being the area under the force–extension graph. Refer to the **Study point** for alternative ways of writing the formulae for W . Releasing the tension and allowing the object to contract allows it to do work in its turn and, because the graph for relaxation is the same as for tensioning (for Hookean materials), the work done by the object in relaxing is the same as the work done on the object during deformation. Hence the same expressions – $\frac{1}{2}F\Delta l$, etc. – give the energy stored in a body by virtue of its deformation. This energy is variously referred to as **strain energy** or elastic **potential energy**.

If the extension of a rubber band is measured when it is loaded (e.g. by hanging 100 g masses) and then unloaded, a load–extension curve similar to that in Fig. 1.5.6 is obtained. The unloading curve is below the loading curve. This phenomenon is called **hysteresis** and is responsible for rolling resistance in car tyres. The work done on the rubber in extension is the area under the loading curve; the work done by the rubber in contracting is the area under the unloading curve.

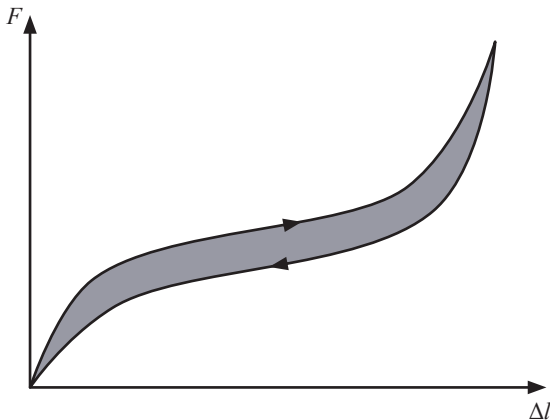


Fig. 1.5.6 Hysteresis in rubber

The area between the curves represents the mechanical energy loss in the cycle: it is transferred to internal energy in the rubber and then lost as heat. The elasticity of rubber is dealt with in Section 1.5.7.

Self-test

1.5.4

Show that the W/V can also be written by $\frac{1}{2} \frac{\sigma^2}{E}$ and $\frac{1}{2} \varepsilon^2 E$

1.5.4 Stress and strain in ductile metals

The mechanical properties of solids depend on their structure, that is the way its particles are arranged and the nature of the forces between them. The next three subsections discuss this for different classes of solids.

(a) Structure

Many metals, such as steel, aluminium and copper, are **ductile**, which means that they can be drawn out into wires. Ductile materials are also **malleable**, especially when hot. These metals are crystalline: they have a periodic structure called a lattice. The lattice particles in metals are positive ions – atoms that have lost one or more electrons. They are held together by a ‘sea’ of negatively charged *delocalised* electrons which are free to move between the ions.

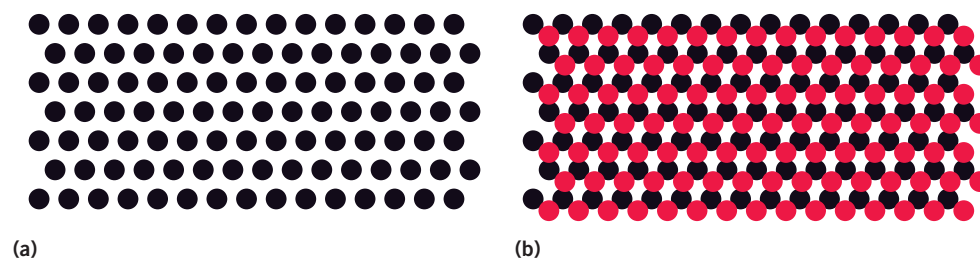


Fig. 1.5.7 Hexagonal packing in metals

Because metal ions are spheres, they pack with the least potential energy into planes with the hexagonal arrangement shown in Fig. 1.5.7(a), with ions in the plane above nestling in the gaps, the red spheres in (b).

Gas-turbine blades have been developed consisting of single crystals of a ‘superalloy’ of nickel. Most metal samples, however, are **polycrystalline**. When they solidify from the molten state, crystallisation occurs at many points independently.

This results in a large number of very small interlocking crystals (grains) as shown in Fig. 1.5.8, which represents a $\sim 100 \mu\text{m}$ square region of a polished and etched surface of a sample of Ti–Al alloy. A notional arrangement of the metal ions is shown in Fig. 1.5.9. The orientation of the crystal planes is random from one grain to the next. Note that the lattice ions are not shown to the same scale and, in reality, a typical grain will have $\sim 10^5$ lattice planes. The grain boundaries have a large component of impurity atoms which are forced out of the grains during crystallisation.

Another important feature of the structure of ductile metals is the presence of irregularities within the lattice. An **edge dislocation** is where an additional $\frac{1}{2}$ -plane of ions is present and a **point defect** is where a lattice ion is missing or a ‘foreign’ atom or just an additional ion is present.

Terms & definitions

Ductile – able to be drawn (into a wire, not ‘drawn by Picasso’).

Malleable – able to be hammered into shape.

Study point

The crystal lattice and non-directional nature of metallic bonding allows for movement of dislocations giving rise to ductility.

Fig. 1.5.8 A titanium-aluminium alloy showing grains

Fig. 1.5.9 A schematic representation of ions

Study point

If you look carefully at Fig. 1.5.10 you'll notice that the ions immediately surrounding the point defect are pulled away from their neighbouring ions by the gap in the structure. This is also the case at the end of the edge dislocation: these are regions of weakness in the lattice.

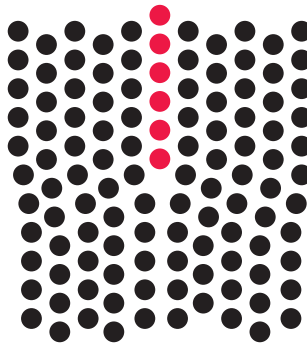


Fig. 1.5.10 Edge dislocation

Fig. 1.5.11 Point defect in a metal lattice

In both types of dislocation the combination of regular lattice, grain boundaries and dislocations is responsible for the mechanical properties of polycrystalline metals.

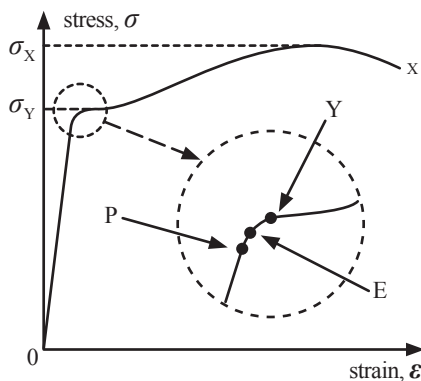


Fig. 1.5.12 σ - ϵ curve for a ductile metal

(b) Stress-strain graphs

Samples of ductile metals typically have stress-strain curves similar to that in Fig. 1.5.12, in which we can identify the following features:

- A linear portion **OP**. **P** is called the **limit of proportionality**. The gradient of this section is the **Young modulus** of the material.
- Point **E** is the **elastic limit**. Only strains up to **P** are **elastic**; beyond **P** they are **plastic**.
- The **yield point**, **Y**, at which the material shows a large increase in strain for little or no increase in stress. The stress here is called the **yield stress**, σ_Y .
- An extensive **plastic** region, **YX**. The maximum stress is called the **breaking stress** or **ultimate tensile strength**, σ_X . X on the graph marks the breaking point.
- The largest strain region of the σ - ϵ curve typically bends downwards. In this region, the sample exhibits **necking**, which is a narrowing of the region where it will eventually break.

The precise shape of the σ - ϵ curve varies with material and also the history of the material (e.g. heat or working treatment).

Stretch & Challenge

The stress, σ , in Fig. 1.5.12 is the so-called **engineering stress**. The tension is divided by the unstressed c.s.a. When necking occurs, the c.s.a. decreases and the true stress is greater than the engineering stress. In a graph of true stress, the σ - ϵ graph does not curve downwards at the end but continues to rise.

Fig. 1.5.13 Ductile metal specimen before and after destructive testing

The 'before and after' image in Fig. 1.5.13 shows the effect of necking and plastic deformation in a specimen of mild steel. A clip of this tensile test is available on YouTube.

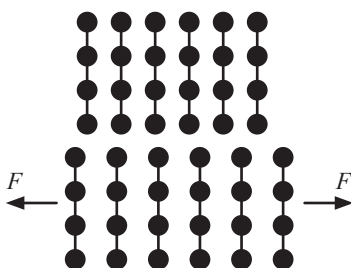


Fig. 1.5.14 Elastic strain

(c) Structure and properties

When a material is put under a low tension, i.e. so that $\sigma < \sigma_E$, the separation between the lattice particles (ions) is increased. This is elastic deformation because the forces between the particles pull them back to their initial position when the tension is removed. This is illustrated in Fig. 1.5.14 – a cubic lattice arrangement has been used as the effect is easier to see – and it is easier to draw!

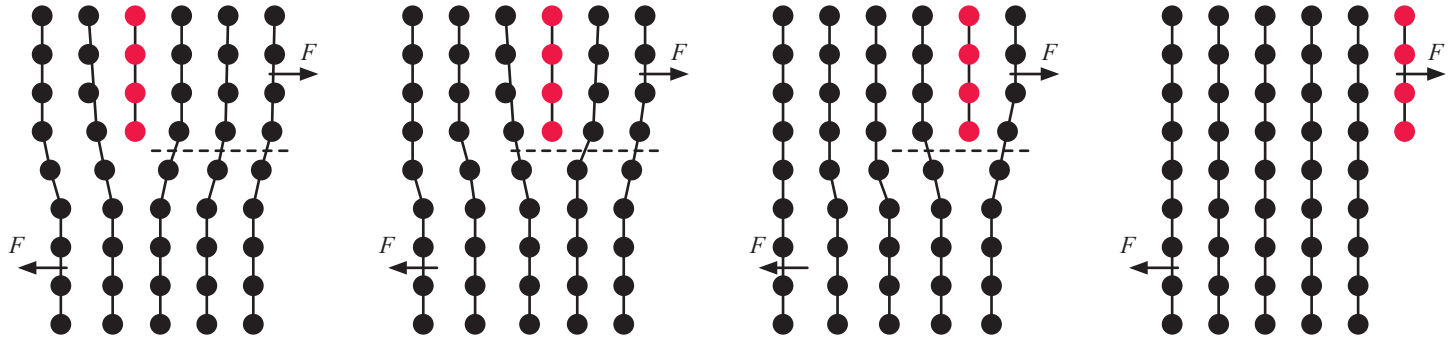


Fig. 1.5.15 Dislocation movement produces plastic strain

Plastic deformation is caused by an irreversible rearrangement of particles. This is made possible by the presence of edge dislocations. The dislocation shown in Fig. 1.5.15 moves to the right under the influence of the applied forces. The individual ions only move slightly; the ions above the dotted line drop into a lower potential energy position in the next plane, so that the extra $\frac{1}{2}$ -plane moves to the right until it reaches the grain boundary; the crystal becomes more elongated. The stress at which this happens is the **yield stress**. The dislocation does not move back when the stress is removed, so this elongation is plastic.

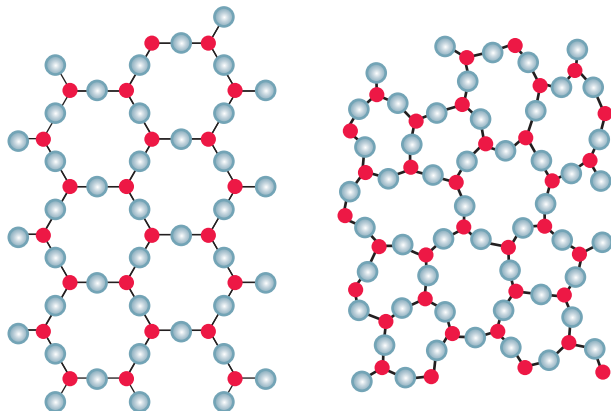
The details of what happens next depend on several factors:

1. Edge locations can get entangled (see Self-test 1.5.5) limiting their movement.
2. The size of the grains – the larger the grains, the greater the freedom of movement of the dislocations.
3. The presence of point dislocations: foreign atoms can inhibit the movement of edge dislocations; a void in the lattice spawns more edge dislocations.

For different metals, especially alloys such as steel, changing the composition can affect all these factors. Heating and quenching regimes, depending on the metal, can make the metal more or less ductile and cold working generally makes a metal stiffer and less ductile because it causes dislocation entanglement.

1.5.5 Stress and strain in brittle materials

Brittle materials, such as cast iron, ceramics and masonry are totally elastic and generally Hookean. Brittle non-metals have an amorphous (non-crystalline) structure. Figs 1.5.17 and 1.5.18 show the distinction between crystalline (quartz) and amorphous (glass) structures for silicon dioxide, SiO_2 , which is covalently bonded.



Glasses form when the SiO_2 cools down too rapidly from the molten state for the molecules to arrange into the crystalline form.

Fig. 1.5.17 Crystalline and 1.5.18 Vitreous (glass) silicon dioxide

Self-test

1.5.5

Sketch a diagram of what is likely to happen next to the crystal with two edge dislocations meeting.

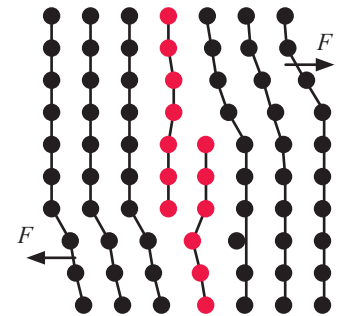


Fig. 1.5.16 σ - ϵ graph for a brittle material

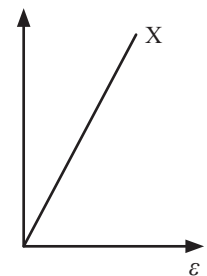


Fig. 1.5.19 Snowflake obsidian

Stretch & Challenge

In covalent structures, atoms are bonded individually to others by the shared electrons. Because of this atoms cannot swap bonding to another atom, so plastic deformation is not possible.

Glasses are not stable in the very long term. Obsidian, a volcanic glass gradually crystallises. The sample of 'snow flake obsidian' in Fig. 1.5.19 is starting to convert to quartz: its molecules are gradually re-arranging themselves. Because of this process, no obsidian is found dating from before the cretaceous period (144–66 Mya).

Amorphous materials are brittle because the absence of a crystal structure means that there can be no dislocations to move and produce plastic deformation. Cast iron is crystalline, but the crystals are very small and the presence of a large fraction of impurity atoms means that dislocations are pinned down.

Brittle fracture

Brittle materials are weak in tension, i.e. their breaking stress is low. Unlike in ductile fracture, the broken pieces can be fitted back together – a broken cup can be glued. This is because of the absence of plastic deformation.

The failure in tension is due to crack propagation. This is shown in Fig. 1.5.20. The so-called stress lines are shown in red. These show how the tension links the atoms in the material. The interatomic forces cannot cross the gap because the atoms are too far apart, so the forces must be transmitted around the tip of the crack, leading to a greatly magnified stress. The brittle material starts to break at the crack tip: the crack extends which increases the stress at the tip and so the crack propagates (at the speed of sound in the material) resulting in catastrophic failure.

Brittle materials can be used in load-bearing structures if the brittle member is:

- always under compression by the design of the structure, as in the Pontcysyllte brickwork pillars which hold up the Llangollen canal,
- or compressively pre-stressed in manufacture.

The bottom of the concrete beam in Fig. 1.5.22 is under tension and is in danger of failure by crack propagation. The diagrams in Fig. 1.5.23 show how a steel pre-tensioning tendon, T, is inserted into a concrete beam:

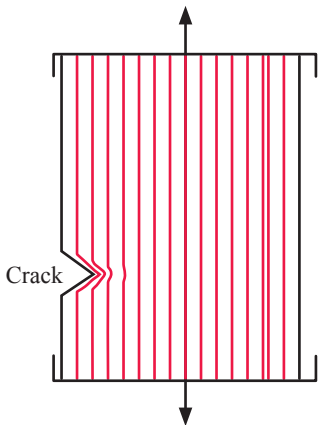


Fig. 1.5.20 Crack failure

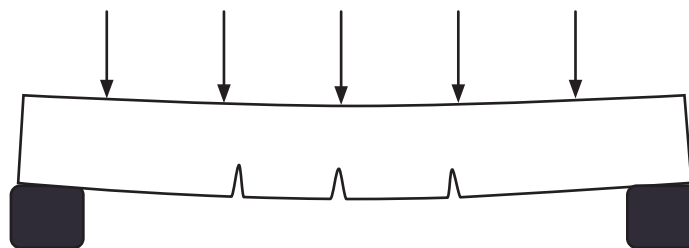


Fig. 1.5.22 Crack failure in a beam

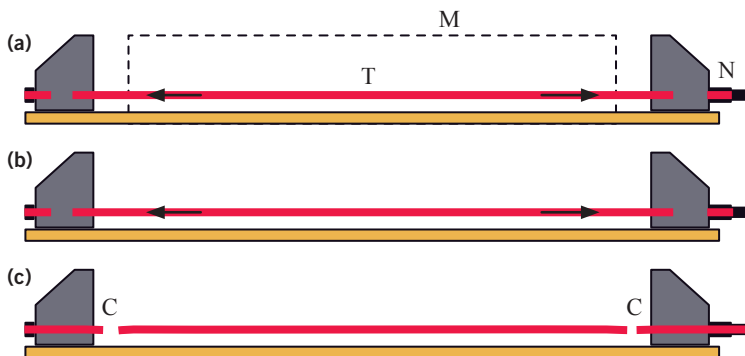


Fig. 1.5.23 Pre-stressing

- The tendon is placed under tension by tightening the nut, N.
- The concrete to be cast is poured into the mould, M, and allowed to set.
- The tendon is then cut and contracts putting the lower surface of the concrete under compression (and flexing it slightly as shown).

If the beam is subsequently loaded at the top as in Fig. 1.5.22, the lower surface of the beam will not be under tension unless a very large load is applied.

1.5.6 Polymers

(a) What are polymers?

Rubber, polythene, melamine and nylon are **polymers**, i.e. their molecules are long chains of repeat units.

The simplest in terms of composition is polythene, which is made from the **monomer** ethane.



Fig. 1.5.24 Ethene, and polythene repeat unit

A significant feature of the C–C bond is that it can rotate so the polymer molecule can assume a huge number of random shapes, which is significant in the stress–strain properties of rubber.

Rubber is polymerised isoprene, $\text{CH}_2=\text{C}(\text{CH}_3)\text{CH}=\text{CH}_2$. In the process of polymerisation, the two double bonds are opened up, enabling the repeat units to link together to produce poly-isoprene. In the process, the two central carbons become double bonded: $\text{C}=\text{C}$.

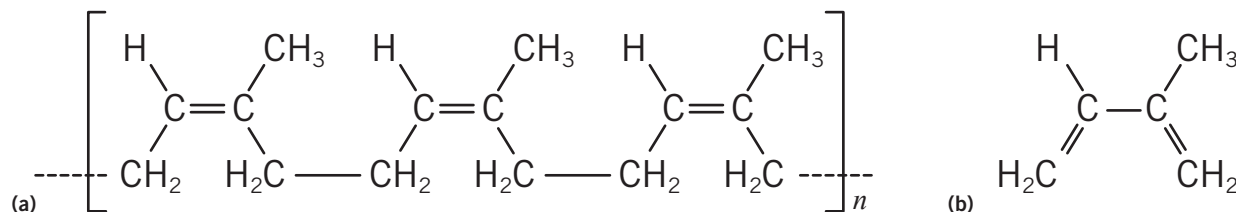


Fig. 1.5.26 Rubber: (a) the polymer; (b) the monomer

(b) Stress, strain and Young modulus of rubber

The load-extension / stress–strain characteristics are as follows:

1. The (engineering) stress–strain graph is non-linear: rubber is stiff for low extensions, becomes less stiff and finally much stiffer. See Fig. 1.5.6.
2. Loading and unloading curves different: called *elastic hysteresis*.
3. It exhibits large strains: strains of up to 5, i.e. the final length is $\sim 6\times$ the original length are possible in some types of rubber
4. The stress values are much lower than most engineering materials. The σ_X value is $\sim 16 \text{ MPa}$, compared to $\sim 80 \text{ MPa}$ for glass and 400 MPa for mild steel.
5. The volume stays roughly constant in spite of its huge extensions. Thus it is important to distinguish between true stress and engineering stress.

Terms & definitions

A **polymer** is a substance whose molecules consist of long chains of identical sections called repeat units.

A **monomer** is a molecule with a double bond which is broken open to form the repeat unit of a polymer.

Study point

An artist's impression of an ethane molecule showing the electron orbitals.

Fig. 1.5.25

Self-test

1.5.6

When rubber stretches, its volume stays roughly constant in spite of its huge extensions. Estimate the change in thickness of a rubber band which is stretched to 4 times its original length.

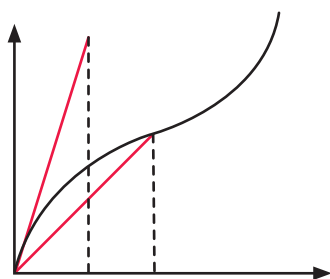


Fig. 1.5.27 Which Young modulus for rubber?

1.5.7

Self-test

A rubber sample has the same dimensions as a steel wire. The same tension is applied to each. Estimate the ratio of the strains of the two objects.

Stretch & Challenge

Statistical random walk theory predicts that, if a route is made of n steps of length Δl , in random directions, the total displacement $\sim \Delta l \sqrt{n}$. A rubber molecule consists of 10^6 repeat units of length $\sim 10^{-9}$ m. Estimate the extent of a rubber molecule.

Because of the non-linearity we need to be careful when referring to the Young modulus of rubber. Depending on the context it could refer to:

- The gradient of the tangent of the σ - ϵ curve at the origin.
- The value of $\frac{\sigma}{\epsilon}$ for a particular stress, e.g. halfway along the almost linear region.

These are the gradients of the red lines in Fig. 1.5.27. Then there is the problem of hysteresis... Whichever we use, the value of E is something like 10–20 MPa; much smaller than the 200 GPa for steel.

(c) Structure and properties of rubber

The angle between adjacent carbon–carbon bonds is approximately 110° . Looking at the structure of the rubber molecule in Fig. 1.5.26, it would seem that its shape should be:



However, this is wrong, or at least, very unlikely.

The molecule can rotate freely around each of the single C–C bonds (but not the double bonds, C=C), so the natural shape of rubber molecules is randomly tangled.

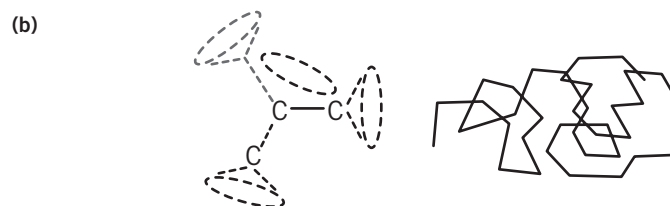


Fig. 1.5.28 (a) Extended and (b) tangled rubber molecules

If a rubber molecule is placed under tension, it responds by straightening out:

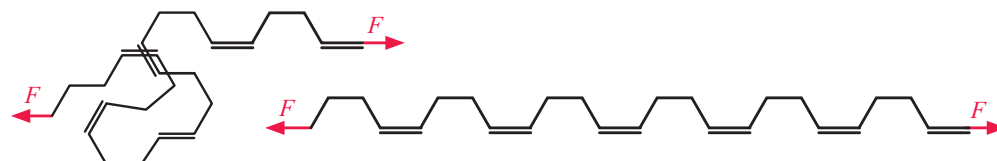


Fig. 1.5.29 Straightening a rubber molecule

This extension allows the rubber (band) to stretch to several times its original length. The force required is much less than when stretching crystalline or amorphous materials because bonds are not being stretched – just rotated. The presence of **cross-linkages** between molecules or just the entanglement of different molecules limits the total extension possible. The thermal motions of the atoms in the molecules, which tend to randomise the shape, provide the opposition to the extension, i.e. the stiffness.

When the tension is removed, random molecular motions of the atoms within the molecules re-randomises the shape the molecules, leading to the rubber contracting. Some energy is converted by intermolecular collisions into random kinetic energy of the molecules, so not as much work is done in contracting, leading to the hysteresis effect.

Study point

Materials scientists often introduce additional S-S cross linkages into rubber in order to make it stiffer. The process is called *vulcanisation*.

1.5.7 Practical work

(a) Determination of the Young modulus of the material of a metal wire

Fig. 1.5.30 shows a common set-up used in A level physics classes for the determination of the Young modulus of metals, such as copper and mild steel, in the form of wires.

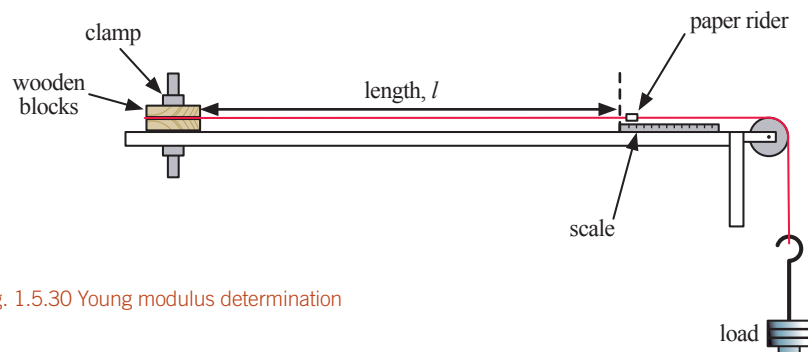


Fig. 1.5.30 Young modulus determination

Procedure:

1. Measure the diameter, d , of the wire using a micrometer or vernier calliper of resolution 0.01 mm . Calculate the c.s.a., A , from $A = \pi\left(\frac{d}{2}\right)^2$.
2. Attach a small load to provide original tension to straighten out the wire.
3. Measure the original length, l_0 , of the wire, using a metre rule with resolution 1 mm , from the blocks to the paper rider.
4. Add a known mass, such as a 10 g / 50 g / 100 g mass, to the hanger.
5. Measure the extension, Δl , from zero load.
6. Repeat 4 and 5 to obtain a series of values of Δl and m .
7. Repeat 4 and 5 with decreasing values of m .
8. Plot a graph of tension, T , calculated from $T = mg$, against Δl .
9. Measure the maximum and minimum gradient of the straight line graph.
10. Calculate E and its uncertainty using $E = \frac{l_0}{A} \times \text{gradient}$.

Notes:

1. The best fit graph might not go through the origin. This is because the wire often has some slight kinks which need straightening.
2. The accuracy of the Δl measurements can be improved by using a hand lens or (better) a travelling microscope.

(b) Investigation of the force–extension relationship for rubber

This is simply a case of loading and unloading the rubber band and measuring the extensions from the original length (actually the length with a very small load so that the band is initially straight). The band should be loaded until its additional extension with additional loads is very small. In this experiment, to reveal the hysteresis, the loading and unloading extensions are plotted separately rather than averaged.

Study point

The Young modulus is given by the equation $E = \frac{Fl_0}{A\Delta l}$. Δl is made large enough to be measurable with a reasonable precision by making l_0 as long as possible, e.g. the length of a lab bench (perhaps 4 m).

Study point

The wire is clamped using the wooden blocks. This prevents damage the wire at the point of clamping.

PRACTICAL TIP

See Section 3.2 for a suggestion on measuring the diameter of a wire.

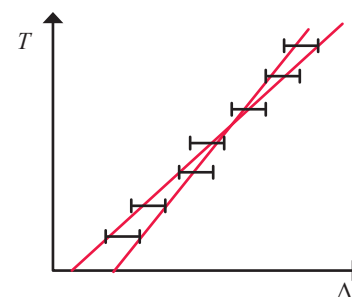


Fig. 1.5.31 Typical T v Δl graph

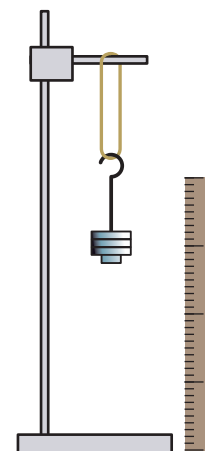
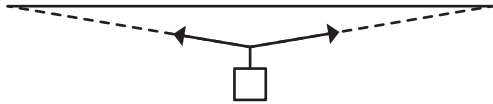


Fig. 1.5.32 Investigating load and extension for rubber

Exercise 1.5

1. A spring extends by 8.0 cm when a load of mass 200 g is hung from it. Calculate
 - (a) the spring constant, k , and
 - (b) the elastic potential energy in the spring with the load,
 - (c) the reduction in the gravitational potential energy of the load when it extends the spring by 8.0 cm.
[Take $g = 10 \text{ m s}^{-2}$]
2. The load in Q1 is hung from another spring which has a spring constant half that in Q1. Without repeating the calculations, write down the extension of the spring and the answers to (a), (b) and (c).
3. The load is placed on the unstretched spring in Q1 and dropped. Calculate
 - (a) the speed of the load when the extension is 8.0 cm and
 - (b) the maximum extension of the spring,
 - (d) the upward acceleration of the load at the point of maximum extension.
4. The constant, k , of a spring is 24 N m^{-1} . Giving your reasoning clearly, determine the constant of two such springs joined
 - (a) end-to-end and
 - (b) side-to-side [sometimes called 'in series' and 'in parallel' respectively].
5. A cylindrical rod of length 50 cm diameter 5 mm is made out of glass with an ultimate tensile strength of 33 MPa and Young modulus 60 GPa. Calculate the maximum tension the rod can take and its increase in length at a tension of 50% of this maximum.
6. A steel wire of diameter 1 mm is fastened horizontally to 2 rigid supports separated by 2 m. The initial tension is negligible as is the sag in the wire.
 

A load of mass 0.10 kg is suspended from the midpoint of the wire, which caused the wire to deflect downwards by 2.0 cm at that point. The diagram is not to scale. Calculate:

 - (a) The tension in the wire.
 - (b) The Young modulus of the material of the wire.
7. By considering a range of loads (e.g. 0.2, 0.3 and 0.4 kg) for the wire in Q6, determine how accurately the deflection in the wire is directly proportional to the applied load. If the yield strain of the steel is 1.0×10^{-3} , what is the maximum load which can be applied to the wire without it passing the yield point?
8. A hawser (a rope) is made from nylon with a Young modulus of 3.0 GPa; it is designed to be used safely at stresses of up to 30 MPa. The diameter of the hawser is 5.0 cm and its length is 1 km.
 - (a) Calculate the maximum load to which the hawser should be subjected.
 - (b) Calculate the energy stored in the hawser with this load.
 - (c) The hawser is attached to a boat of mass 20 tonnes [$1 \text{ t} = 10^3 \text{ kg}$] which is drifting away at 4 m s^{-1} , to bring it to a halt. Calculate how far the hawser stretches before stopping the boat.
 - (d) Determine the shortest length of hawser which would stop the boat without exceeding its maximum safe stress.
9. Pre-stressed glass, also known as *sight glass*, is manufactured in sheets in such a way that its outside layers are under compression and the middle is under tension. Explain why this glass is more resistant to breaking than ordinary glass.
10. Two cylinders of identical length and diameter are joined firmly end to end. The Young modulus of the material of one cylinder is twice that of the other. The composite cylinder is put under tension. Compare the values of the following quantities for the two sections: the tension; the extension; the stress; the strain; the energy stored.

11. Calculate the effective Young modulus of the composite cylinder in Q10 in terms of the lower Young modulus, E , of the two components. [Hint: Calculate the total extension for a given stress, σ .]
12. A cylinder is made of a composite of a cylinder inside a second cylindrical shell. The cross sectional area of one component is twice that of the other. The Young modulus of the larger area component is E_1 and that of the other is E_2 . The composite cylinder is put under tension. Compare the values of the following quantities for the two sections: the tension; the extension; the stress; the strain; the energy stored.
13. Calculate the effective Young modulus of the composite cylinder in Q12 in terms of E_1 and E_2 .

Stretch & Challenge

A student hangs a series of 500 g masses on a catapult elastic and measures its extension. These are her results:

Mass/kg	0	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
Extension/cm	0.0	0.7	1.5	2.8	5.0	12.2	17.0	21.2	23.0	24.1	25.0

She then removes the masses one by one and measures the extension and obtains the following data:

Mass/kg	0	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
Extension/cm	0.0	1.0	2.0	3.6	6.5	14.8	20.0	22.3	23.5	24.3	25.0

- (a) By drawing suitable graphs, use these results to calculate:
 - (i) The work done on the elastic in stretching it from 0 to 25 cm.
 - (ii) The work done by the elastic in contracting by 25 cm to 0.
 - (iii) The change in total potential energy if a 5 kg mass is suspended from the elastic and gently lowered by 25 cm.
- (b) The student places a bullet of mass 5.0 g into a catapult, draws the elastic back by 25 cm and releases the bullet at an angle of 30° to the horizontal. Ignoring air resistance, calculate the horizontal distance that the bullet travels.
- (c) Estimate the magnitude of the effect of air resistance on your answer to (b).

[Hint: See Section 1.3.5(c) and take $c_d \sim 0.5$ for a rapidly moving sphere.]

1.6 Using radiation to investigate stars

Almost all the information we have about the universe comes from electromagnetic radiation. The image of the constellation of Taurus was taken using visible radiation, i.e. with wavelengths between ~ 400 and 700 nm. Until recently this was the only wavelength range available to us because the Earth's atmosphere is opaque to most of the electromagnetic spectrum. However, space telescopes such as Spitzer and Chandra, have allowed us to 'see' the universe across the complete range from radio to gamma rays. Multi-wavelength astronomy gives us a much more complete understanding of the processes in the universe.

The following images of the Sun, Fig. 1.6.2, were taken in visible light (450 nm) and ultraviolet (17.1 nm) from the Solar Dynamics Observatory 10 October 2014.

Fig. 1.6.1 Taurus

Study point

The sites of the sunspots which are visible on the 450 nm image are also noticeably sites where violent processes are seen on the 17.1 nm image.

Stretch & Challenge

Several neutrinos were received from supernova SN1987A providing useful information about the supernova and testing our understanding of particle physics. Astronomers are also attempting to detect gravitational waves.

Terms & definitions

Diffraction grating – see Section 2.3.

Terms & definitions

A **continuous spectrum** consists of all wavelengths within a range.

A **line spectrum** consists of a series of individual wavelengths (or, more accurately, a series of very narrow wavelength bands).

Terms & definitions

The plural of spectrum is **spectra**.

Fig. 1.6.2 The Sun in (a) visible and (b) UV light

Section 1.6.4 considers multi-wavelength astronomy in more detail.

Scientists analyse the light from stars by separating out the different wavelengths, e.g. by passing it through a prism or a **diffraction grating**, and then either making an image or plotting the energy density at different wavelengths. Examples of both of these spectra for our nearest star, the Sun, are shown on the next page in Fig. 1.6.3.

The graphical spectrum of the Sun, published by the World Meteorological Organisation, is the continuous graph in the upper part of the figure. The wavelength range is ~ 200 – 1500 nm. We'll ignore the dashed curve for the moment. The colour image is the appearance of visible solar spectrum. This is what you can actually see (or image) if sunlight is dispersed through a prism or passed through a diffraction grating. The wavelength range is approximately 400 – 700 nm and its relationship with the spectrum graph is also shown.

The spectrum of the Sun consists of two parts:

- A **continuous spectrum** (the bright band).
- A **line spectrum** (the dark lines, also visible on the graph).

These spectra contain a huge quantity of information about the star, in particular the temperature of its outer layer and its chemical composition.

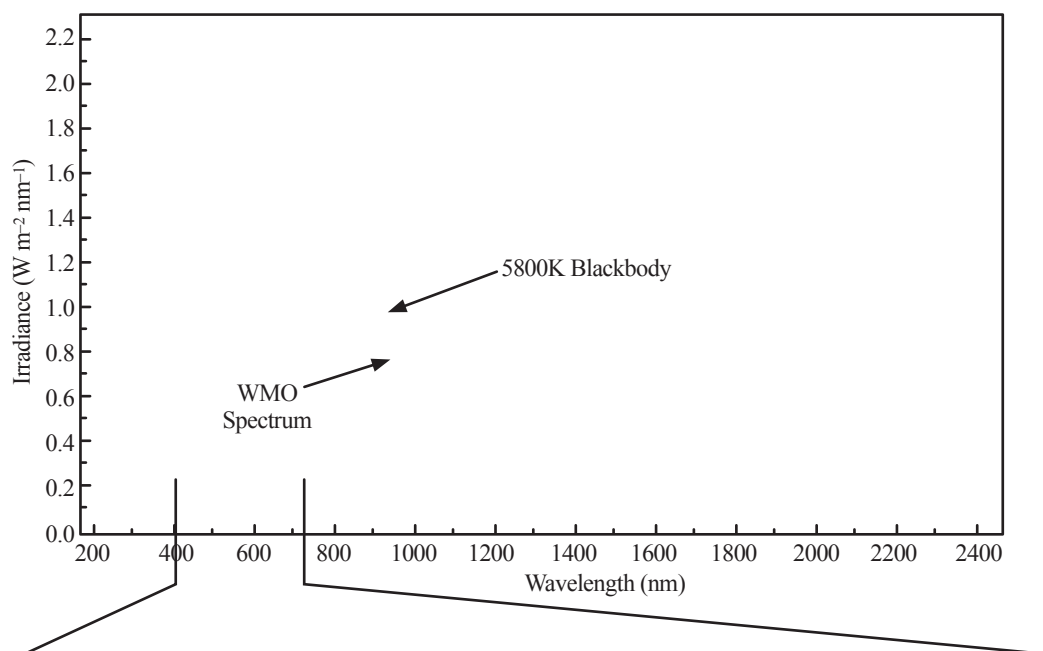


Fig. 1.6.3 The solar spectrum

This information is contained in both the continuous and line spectra. Using the shape of the continuous spectrum and the positions of the dark lines (the *Fraunhofer* lines), astronomers can compare stars and also obtain evidence about their motion and even the age of the universe. This section will explore how such information is obtained.

1.6.1 Black body radiation

When a farrier heats up a horseshoe in a forge, it starts to give out visible radiation. Initially it glows faintly dull red; as the temperature increases, the brightness increases and the colour changes from red to orange to yellow, as shown in Fig. 1.6.4. The precise details of the spectrum of the emitted radiation vary from material to material: in general, the more a material absorbs radiation, the more it will emit. Scientists use the idea of a perfect **black body** (see box) as a theoretical standard against which to compare other bodies.

As an excellent approximation to perfect black body radiation, scientists have made measurements of the radiation coming from a small hole in the side of a furnace. This radiation is also known as **cavity radiation**. Fig. 1.6.5 shows the principle of this.

If radiation enters the cavity through the opening, it will undergo multiple reflections. If the material lining the cavity is very dark, it will absorb most of the radiation at each reflection so hardly any of the incident radiation will re-emerge from the aperture.

Study point

The dark lines in the solar spectrum are called *Fraunhofer* lines after the German scientist who noticed them in 1814. The English chemist, William Hyde Wollaston, had discovered them in 1802.

Fig. 1.6.4 Thermal radiation

Terms & definitions

A **black body** is defined as one that absorbs all radiation which is incident upon it. It also emits more radiation at any wavelength than a non-black body in the continuous spectrum.

Study point

The **albedo** of an object is the fraction of e-m radiation that it reflects. The optical albedo of the Earth is ~0.3. Some trans-neptunian objects (TNOs) and comet-like outer solar system objects (centaurs) have measured albedos as low as 0.02.

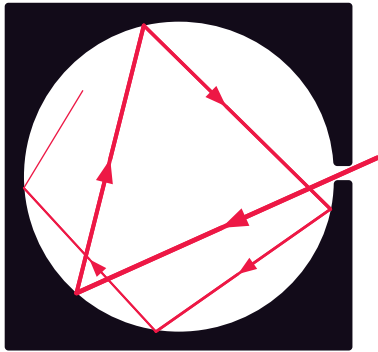


Fig. 1.6.5 A cavity absorbs almost all incident radiation

Terms & definitions

Absolute temperature, T , is expressed in kelvin (K).

Celsius temperature, θ , is defined by:

$$\theta / ^\circ\text{C} = T / \text{K} - 273.15$$

On the **kelvin scale**, ice melts at 273.15 K, water boils at 373.15 K and absolute zero is 0 K.

UNIT

Notice that the unit of the Wien constant is m K [metre kelvin] not mK [millikelvin]. Sometimes a space is important!

1.6.1

Self-test

In the example:

- Express λ_{max} in nm.
- In what part of the e-m spectrum is λ_{max} ?
- Without using a calculator, estimate P and λ_{max} for a similar sphere at 6000 K.

Study point

Not all **radiation** is black body, e.g. the pulses emitted by pulsars arise from electrons spiralling around magnetic field lines (so-called *synchrotron radiation*).

Because the furnace is hot, it also emits radiation, some of which escapes through the hole. The spectra of this radiation at various temperatures are in Fig. 1.6.6. These results fit in with the observations of the glowing wire:

- Below about 1000°C no visible radiation is seen.
- At 1400°C a small amount of red light is emitted.
- At 1800°C much more visible radiation is emitted, mainly at the long wavelength (red) end but with some shorter wavelengths.

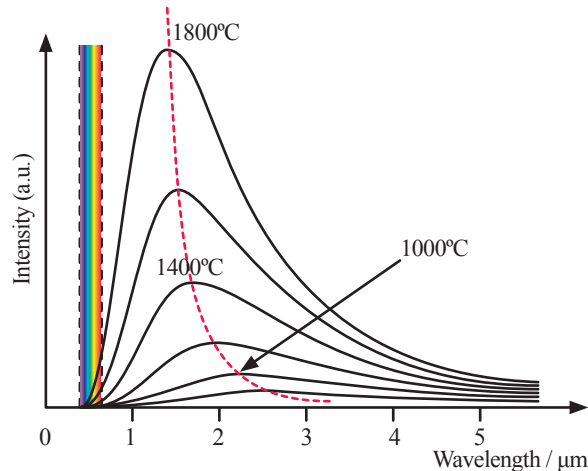


Fig. 1.6.6 Black body spectra

Studies of these spectra in the 19th century produced two empirical laws which were later explained theoretically by the German physicist, Max Planck.

- The **Wien displacement law**: The peak wavelength of radiation emitted by a black body is inversely proportional to the absolute temperature of the body, i.e. $\lambda_{\text{max}} = \frac{W}{T}$, where T is the kelvin temperature and W is a constant, now called the Wien constant, which has a value of $2.898 \times 10^{-3} \text{ m K}$.
- The **Stefan-Boltzmann law**: The total power of the radiation emitted by a black body, per unit area is directly proportional to T^4 , i.e. the fourth power of the absolute temperature, i.e. $P = A\sigma T^4$, where A is the surface area and σ a constant, called the Stefan constant, with a value of $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

Example

A sphere of tungsten of diameter 1.0 cm has a temperature of 3000 K. Assuming the sphere acts as a black body, calculate (a) the power of the radiation emitted, and (b) the wavelength of the radiation with the greatest intensity.

Answer

$$(a) \quad P = A\sigma T^4 = 4\pi \times (0.005 \text{ m})^2 \times (5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}) \times (3000 \text{ K})^4$$

$$\therefore \text{Power} = 1.70 \text{ kW}$$

$$(b) \quad \lambda_{\text{max}} = \frac{W}{T} = \frac{2.898 \times 10^{-3} \text{ m K}}{3000 \text{ K}} = 0.97 \text{ } \mu\text{m}$$

The importance of these results for astronomers is that many astronomical objects emit thermal radiation, which approximates to black body radiation. Examples are: stars, with surface temperatures up to tens of thousands of kelvin; the accretion discs around black holes (up to 10^6 K); the cosmic microwave background radiation (2.713 K).

1.6.2 Luminosity, intensity and distance

(a) The inverse square law

If we look back at the stars in *Taurus* in Fig. 1.6.1, we notice that they have a range of brightness (and colour). Are they really different or are they just at different distances – with the fainter-seeming ones just being further away? The diagram in Fig. 1.6.7 shows how radiation from a small source, such as a star, spreads out as it moves out. The further from the star, the bigger the area the same quantity of radiation has to cover, so the lower the **intensity** of the radiation.

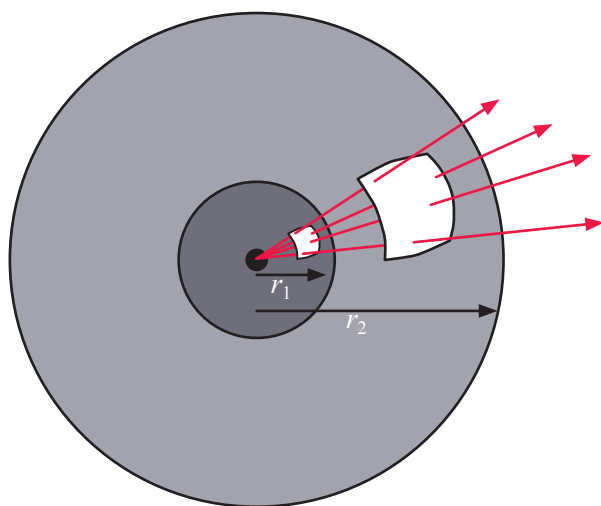


Fig. 1.6.7 Inverse square law

So the intensity of the radiation decreases as the inverse square of the distance:

a star 10 × as far away as an identical one will appear only $\frac{1}{100}$ as bright.

(b) Measuring the distance to a star

Apart from the Sun, stars are vast distances away. How far? For distances up to ~1000 ly astronomers can use the fact that nearby stars appear to shift position as the Earth moves around in its orbit. Fig. 1.6.8 shows this – but is rather exaggerated!

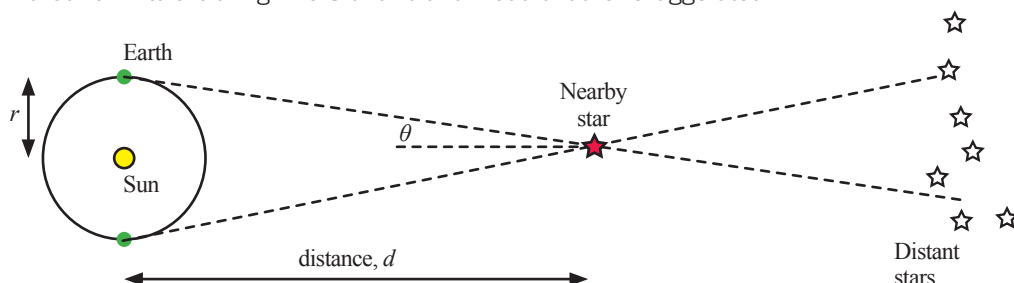


Fig. 1.6.8

Over a period of half a year, the nearby star appears to shift when seen against the background of much more distant stars because the Earth moves around in its orbit. If we know r , the radius of the orbit and we can measure the angle, θ , we can calculate the distance d to the star.

Terms & definitions

The **luminosity**, L , of a star is the total energy emitted per unit time. The unit is W.

Terms & definitions

The **intensity**, I , of radiation is the power per unit area incident on a surface at right angles to the radiation. The unit is W m^{-2} .

Self-test

1.6.2

A candle gives out light with a power of 0.1 W. What will be the intensity of its radiation at a distance of: (a) 1 m, (b) 10 m and (c) 1 km?

Study point

The luminosity and brightness of a star are not the same. A close faint star can appear as bright as a distant luminous one.

Self-test

1.6.3

Stars A and B appear to have the same brightness. Star B is twice as far away as star A. Compare the luminosity of the two stars.

Self-test

1.6.4

The distance to star X is measured as 10 ly [$1 \text{ ly} = 9.5 \times 10^{15} \text{ m}$]. The intensity of radiation from it is measured as 42.8 nW m^{-2} . Show that its luminosity is about $5 \times 10^{27} \text{ W}$.

MATHS CHECK

See the maths chapter for angles in radians and for the small-angle approximation, $\theta = \tan \theta$

Fig. 1.6.9 ESA's Gaia

1.6.5

Self-test

The Gaia satellite measures the annual stellar parallax of a star to be 2.0×10^{-10} radian. Calculate the distance of the star (a) in m and (b) in ly.

Stretch & Challenge

Astronomers measure stellar parallax in *arcseconds* (as) and distances in *parsecs* (pc). Research these units and answer Self-test 1.6.5 using them.

1.6.6

Self-test

Use answers to (a) and (b) in the example together with the Stefan-Boltzmann law to estimate the diameter of the Sun.

1.6.7

Self-test

The mean apparent diameter of the Sun in the sky is 0.535° (9.34×10^{-3} rad). Show that this is consistent with the answer to Self-test 1.6.6.

1.6.8

Self-test

The peak wavelength of star X in Self-test 1.6.4, is 788 nm. Estimate the star's temperature and radius.

Using trigonometry: $\tan \theta = \frac{r}{d}$, so $d = \frac{r}{\tan \theta}$. As θ is very small (typically 10^{-8} rad) we can

use the approximation $\tan \theta \approx \theta$ (with θ in radians) to calculate the distance using

$$d = \frac{r}{\theta/\text{rad}}, \text{ using the Earth's orbital radius of } 1.50 \times 10^{11} \text{ km.}$$

Using ground-based telescopes astronomers can measure θ to a precision of about 10^{-8} radians. The European Space Agency Gaia mission, the space telescope launched in December 2013, will measure stellar parallaxes to $\sim 5 \times 10^{-11}$ rad enabling distances up to 10s of thousands of light-years to be measured.

To measure larger distances, astronomers use objects of known brightness. If you know the true luminosity of an object and measure the intensity of radiation received, the inverse square law can be used to determine its distance. For distances to relatively nearby galaxies, astronomers can use *Cepheid variable* stars; for more distant galaxies they use *Type 1a supernovae*.

1.6.3 Black body radiation and stars

We are now in a position to interpret the graphical part of Fig. 1.6.3. The dotted line gives the shape of the spectrum for a perfect black body with a temperature of 5800 K. For wavelengths from 400–1000 nm, i.e. the visible and near infrared region, the overall shape of the observed spectrum matches the 5800 K spectrum reasonably closely. This suggests that the Sun and other stars emit radiation in a manner which is very similar too, but not exactly like, a black body. It might seem a little strange to regard the Sun as a *black* body. However, in radiation terms, that is exactly (or nearly) what it is! From the graph, the effective temperature of the Sun's photosphere (its outer layer) appears to be a little lower than 5800 K. Recent measurements give a best-fit estimate of the Sun's temperature of 5770 K. There is clearly more to be said about this. We'll consider the Fraunhofer lines in Section 1.6.4.

The assumption that we can treat stars as black bodies allows us to determine the temperature and diameter of a star from measurements of its spectrum, as long as we know its distance. The example does this for the Sun.

Example

Use the spectrum in Fig. 1.6.3 and the following data to estimate the Sun's (a) temperature, (b) power and (c) diameter:

- The mean **intensity** of the solar radiation at the Earth = 1.36 kW m^{-2} .
- The mean radius of the Earth's orbit = $1.50 \times 10^{11} \text{ m}$

Answer

(a) From Fig. 1.6.3 the peak wavelength $\lambda_{\text{max}} = 500 \text{ nm}$

$$\therefore \text{Using Wien's law, the temperature, } T = \frac{W}{\lambda_{\text{max}}} = \frac{2.898 \times 10^{-3} \text{ m K}}{500 \times 10^{-9} \text{ m}} = 5796 \text{ K}$$

(b) At the Earth's orbit, the Sun's radiation is spread out over the surface of a sphere of radius $1.49 \times 10^{11} \text{ m}$. Area of a sphere = $4\pi r^2$.

$$\therefore \text{Power of Sun} = 1.36 \times 10^3 \text{ W m}^{-2} \times 4\pi \times (1.50 \times 10^{11} \text{ m})^2 = 3.85 \times 10^{26} \text{ W}$$

1.6.4 Line spectra: Fraunhofer lines

(a) Emission spectra

Chemists identify ions using the flame test. The flame in Fig. 1.6.10 is ‘brick red’ revealing the presence of calcium ions. Similarly the glowing pink regions in the spiral arms of the Whirlpool galaxy, M51, in Fig. 1.6.11 pick out clouds of atomic hydrogen, that astronomers call H I clouds, which identify regions where active star formation is occurring.

The sources of the light could hardly be more different but, in fact, the physics by which the particles in the flame and the gas clouds emit light is the same in the two cases. The source of energy is different: the calcium ions in the flame derive their energy from impacts with other particles in the flame; the hydrogen atoms are energised by ultraviolet radiation from newly formed giant stars in the centre of the gas clouds.

Fig. 1.6.11
M51, the Whirlpool galaxy

These sources produce light which is very different from black body radiation as the spectra of atomic hydrogen and calcium show: the spectra are called *line spectra* for obvious reasons. For comparison, Fig. 1.6.12 includes a black body spectrum corresponding approximately to the Sun’s temperature – about **5800 K**. The colour of the H I clouds is due to a combination of the red and blue lines, the ‘brick-red’ of the calcium flame from all the lines in its spectrum.

The reason that low density gases, whether in the Bunsen flame or in the galactic H I clouds, produce only discrete wavelengths rather than a continuous spectrum is explored in detail in Section 2.7. Importantly for astronomers (and chemists) different elements emit different combinations of wavelengths, so the lines act as a spectral fingerprint and we can use them to identify the gases present.

(b) Absorption spectra

In order to reach us, the Sun’s radiation has to pass through the low pressure gas of its ‘atmosphere’ – the chromosphere and corona. These are normally only visible during a total solar eclipse because, although they emit light, the photosphere of the Sun is overwhelmingly brighter. Fig. 1.6.14 was taken during an eclipse in India in 1980 and Fig. 1.6.15 in France in 1999. Note that the pinkish colour of the *prominences* in the chromosphere is just like the H I regions in M51, for the reason that they are produced by the same process – they are glowing hydrogen gas.

Just as glowing hydrogen emits light of only a few characteristic wavelengths, the gas also absorbs light at just the same wavelengths. If visible radiation with a continuous spectrum passes through a gas, the gas absorbs just these wavelengths.

Fig. 1.6.10 Flame test showing calcium and the standard colours of some other ions

Colour of the flame	Element
Colourless	Mg, Be
Red	Li
Crimson	Sr
Brick red	Ca
Red-purple	Rb
Purple	K
Yellow	Na
Apple green	Ba
Dark green	Cu
Blue	Cs

Black body

Hydrogen

Calcium

Fig. 1.6.12 Atomic emission spectra

Terms & definitions

The variation in intensity of radiation with wavelength due to absorption by a material is called the **absorption spectrum**.

Fig. 1.6.13 shows the visible part of the **absorption spectrum** of hydrogen (it also emits and absorbs in the UV and IR). The diagram also shows the relationship between the image spectrum and the graphical representation.

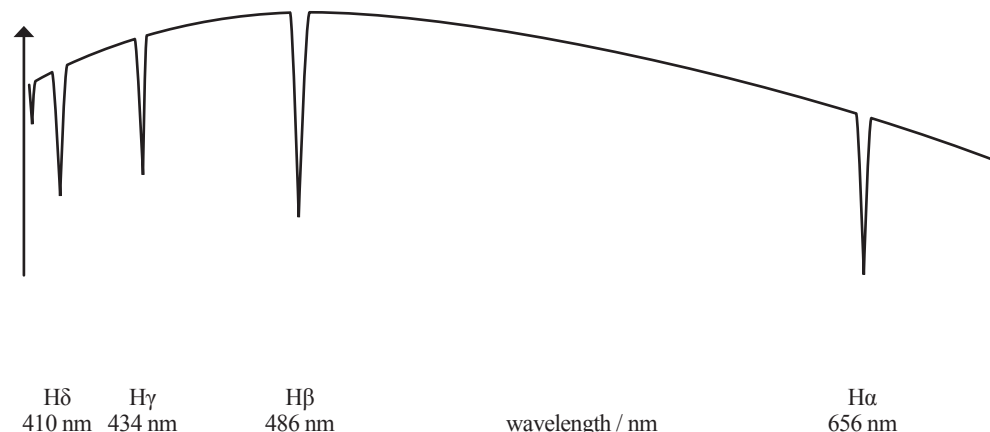


Fig. 1.6.13 Hydrogen absorption spectrum

Fig. 1.6.14 Sun's corona

Fig. 1.6.15 Sun's chromosphere

1.6.9

Self-test

Use the following emission spectra wavelengths (in nm) to identify the labelled lines in Fig. 1.6.16:

- Hydrogen ($H\alpha$) 656, ($H\beta$) 486
- Oxygen (O_2) 759, 687
- Sodium (NaI) 589, 590
- Iron (FeI) 440, 441, 452, 489, 492, 496, 525, 527
- Calcium (CaI) 610, 612
- Ionised calcium ($CaII$) 397, 393
- Ionised barium ($BaII$) 455
- Magnesium (MgI) 470, 518, 552

The labels, $H\alpha$ - $H\delta$, are the names that astronomers give to the absorption lines. The lines clearly form the same pattern as those of the emission spectrum above.

The solar spectrum in Fig. 1.6.3 shows a vast number of **Fraunhofer lines** because of the large number of elements present in the solar atmosphere. The next diagram shows a simplified solar spectrum with the most prominent Fraunhofer lines. Self-test 1.6.8 gives some wavelengths in the spectra of various elements, which you can use to identify elements present in the Sun.

Fig. 1.6.16 Simplified solar spectrum

(c) 'Missing lines' and temperature

From Self-test 1.6.9, you'll notice that some of the lines which exist in the emission spectrum of an element (or ion) do not apparently occur in the solar spectrum, although other lines of the same element do. For example, the **470 nm** line for atomic magnesium is missing but the **518 nm** is identifiable. The reason for this is that, in order to absorb a particular photon the atom must be in the lower of two energy states which have a difference in energy equal to the energy of the photon. This is dealt with in detail in Section 2.7. If the temperature is too high, there will be very few atoms in the lower energy state (violent collisions will put them into a more excited state) so they won't be available for absorbing the photon; if the temperature is too low, even the lower energy state might be too high to have any significant population.

Observing which lines are present, and their prominence, gives astronomers information of the temperature of the gas which is responsible for the absorption spectrum.

Stretch & Challenge

Planck derived the formula $\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$ for Stefan's constant, where the energy E_f of a photon is $E_f = hf$, and the KE, E_k , of a molecule is given by $E_k = \frac{3}{2} kT$. Investigate this dimensionally and numerically.

1.6.5 Radio astronomy

This is far too big a topic to do it justice here. In Fig. 1.6.17 we see the famous Jodrell Bank telescope which was instrumental in establishing radio astronomy as an academic field. Most observations are made in the microwave window – the atmosphere is transparent to short wavelength radio waves and microwaves.

(a) Cosmic microwave background radiation (CMBR)

Previously predicted, this was accidentally discovered by two US radio astronomers, Penzias and Wilson, in 1964. It represents the extremely red-shifted radiation from the Big Bang which permeates space. When the universe was younger than 380 000 years, all its atoms were ionised because of its high temperature. At this time the universe cooled sufficiently (~ 3000 K) for hydrogen atoms to form and space became transparent to radiation. The CMBR is this radiation, the spectrum of which is shown in Fig. 1.6.18.

The graph corresponds closely to that of a black body with a temperature of 2.73 K. The error bars for the observations in Fig. 1.6.18 are 400 $\times$ the normal length – otherwise they would not be visible. The effective temperature of the CMBR is the same to 1 part in 10^5 in different directions, which is shown in Fig. 1.6.19: the small fluctuations are the seeds from which the structures we observe in the universe (galaxies, clusters of galaxies) have grown.

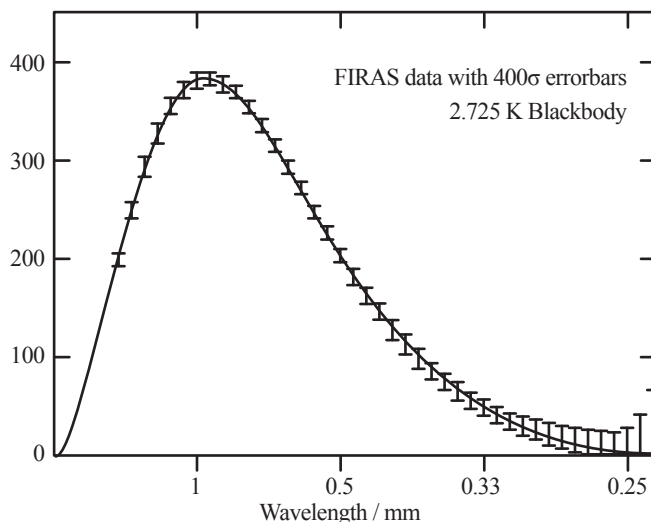


Fig. 1.6.18 CMBR spectrum

(b) The 21 cm hydrogen line

In its ground state, the electron of atomic hydrogen can either have its spin parallel to that of the nucleus or antiparallel (i.e. in the opposite direction). As the atom flips between these states it either absorbs or emits radiation with the specific wavelength of 21.1 cm (3 s.f.). Radiation with this wavelength indicates the presence of atomic hydrogen (HI) and the sensitivity of radio telescopes enables astronomers to detect very tenuous hydrogen clouds on the edges of galaxies – and their orbital velocity can be measured using the **Doppler shift** and provides evidence for **dark matter**.

Fig. 1.6.17 Jodrell Bank radio telescope

Self-test

1.6.10

Use Fig. 1.6.18 to show that the black body temperature of the CMBR is ~ 2.7 K.

Stretch & Challenge

By what factor has the universe grown since it was 380 000 years old?

[Hint: as the universe grows, the wavelength of any light travelling through it stretches in proportion. Use the decoupling temperature of matter and radiation.]

Fig. 1.6.19 10^{-5} K temperature fluctuations

Terms & definitions

Doppler shift is a change in the wavelength (or frequency) of radiation caused by the motion of the source.

Terms & definitions

Dark matter is hypothesised material, detectable only by its gravitational effects, needed to account for the rate of rotation of galaxies.

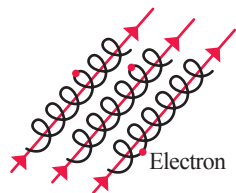


Fig. 1.6.20 Spiralling electrons emit radiation

(c) Synchrotron radiation

As with 21 cm HI radiation, this is not thermal radiation so it doesn't have the Planck spectrum. It is caused by electrons spiralling around magnetic field lines as in Figure 1.6.20. Wide varieties of astronomical objects generate intense magnetic fields and give off synchrotron radiation. Planets generate synchrotron radiation at radio wavelengths. Neutron stars have strong magnetic fields and give off 'searchlight beams' of synchrotron radiation: if these beams flash over the earth we see these objects as pulsars. In the intense magnetic fields generated by supernovae, the synchrotron radiation can be in the visible and even UV part of the spectrum.

1.6.6 Multiwavelength astronomy

The various regions of the electromagnetic spectrum give us information about different processes in the universe. We have seen that most of the power of the Sun is emitted in the form of near infrared, visible and near ultraviolet radiation. This is because the temperature of the photosphere is approximately 5800 K. The higher the temperature of an object, the shorter the wavelengths of the continuous spectrum the object emits.

The lower levels of the Sun's chromosphere have temperatures similar to that of the photosphere but the temperature rises with distance from the Sun's surface and the solar corona reaches over 10^6 K. In a solar flare, the temperature can reach tens of millions of K.

We saw in Section 1.6.5 that some non-thermal processes result in the emission of radiation: 21 cm HI and synchrotron radiation. These can give us additional information about hydrogen clouds and about magnetic fields. So a study of radiation across the e-m spectrum provides us with much more information than observations in one spectral region alone.

Consider the images in Fig. 1.6.21 of the spiral galaxy, M81. The different regions of the electromagnetic spectrum reveal different processes.

(a) X-ray

(b) UV

(c) visible

(d) Infrared

(e) 21cm HI

Fig. 1.6.21 Images of M81 in various wavebands

Image (c) in visible light is the familiar astronomical image of a spiral galaxy. It is also high definition. The spiral arms are well shown, as is the central bulge. The stars in the centre are predominantly old, low mass stars and appear yellowish. Lanes of dust are also visible. The ultraviolet image, on the other hand, picks out hotter regions and the image shows knots of young giant stars forming well away from the centre. The infrared image shows regions where stars are heating up dust especially in the spiral arms.

The X-ray image only displays very high temperature regions. The bright knot in the centre is heated by matter spiralling into the giant black hole at the heart of the galaxy. The other two bright blobs below are actually not part of the galaxy at all. These are vastly more distant *quasars* which happen to be behind M81. They are not visible in any of the other images. Image (e) shows neutral hydrogen by its 21 cm signature emission. It is clearly missing from the centre of the galaxy.

Some indication of the ability of 21 cm radio astronomy to reveal processes which are not detectable at other wavelengths is in Fig. 1.6.22. This shows M81 again but this time with some smaller neighbouring galaxies. Encounters between the galaxies have resulted in long filaments of hydrogen being pulled out into intergalactic space. Only the sensitivity of 21 cm radio telescopes allows this to be imaged and the dynamics of galactic tidal interactions to be studied.

Exercise 1.6

1. A white dwarf star has a temperature of 24 000 K and a diameter of 14 000 km. Calculate
 - (a) its luminosity and
 - (b) the peak wavelength of its spectrum.
2. Without using a calculator, compare the luminosity and peak wavelength of the white dwarf in Q1 with those of the Sun. Take the temperature and diameter of the Sun to be 6 000 K and 1.4 million km respectively.
3. By calculation suggest the regions of the e-m spectrum which are appropriate for studying processes which take place at:
 - (a) 10 K, (b) 10^3 K, (c) 10^5 K (d) 10^7 K.
4. How would the scales of Fig. 1.6.6 need to be changed to include a black body spectrum of 6 000 K (the approximate temperature of the Sun).
5. A red giant star has a diameter 1000 times that of a red dwarf star of the same surface temperature. Compare their distances from the Earth given that the red giant appears 100 times as bright. Show your working.
6. The central bulges of spiral galaxies consist mainly of old stars. Very little star formation is taking place. How does this tie in with the absence of 21 cm emission from the centre of M81?
7. Interstellar dust particles are typically 0.1–1 μm in size. Stars are formed from cold molecular clouds which contain dust particles. Explain why stellar formation is most easily observed using infrared radiation.
8. The discs of dust and gas around young stars and from which planetary systems are thought to develop are heated up (to several 100 K) by their parent star. Suggest how this is detected in the spectrum of the star?
9. The radiation from young hot stars heats up nearby clouds of atomic hydrogen (HI). Explain the appearance of the HI regions in M51 (Fig. 1.6.11) in terms of the emission spectrum of hydrogen (Fig. 1.6.12).
10. An X-ray pulsar is a neutron star (the remnant of a supernova) which pulls gas from the surface of its red giant companion star. This gas spirals in to the neutron star in an *accretion disc*, which is in the plane of equator of the star. The point of impact with the surface of the neutron star is heated to $\sim 10^7$ K, the position of which rotates with the star, which has a rotation period of less than 1 s. Describe how this would appear to a distant astronomer who observes from an angle well away from the axis of rotation.

Fig. 1.6.22 Tidal effects in the M81 group

1.7 Particles and nuclear structure

Terms & definitions

The word **atom** comes from the Greek word *atomos* (atomos) which means indivisible.

Terms & definitions

A particle is **elementary** (or fundamental) if it is not a combination of other particles.

Until the late 19th century, the atom was regarded as an elementary particle. The periodic table of the elements, first published by the Russian chemist, Dmitri Ivanovich Mendeleev in the 1860s, strongly suggested an underlying structure to atoms and by the end of the century, the negatively charged electron had been identified as a universal component of atoms. The positively charged atomic nucleus, which contained virtually all the mass of the atom, was discovered from the work of Rutherford, Geiger and Marsden between 1908 and 1913 and both its main constituents – protons and neutrons – identified by the early 1930s and initially thought to be elementary.

The development of understanding of atomic structure since that time is shown schematically for a deuterium (heavy-hydrogen) atom in Fig. 1.7.1. With the development of quantum theory in the 1920s it was realised that electrons occupied a region a fraction of a nanometre across which is typically $100\,000 \times$ the diameter of the nucleus. This was shown in the 1930s to consist of protons (p) and neutrons (n), which are collectively known as nucleons. The results of collision experiments known as *deep inelastic scattering* in the 1960s and 1970s showed that nucleons are composed of 3 particles called quarks bound together by the so-called *strong interaction*. These quarks are thought to be elementary particles.

The middle decades of the 20th century produced a veritable smorgasbord of particles, called variously hadrons, fermions, bosons, mesons, nucleons, baryons, neutrinos.... The standard model of particle physics goes a long way towards simplifying this picture: this will be described briefly here.

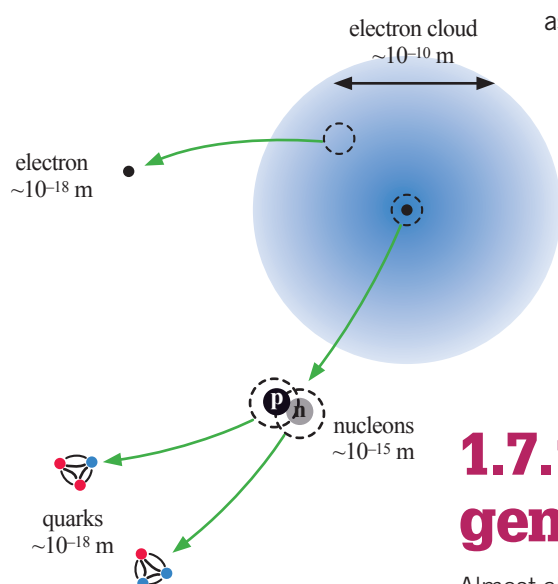


Fig. 1.7.1 The structure of a deuterium atom

1.7.1 The standard model – three generations of leptons and quarks

Almost all the normal matter in the universe (that is, ignoring the mysterious *dark matter* which is dealt with later) is composed of heavy protons and neutrons and the much lighter electrons. We infer the existence of other almost massless particles called neutrinos (see Section 1.7.4) and notice that, if we bash protons and neutrons together, we produce showers of other intermediate mass particles, which we call **mesons**. When cosmic ray particles collide with atoms in the upper atmosphere they produce showers of particles, called muons, which form part of the background radiation which we can detect using a Geiger-Müller tube. We now know that the electrons, muons and neutrinos (and some others) are elementary particles, which we call **leptons**. The other particles, the heavy ones, are called **hadrons**, and are not elementary – they are composed of combinations of quarks.

Table 1.7.1 contains the elementary particles in the **standard model**.

1.7.2 Units of mass and energy

Particle physicists usually express energy in **electron volts** (eV) or its multiples, keV, MeV, GeV and TeV. This makes calculations of kinetic energy gain very straightforward: an electron accelerated through a pd of 100 V gains a kinetic energy of 100 eV; a particle with a charge of $2e$ (e.g. an alpha particle) would gain a kinetic energy of $2 \times 100 = 200$ eV.

Terms & definitions

Lepton – low mass, elementary particles, e.g. electron, neutrino.

Quark – elementary particle, not found in isolation, which combines to form hadrons and baryons, e.g. up, down.

Hadron – high mass particle consisting of *quarks* and/or *anti-quarks*.

Baryon – hadron composed of 3 quarks, e.g. proton, neutron.

Antibaryon – hadron composed of 3 antiquarks, e.g. antiproton.

Meson – hadron composed of a quark and an antiquark, e.g. pion.

Tab. 1.7.1 Standard model particles

Generation	Leptons		Quarks	
1st	electron Symbol: e^- charge: $-e$	electron neutrino Symbol: ν_e charge: 0	up Symbol: u charge: $\frac{2}{3}e$	down Symbol: d charge: $-\frac{1}{3}e$
2nd	muon Symbol: μ^- charge: $-e$	muon neutrino Symbol: ν_μ charge: 0	charm Symbol: c charge: $\frac{2}{3}e$	strange Symbol: s charge: $-\frac{1}{3}e$
3rd	tauon Symbol: τ^- charge: $-e$	tauon neutrino Symbol: ν_τ charge: 0	top Symbol: t charge: $\frac{2}{3}e$	bottom Symbol: b charge: $-\frac{1}{3}e$

Particle masses are also expressed in a combination of energy units and the speed of light c , using the Einstein relationship, $E = mc^2$.

Rearranging this gives $m = \frac{E}{c^2}$, which allows us to write the mass of a particle in the unit eV/c^2 . In these units, the electronic mass, m_e is $0.511 \frac{\text{MeV}}{c^2}$. The conversion to SI units proceeds as follows:

$$m_e = 0.511 \frac{\text{MeV}}{c^2} = \frac{0.511 \times 1.602 \times 10^{-13} \text{ J}}{(2.998 \times 10^8 \text{ m s}^{-1})^2} = 9.11 \times 10^{-31} \text{ kg}.$$

1.7.3 Antiparticles

Antimatter is not only the stuff of science fiction. For each of the particles in Table 1.7.1, there is corresponding antiparticle with an identical mass; if the particle has a charge, the antiparticle has an equal and opposite charge. The symbol for most antiparticles that you will meet is formed by putting a bar over the symbol for the particle, e.g. $\bar{u}$, $\bar{\nu}_e$, $\bar{p}$ for the anti-up quark, the electron antineutrino [or 'anti-electron neutrino'] and the antiproton respectively. The exceptions are the antiparticles of the electron, muon and tauon which are written e^+ , μ^+ and τ^+ respectively. The antielectron has its own name: the *positron*.

When a particle and its antiparticle interact they annihilate each other; that is they disappear and their mass-energy manifests itself as two photons of electromagnetic radiation. These photons are given the symbol γ because they are at the very high energy end of the e-m spectrum. The total energy of the photons is equal to the sum of the mass-energy and kinetic energy of the annihilating particles.

Example

An electron and a positron collide head on and annihilate. Each particle has a kinetic energy of 100 keV. Calculate the energy of each of the photons produced.

Answer

Electron mass = $511 \text{ keV} / c^2$, so the mass energy is 511 keV.

$$\begin{aligned} \therefore \text{Total energy} &= \text{total mass energy} + \text{total kinetic energy} \\ &= 2 \times 511 \text{ keV} + 2 \times 100 \text{ keV} \\ &= 1222 \text{ keV} \end{aligned}$$

$\therefore$ Each photon has energy $\frac{1}{2} \times 1222 \text{ keV} = 611 \text{ keV}$

Exam tip

The Eduqas specification requires knowledge that there are three generations of particles but questions involving interactions will be restricted to the first.

Terms & definitions

The **elementary charge**, e , is the charge on the proton and has a value of $1.602 \times 10^{-19} \text{ C}$ (4 s.f.). The charge on the electron is $-e$.

Terms & definitions

The **electron volt** is the energy transfer when an electron moves through a potential difference of one volt. Using the definition of a volt, $W = QV$,

$$\therefore 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$\text{Also } 1 \text{ keV} = 1.602 \times 10^{-16} \text{ J}$$

$$\text{and } 1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}$$

Self-test

1.7.1

To convert from MeV/c^2 to kg we multiply by 10^6 and divide by c^2 . Invert this method to express the proton mass of $1.672 \times 10^{-27} \text{ kg}$ in GeV/c^2 .

Study point

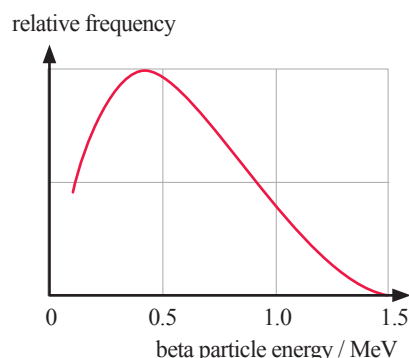
If an electron and a positron annihilate, they produce two γ photons. These are emitted in opposite directions – otherwise momentum would not be conserved.

Self-test

1.7.2

Calculate the wavelength, frequency and momentum of the photons in the example. [See Section 2.7 for the momentum of a photon.]

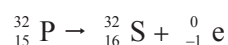
The opposite process can also happen: if it possesses enough energy, a high energy photon can create an electron-positron pair. It also needs to interact with another particle (usually an atomic nucleus) to enable energy and momentum to be simultaneously conserved. In Fig. 1.7.2 a high energy photon enters from the top and interacts with a hydrogen atom at A (it is in a bubble chamber, which consists of a tank of liquid hydrogen), ejecting a high energy electron, creating a low energy electron-positron pair and a second photon which continues to B where it creates a second (higher energy) e^-e^+ pair. A magnetic field at right angles to the page makes the charged particles travel in curves: the opposite charges of electrons and positrons results in the typical 'ram's horn' effect at A.

Fig. 1.7.2 e^-e^+ pair productionFig. 1.7.3 P-32 β -spectrum

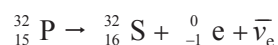
1.7.4 The evidence for neutrinos

Neutrinos are neutral particles of very low mass which only interact via the weak force. This means that they need to come within $\sim 10^{-18}$ m to interact, so interactions hardly ever happen: for example, a typical solar neutrino could expect to penetrate 1–2 light-years thickness of lead!

The first evidence for their existence came from studies in the 1930s of the energy spectrum of beta particles. Phosphorus-32, $^{32}_{15}\text{P}$, decays by β^- emission. Before neutrinos were known the complete reaction was expected to be:



The energy release in the decay is 1.5 MeV. If we apply the principle of conservation of momentum, we can calculate that the beta particles should take nearly all the energy (>99.9%), with the much heavier sulphur nucleus taking a tiny fraction. Compare this with the actual energy spectrum in Fig. 1.7.3: 1.5 MeV is indeed the maximum beta particle energy but there is a continuous spectrum of energies with the peak being less than 0.5 MeV. This energy spectrum is only possible if a third particle is also produced, which can share the energy with the beta particle. This particle is called the neutrino (strictly, the electron anti-neutrino) and the complete reaction is:



The photograph in Fig. 1.7.4 is of the β -decay of a nucleus of He-6. As with Fig. 1.7.2 this is in a bubble chamber. The curved track is the β -particle; the short fat track, the recoil of the resulting Li-7 nucleus. In order to conserve momentum a particle (a neutrino) must be emitted downwards in the photograph. The neutrino doesn't interact and so leaves no track.

Fig. 1.7.4 He-6 decay in a bubble chamber

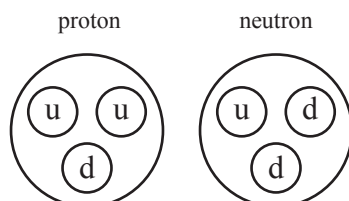


Fig. 1.7.5 Quark structure of nucleons

1.7.5 Building heavy particles

Electrons, being leptons, are elementary particles; that is they are not composed of other particles. **Hadrons**, e.g. protons and neutrons (which are together also called nucleons), on the other hand, are composed of quarks, bound together with the **strong force** (see Section 1.7.6). Evidence for the existence of quarks is indirect. Single quarks are never detected. They are always seen in combination (see Section 1.7.8).

There are three different types of hadron:

- **Baryons**, such as protons and neutrons, are composed of three quarks. First generation baryons are composed entirely of a mixture of up (u) and down (d) quarks.
- **Antibaryons** such as antiprotons are composed of three antiquarks.
- **Mesons**, are composed of a quark and an antiquark.

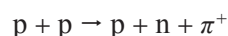
The quark structure of protons and neutrons is shown schematically in Fig. 1.7.5 and can be summarised by

$$p = uud \quad n = udd$$

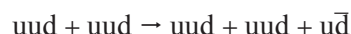
Note that, in particle physics, the proton has the symbol **p** rather than ${}^1_1\text{H}$, which is usual in nuclear physics; the neutron is **n** rather than ${}^1_0\text{n}$. Also the order of writing the quarks is arbitrary: **p** = **udu** and **n** = **ddu**, etc., are perfectly good ways of writing the structure. Protons are the only stable baryons: there are theories which suggest they could be unstable with a half life of around 10^{32} years!

Mesons are created in copious numbers when baryons are collided at moderate to high energies (more than a few hundred MeV). The first generation mesons are called **pions** (or pi mesons). Their names and quark structure are given in the definition:

A typical meson-generating reaction is



which, at the quark level, can be written:



There are only six first-generation baryons. They are summarised in Table 1.7.2:

Tab. 1.7.2 1st generation baryons

Family	baryons
Nucleons	p (uud); n (udd)
Δ particles	Δ^{++} (uuu); Δ^+ (uud); Δ^0 (udd); Δ^- (ddd)

The symbol, Δ , is the Greek letter capital delta, so the Δ family is called delta double plus, delta plus, etc. Note that the quark structures of Δ^+ and Δ^0 are the same as those of **p** and **n** respectively but the mass of the Δ^+ is $1232 \text{ MeV}/c^2$ against $938 \text{ MeV}/c^2$ for the proton. The Δ^+ can be regarded as an excited state of the proton: similarly for Δ^0 and the neutron. This is discussed in the Stretch & challenge.

1.7.6 Interactions (forces) between particles

Macroscopic objects are subject to two types of force: gravitational and electromagnetic. Subatomic particles are also affected by two other forces: the strong and the weak interactions. These are not experienced at all on the everyday scale because their range is so small. The four forces are summarised in Table 1.7.3 in order of increasing strength.

Tab. 1.7.3 Interactions summarised

Interaction	Affects	Range	Comments
gravitational	all matter	infinite	negligible for subatomic particles
weak	all particles	$\sim 10^{-18} \text{ m}$	only significant when e-m and strong interactions not involved
electromagnetic (e-m)	all charged particles	infinite	also affects neutral hadrons because quarks have charges
strong	all quarks	$\sim 10^{-15} \text{ m}$	also affects interactions between hadrons (e.g. nuclear binding)

Study point

Because mesons are composed of a quark and an antiquark, there is no need to define a separate category of 'antimeson'.

Study point

The antiproton and antineutron have the following structures:

$$\bar{p} = \bar{u}\bar{u}\bar{d}; \quad \bar{n} = \bar{u}\bar{d}\bar{d}.$$

Study point

Notice that the charge on the two sides of the reaction is the same, as is the number of u-quarks (4); there are 2 d-quarks on the left and 3 d + 1 anti-d on the right. This is explored further in Section 1.7.6.

Terms & definitions

Pions have the following names and structure:

$$\pi^+ \text{ (pi plus)} = u\bar{d}$$

$$\pi^- \text{ (pi minus)} = d\bar{u} \text{ (or } u\bar{d}!)$$

π^0 (pi zero) = $u\bar{u}$ or $d\bar{d}$. A beam of π^0 mesons is composed of a mixture of the two!

Stretch & Challenge

The additional mass is related to the excitation energy. This effect also occurs in atomic energy levels. The first excitation energy of atomic hydrogen is 10.2 eV. By what fraction does the mass of a hydrogen atom increase if it is put into its first excited state?

Self-test

1.7.3

Heavier nuclei need a greater fraction of neutrons to overcome the increased e-m repulsion of the protons. Illustrate this from the fraction of neutrons in the stable nuclei, ${}^{12}_6\text{C}$, ${}^{56}_{26}\text{Fe}$ and ${}^{197}_{79}\text{Au}$.

Study point

The word *interaction* is often preferred to *force* because it has a wider implication than just attraction or repulsion. It includes the control of the creation of particles or their decay.

Exam tip

In considering which force is responsible for a reaction we also have to have an eye on the conservation laws (Section 1.7.7).

Exam tip

In a decay reaction, the stronger the force, the shorter is the decay time.
In a collision reaction, the stronger the force, the more likely the reaction is to occur.

1.7.4

Self-test

In terms of energy conservation, why can a neutron (mass $939.6 \text{ MeV}/c^2$) decay into a proton ($938.3 \text{ MeV}/c^2$) an electron ($511 \text{ keV}/c^2$) and a neutrino, but an isolated proton cannot decay into a neutron, a positron and a neutrino?

The stability of atoms arises from the three non-gravitational forces:

- Electrons are bound to the nucleus by the electromagnetic force.
- Protons and neutrons are held together in the nucleus by the strong force which opposes the e-m repulsion of the protons.
- The weak force is responsible for the decay of neutrons in neutron-rich nuclei, giving rise to β^- decay.

Generally the interaction responsible for any particular interaction is the strongest one which is felt by all the particles on both sides of the equation. For example:

- All the particles in the reaction in Section 1.7.5 are composed of quarks. It is controlled by the strong interaction meaning that it is likely to happen.
- Neutrinos only feel the weak force, so any interaction with neutrons in it (e.g. β -decay) must be weak. This results in the high ability of neutrinos to penetrate matter.

The different strengths of the interactions are illustrated by the different decay times of particles and the force responsible. The following are examples:

Strong	$\Delta^- (\text{ddd}) \rightarrow \text{n} + \pi^-$	lifetime $\sim 10^{-24} \text{ s}$
Electromagnetic	$\pi^0 (\text{u}\bar{\text{u}}) \rightarrow \gamma + \gamma$	lifetime $\sim 10^{-12} \text{ s}$
Weak	$\text{n} (\text{udd}) \rightarrow \text{p} + \text{e}^- + \bar{\nu}_{\text{e}}$	lifetime $\sim 15 \text{ min}$

1.7.7 Conservation laws in particle physics

The familiar conservation laws of energy and momentum apply in particle physics, though they have to take into account the relativistic speeds of the particles. The conservation of charge is a universal law and there are additional rules – one of which is sometimes broken!

(a) Conservation of lepton number

For the first generation of leptons, i.e. the electron family, we assign each of them a number, which we call the *electron lepton number*, (L_{e}), as shown in the table. The value of L_{e} for all other particles is zero. We find experimentally that this lepton number is always conserved, i.e. if we add the lepton number for the reactants and for the products, the numbers are always the same.

Particle	L_{e}
e^-	-1
e^+	-1
ν_{e}	1
$\bar{\nu}_{\text{e}}$	-1

Conversely, if a proposed reaction would violate the conservation of L_{e} we can be just as sure that it is impossible as if it violated the conservation of energy.

The other generations of leptons, the muon and tauon families, have their own lepton numbers, L_{μ} and L_{τ} , which are defined in the same pattern as L_{e} . These are separately conserved, as shown by the muon decay reaction:

$$\mu^- \rightarrow \text{e}^- + \bar{\nu}_{\text{e}} + \nu_{\mu}$$

$L_{\text{e}} = 0$ on both sides of the equation and $L_{\mu} = 1$ on both sides; thus both numbers are conserved.

1.7.5

Self-test

Show that neutron decay (Section 1.7.5) is allowed by the conservation of electron lepton number. Hint: quarks have zero lepton number.

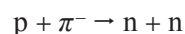
1.7.6

Self-test

Positive muons (μ^+) decay into positrons and neutrinos. Write the decay equation and show how L_{e} and L_{μ} are each conserved.

(b) Conservation of baryon and quark numbers

Similarly to lepton number we define a baryon number, B . Each baryon, e.g. proton, has $B = 1$; antibaryons, have $B = -1$; leptons and mesons have $B = 0$. Again, baryon number is always conserved. The following reaction could not happen:

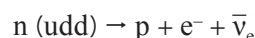


even if the p and π^- had enough kinetic energy and even though it conserves charge and lepton number. Why not? Because the total baryon number on the left is 1 and on the right it is 2!

The conservation of baryon number is really a special case of the conservation of quark number, Q . Looking again at our 'impossible reaction' and assigning a quark number of -1 to antiquarks, we can tally the quarks as follows:

$$\text{Left-hand side: } Q = 3 + (1 - 1) = 3 \quad \text{Right-hand side: } Q = 3 + 3 = 6$$

On the other hand the Q totals for the $p + p \rightarrow p + n + \pi^+$ are the same on both sides. Looking more closely we see that the individual quark numbers, U (up) and D (down) are also conserved, with the usual convention that an antiquark has a value -1 : on each side $U = 4$ and $D = 2$. Individual quark numbers are conserved in strong and e-m interactions, but **can be changed by ± 1 in weak interactions**. Looking again at neutron decay:



Writing this in terms of quarks: $udd \rightarrow uud + e^- + \bar{\nu}_e$,

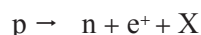
$$U = 1 \quad 2 \quad 0 \quad 0$$

$$D = 2 \quad 1 \quad 0 \quad 0$$

We see that U changes from 1 to 2 and D from 2 to 1. The total quark number Q is 3 on both sides but one of the quarks has changed its flavour from down to up.

Exercise 1.7

1. A proton, an electron and a helium nucleus are each accelerated through a potential difference of 500 V . State the increase in kinetic energy of each
 - (a) in eV and
 - (b) in J .
2. The masses of atomic particles are often expressed in terms of the *atomic mass unit*, u which has a value of $1.660 \times 10^{-27} \text{ kg}$. The neutron has a mass of $1.008\,665 \text{ u}$. Calculate the mass of a neutron
 - (a) in kg ,
 - (b) in MeV/c^2 [$c = 2.998 \times 10^8 \text{ m s}^{-1}$].
3. The radioactive isotope $^{13}_7\text{N}$ decays by the emission of a positron, when one of the protons in its nucleus transforms into a neutron. The equation is:

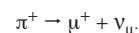


where X is an unidentified particle. The half life of the decay is 10.1 minutes.

- (a) Identify X and justify your choice in terms of relevant conservation laws.
- (b) State which of the interactions controls this decay. Justify your choice.
- (c) An isolated proton cannot decay into a neutron in this way. Explain which conservation law would be violated.

Study point

The number of mesons is not conserved. The most common π^+ decay mode (>99%) is:



The reason for non-conservation of mesons is that they each consist of a $q\bar{q}$ pair so $Q = 0$.

Self-test

1.7.7

For the decay: $\pi^+ \rightarrow \mu^+ + \nu_\mu$,

- (a) Explain what interaction is responsible.
- (b) Show what conservation laws are demonstrated.

Terms & definitions

The different types of quark, up, down, etc., are said to possess different **flavours**. This strange (!) use of the word is possibly related to the German sour milk product which is often flavoured with fruit.

Exam tip

Indications of a weak force are:

1. Leptons are involved: they don't feel the strong force.
2. Neutrinos are involved: they don't feel the e-m force.
3. If quarks are involved, a change in quark flavour occurs.
4. If it is a decay, the lifetime is greater than $\sim 10^{-10} \text{ s}$.

4. The delta particles generally usually decay into either a proton or a neutron plus a charged pion (π^+ or π^-) in about 10^{-24} s. See the end of Section 1.7.6 for the decay of Δ^- . This decay can be written in two ways:

$$\Delta^- \rightarrow n + \pi^- \quad \text{and} \quad ddd \rightarrow udd + \bar{u}d$$

Write equations for the decays of the other Δ particles in the same way. What indications are there that these decays take place by the strong interaction?

5. A second generation baryon has the quark structure uus. It decays into first generation particles (a baryon and a meson) with a decay time of $\sim 2.6 \times 10^{-10}$ s.

- What indications are there that this decay proceeds via the weak interaction?
- There are two modes of decay: into two charged particles or into two uncharged particles. Write these equations at both the composite particle and the quark level.
- Following one decay, the meson subsequently decays into two photons. Identify, giving a reason, which of the two modes in (b) was followed.

6. One method of detecting neutrinos is to use dry-cleaning fluid! This fluid is chlorine rich: 25% of chlorine atoms are the Cl-37 isotope. Occasionally an electron neutrino (e.g. from the Sun) interacts with a particle in a Cl-37 nucleus, converting the nucleus into Ar-37. The proton numbers for chlorine and argon are 17 and 18 respectively.

- Identify the particle in the chlorine nucleus which is changed and the particle it is changed into. Explain your answer.
- Write the equation for the interaction just including the relevant particles (i.e. ignore the rest of the nuclei).
- Huge numbers of solar neutrino pass through the Earth every second. Why is this only an 'occasional' interaction?

7. Only one-third of solar neutrinos, arriving at the Earth, consist of electron neutrinos. The other two-thirds consist of muon and tauon neutrinos in roughly equal numbers. Suggest why the muon and tauon neutrinos cannot be detected as in Q6. [Masses in MeV/c^2 : $m_e \sim 0.5$; $m_\mu \sim 105$; $m_\tau \sim 1800$.]

8. The K^+ particle is a second generation meson with quark composition $u\bar{s}$. It decays into a π^+ meson and X which is another first generation particle.

- Identify X by considering the relevant conservation laws.
- Write the decay reaction, $K^+ \rightarrow \pi^+ \dots$ at the quark level.

Stretch & Challenge

The figure shows the energy spectra of the solar neutrinos arising from the following reactions:

pp: $p + p \rightarrow d + e^+ + \nu_e$

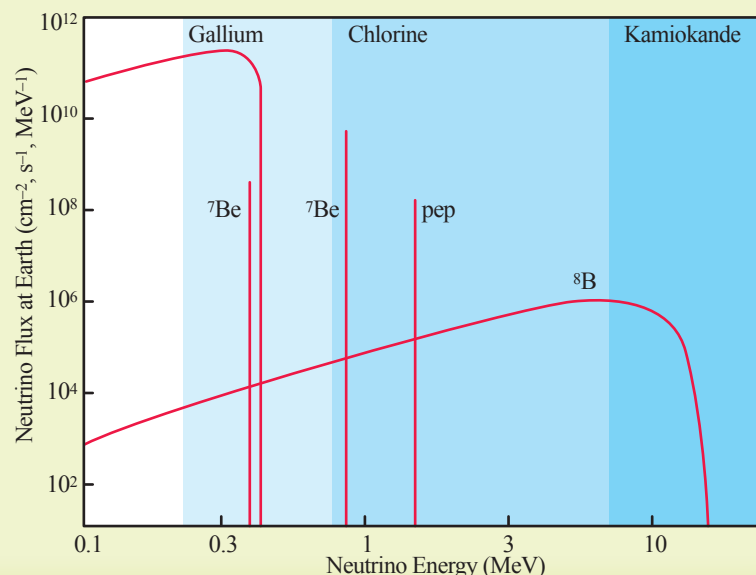
pep: $p + e^- + p \rightarrow d + \nu_e$

${}^7\text{Be}$: ${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$

${}^8\text{B}$: ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$

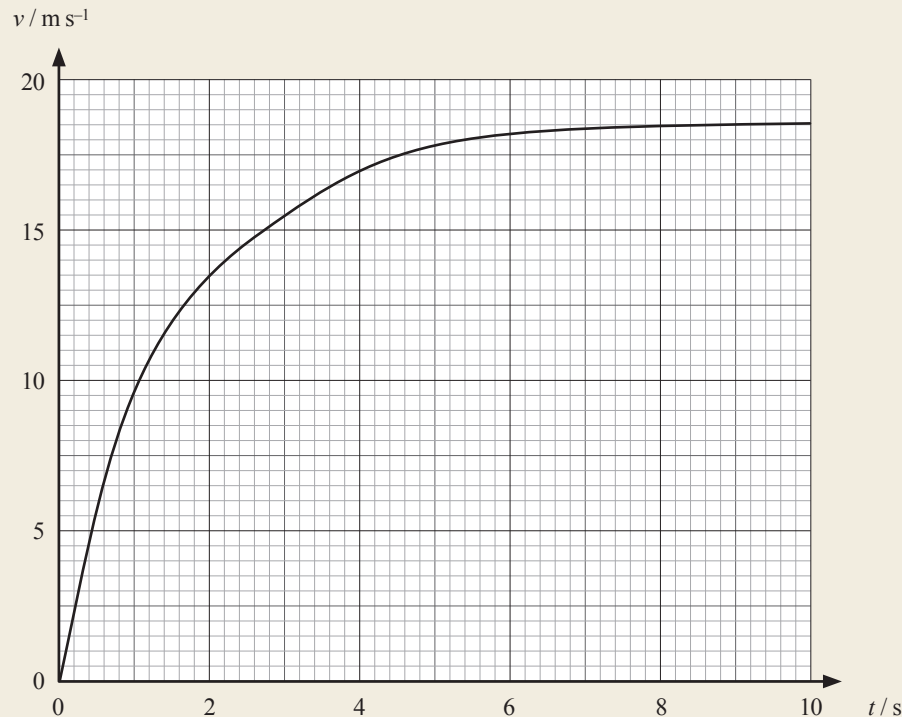
where d is a deuteron, the heavy hydrogen nucleus, ${}^2_1\text{H}$.

- Account for the fact that the pp and 8B spectra are continuous but the others each consist of single lines.
- The left-hand edge of each of the blue areas gives the low energy limit of detection of three types of neutrino detectors: gallium, chlorine and Kamiokande. By researching the detection methods, explain the energies of these lower limits.



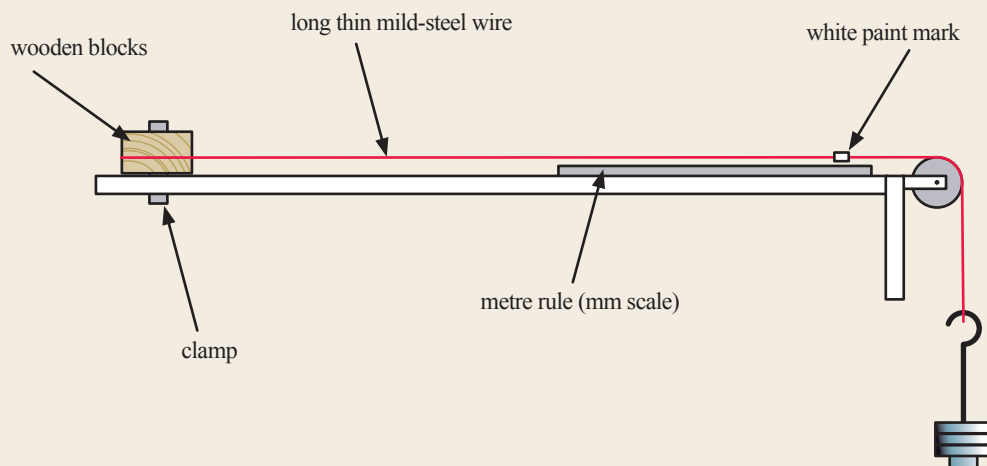
Exam practice questions

1. (a) (i) Define mean speed. [1]
 (ii) A motorcyclist enters an 80 km/h restricted zone on a motorway. There is an average speed check on this 10 km section. After 6 km the cyclist realises that he has been doing 90 km/h all the time. What must his top speed be for the remainder of the restricted section so that the mean speed is less than 80 km/h? [3]
- (b) The combined mass of the motorcycle and rider is 350 kg. The graph shows the variation of velocity, v , with time, t , over a 10 s period.



- (i) Describe, without calculation, how the **resultant force** acting on the motorcycle varies over this 10 second interval. [3]
- (ii) Determine the resultant force acting on the motorcycle at 2.0 s. [3]
 [mass of motorcycle and rider = 350 kg].
- (c) (i) A force F acts on a body moving with a velocity v . F and v are in the same direction. Starting from the definition of power, show that the power, P , is given by $P = Fv$. [3]
- (ii) When the motorcycle in part (b) is travelling at the steady velocity shown in the last part of the graph, the useful power output by the engine is 2.45 kW. Calculate the **driving force** required to maintain this velocity. [1]
- (iii) Assuming that the resistive force is proportional to the velocity, calculate the power output of the engine at 2.0 s. [3]
- (d) At a later time the motorcycle in part (b) brakes to a halt in a distance of 25 m. Calculate the mean braking force. [3]

2. A group of students proposes to determine a value for the Young modulus of mild steel using the following setup:



- (a) Starting from the definitions of stress, strain and the Young modulus, E , show that the extension, Δ , of a wire of initial length l_0 and diameter d , subject to a tensile load provided by the weight of a mass, m is given by:

$$\Delta l = \frac{4l_0 mg}{\pi d^2 E} \quad [3]$$

- (b) The students read that the value of E is of the order of 200 GPa and the yield stress is about 300 MPa. The students propose to use a wire of length (from the blocks to the mark) of ~4 m and diameter ~0.3 mm. A range of masses up to 10 kg is available.

- (i) Label the length, l , of wire which is to be used in the calculations of E . [1]

- (ii) Estimate the required mass of the load to achieve a stress in the wire of 250 MPa, i.e. close to but significantly less than the yield stress. [2]

- (iii) Consider whether these proposals are appropriate by estimating the likely values of the extension, Δl . [3]

- (iv) The students planned how to decrease the uncertainty in the final value of E . I. State why increasing the length of the wire decreases the uncertainty in E . [1]

- II. Discuss whether using a wire with a smaller diameter would result in a lower uncertainty in E . [2]

- (c) Using a different arrangement, another group of students obtained the following values:

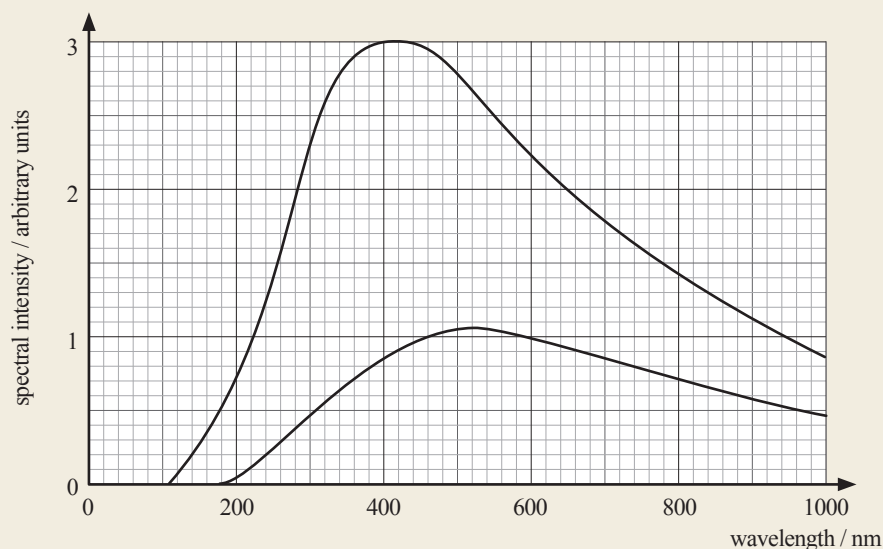
$$d = 0.272 \pm 0.012 \text{ mm};$$

$$l_0 = 1535 \pm 2 \text{ mm}$$

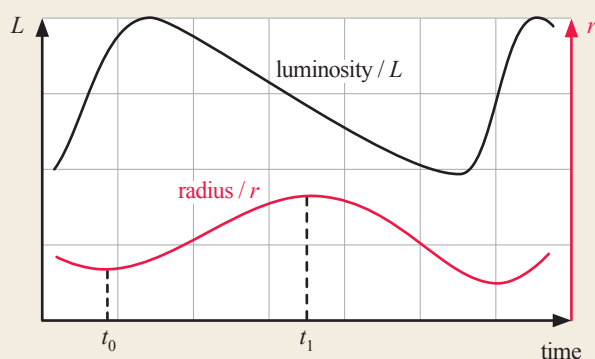
$$\text{gradient graph of } \Delta l \text{ against } m = 1.27 \pm 0.06 \text{ mm kg}^{-1}$$

- Calculate their value of E together with its uncertainty. [4]

3. Delta Cephei (δ Cep) is a variable star whose surface temperature varies between fixed maximum and minimum values. Its continuous spectrum is given below for the maximum temperature and minimum temperature.



- (a) (i) Show that δ Cep's maximum temperature, $T_{\max}$, is approximately 7 000 K. [2]
 (ii) Calculate the minimum temperature. [1]
 (iii) Apart from changes in brightness, how would expect the appearance of δ Cep to vary between its maximum and minimum temperatures. [2]
 (b) The luminosity of δ Cep at $T_{\max}$ is 1.46×10^{30} W. Calculate its diameter at this temperature. [4]
 (c) According to an accepted theory, the reason for the changes in the temperature and brightness is that δ Cep is a *pulsating* star, i.e. its periodically swells and shrinks. The graphs show the variation in luminosity, L , and radius, r , over a little more than one cycle.



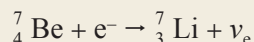
- (i) The times t_0 and t_1 are marked. Describe what is happening to the radius and luminosity around these times and use the descriptions to infer how the temperature is changing. [3]
 (ii) Mark on the graph estimates of the times of maximum and minimum temperature, $t_{\max}$ and $t_{\min}$. Justify your estimates. [2]

4. (a) The electron, the proton, the electron neutrino and the π^- meson are all first-generation particles. Some of them are fundamental particles, the others being composite particles.

Identify the fundamental particles and give the makeup of the others.

[3]

- (b) This is one of the reactions occurring in the proton-proton chain in the Sun:

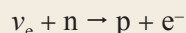


- (i) Only one of the particles in the beryllium nucleus is involved in the reaction. Identify this particle and write the reaction in terms of the quarks involved.
- (ii) Which of the nuclear interactions, strong, weak or electromagnetic, is involved in this reaction? Give two reasons for your answer.
- (iii) Which conservation laws are illustrated by this reaction? Justify your answers.
- (c) Most neutrinos produced in the core of the Sun, escape from the Sun. Some of them can be detected on Earth using radiochemical methods in which a neutrino hits a neutron in the nucleus of a suitable atom, e.g. Cl-17, and converts it into a proton:

[2]

[2]

[2]



The reaction is detected when the new radioactive nucleus subsequently decays.

The new atom, Ar-17, is more massive than the first atom, so the reaction can only proceed if the neutrino has enough kinetic energy to produce the extra mass, according to Einstein's equation $E = mc^2$.

The neutrinos produced in the reaction in part (b) have an energy of 1.38×10^{-13} J. Use the following data to decide whether these neutrinos can be detected by Cl-17.

[2]

Mass of a Cl-17 atom = 36.965 90 u ; Mass of an Ar-17 atom = 36.966 77 u;

1 u = 1.66×10^{-27} kg; $c = 3.00 \times 10^8$ m s⁻¹.

5. (a) Two of the equations of motion for constant acceleration are

$$v = u + at \text{ and } x = \frac{u+v}{2} t$$

Use these equations to show that, for an object accelerating from rest,

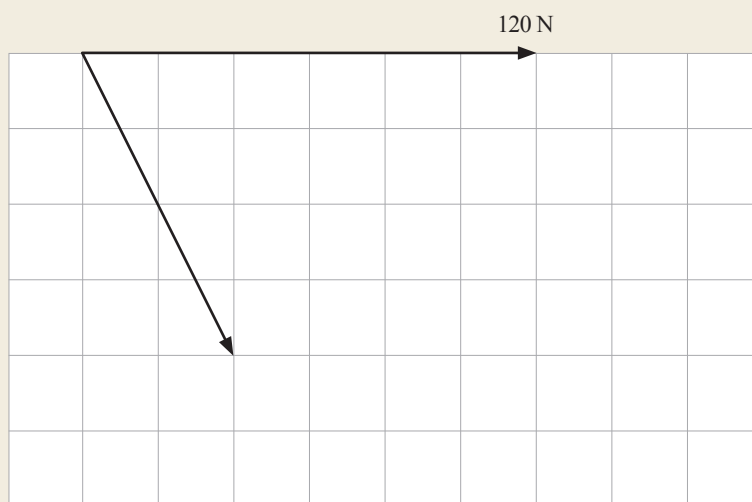
$$v^2 = 2ax$$

$$\text{and } x = \frac{1}{2}at^2$$

[4]

- (b) The combination of two velocities depends not only upon their magnitudes but also their directions.

- (i) The diagram shows a 120 N force and another force drawn to the same scale.



Complete the diagram to show the resultant force and state its magnitude and direction.

[2]

- (ii) Give the general name of quantities which add in this way.

[1]

- (c) A stone is thrown horizontally from a cliff of height 40 m. It hits the ground below at an angle of 60° to the horizontal. Ignoring the effect of air resistance, calculate the speed of projection.

[3]

Overview: Component 2 Electricity and Light

The conduction of electricity

p94

- The electronic charge, e , and the coulomb.
- Electric current is the rate of flow of charge and is expressed in amperes (A), where $A = C\ s^{-1}$.
- The mechanism of conduction in metals and the derivation and use of $I = nAve$.

Resistance

p100

- Potential difference and the unit of pd; power calculations.
- The I – V characteristics for a filament lamp and a metal wire at constant temperature.
- Ohm's law; definition and unit of resistance.
- Resistance and energy transfer in terms of free-electron collisions; temperature variation of resistance for metals.
- Resistivity.
- Superconductivity, and superconducting transition temperature; uses of superconductors; high temperature superconductors.

PRACTICAL WORK

- Investigation of the I – V characteristics of a lamp and a metal wire at constant temperature.
- Determination of resistivity of a metal.
- Investigation of the variation of resistance with temperature for a metal wire.

DC circuits

p112

- Conservation of energy and charge in a circuit; current and pd in series and parallel circuits.
- Series and parallel resistance combinations.
- Potential divider circuits, including non-linear components.
- Emf and internal resistance of a power source; calculating current and pd in circuits with one power supply or series combination of supplies.

PRACTICAL WORK

- Determination of the internal resistance of a cell.

The nature of waves

p124

- Transverse and longitudinal waves, energy transfer and polarisation.
- The terms displacement, amplitude, wavelength, frequency, period and velocity of a wave; phase differences; the wave equation $c = f\lambda$.
- Displacement–time and displacement–position graphs for transverse waves.
- Wavefronts at right angles to the direction of propagation, having all points in phase.

PRACTICAL WORK

- Investigating polarisation.

Wave properties

p130

- Diffraction – the dependence on wavelength and obstacle size.
- Two-source interference and the principle of superposition; coherence; coherent sources.
- Young's experiment, historical importance, path difference rules; Young's slits formula.
- Diffraction gratings; the derivation and use of the diffraction grating formula.
- Stationary and progressive waves; stationary waves regarded as a superposition of two progressive waves; nodes and intermodal distance.

PRACTICAL WORK

- Determination of λ using Young's double slits.
- Determination of λ using a diffraction grating.
- Determination of the speed of sound using stationary waves.

Refraction of light

p144

- Snell's law and refractive index; Snell's law related to the wave model of light propagation.
- Total internal reflection; derivation of the equation for critical angle.
- Multimode optical fibres and total internal reflection; multimode dispersion; monomode optical fibres and data transmission rates.

PRACTICAL WORK

- Measurement of the refractive index of a material.

Photons

p154

- The demonstration of and analysis of the photoelectric effect; photon explanation leading to Einstein's photoelectric equation.
- The orders of magnitude of the wavelengths and photon energies of the electromagnetic spectrum.
- Production of atomic line emission and line absorption spectra using a diffraction grating.
- Atomic energy level diagrams; ionisation energy.
- Electron diffraction, particle wave properties; the de Broglie relationship, radiation pressure.

PRACTICAL WORK

- Determination of h using LEDs.

Lasers

p166

- Stimulated emission gives coherent light.
- The need for population inversion; its achievement by pumping in 3- and 4-level systems.
- Laser structure; the advantages and uses of semiconductor lasers.

Electricity and Light

Component 2

There are three distinct, albeit interacting, areas of focus of this unit:

- **Electricity**

The nature of an electric current is explored, together with the way in which different materials and devices respond to currents. The properties of circuits and power supplies are examined, allowing students to predict their behaviours.

- **Waves**

Like electricity, wave motion forms a cornerstone of modern physics. Waves are classified and their properties examined in mathematical detail. The wave model of light is used to explain the phenomena of refraction, diffraction and interference.

- **Photons**

We now understand that, as well as having wave properties, electromagnetic radiation behaves like a stream of particles, called photons. Evidence for this is presented and this model is used, together with knowledge of atoms, to account for the atomic absorption spectra, which were introduced in Component 1. The basics of laser operation are explored.

Content

- 2.1 Conduction of electricity
- 2.2 Resistance
- 2.3 DC Circuits
- 2.4 The nature of waves
- 2.5 Wave properties
- 2.6 Refraction of light
- 2.7 Photons
- 2.8 Lasers

Practical work

Component 2 provides a wealth of opportunities, especially in the electricity and wave sections, for students to continue developing their practical skills.

2.1 Conduction of electricity

This short section gives the elementary facts about electric charge (forces between charges, why we say that electrons have a *negative* charge, conservation of charge and so on). We're then able to discuss moving charges and to relate quantitatively the velocity of charges moving in a wire to the current in the wire.

2.1.1 Electric charge

Terms & definitions

The law of conservation of charge

The net charge in a system remains constant (provided charges can't enter or leave).

2.1.1

Self-test

Calculate the number of electrons with a total charge of 1.0 coulomb.

2.1.2

Self-test

A polythene rod acquires a negative charge of 3.2 nC when rubbed with fur. Explain what happens in terms of electrons, calculating how many are involved.

Study point

The coulomb is named after Charles-Augustin de Coulomb, who discovered (in the 1780s) how the strength of the forces between small charged bodies depended on their separation.

This section is mainly about *electric charge* (simply charge from now on) flowing in electrical conductors. An electrical *conductor*, as opposed to an electrical *insulator* such as air, is indeed *defined* as a material, or a piece of material, through which a charge can flow.

Charge was first investigated with *static* – residing on the surfaces of materials that had been rubbed with other materials. [If the rubbed materials are conductors, they have to be held with insulators, so that their charges aren't conducted away, by hands, for example.]

Two sorts of charge were proposed. These were enough to account for the attractive and repulsive forces observed between *any* rubbed materials.

- Glass, after rubbing with silk, was said to have a *positive* charge.
- Amber, after rubbing with fur, was said to have a *negative* charge.

The attractive and repulsive forces then fitted the rule (see Fig. 2.1.1):

Like charges (e.g. two positives) repel, unlike charges attract.

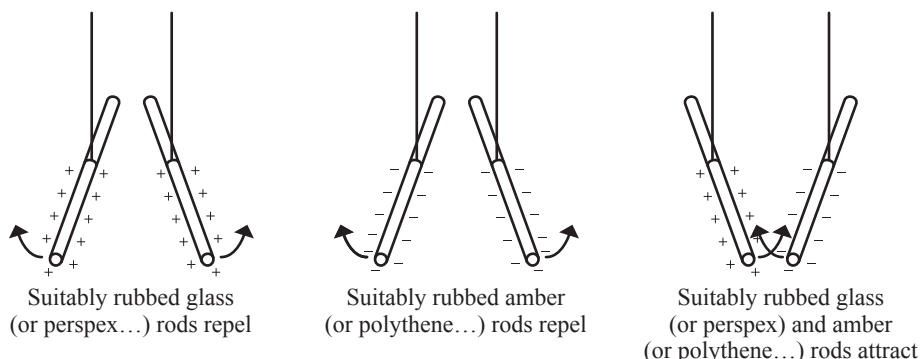


Fig. 2.1.1

Positive and *negative* are apt names. For one thing, the different charges can cancel, or neutralise, as when oppositely charged metals touch.

Over a century after these discoveries had been made, the particles inside the atom were discovered. Protons have a positive charge (according to the glass-rubbed-with-silk definition) and electrons a negative charge. So we now picture the charging of the glass as some electrons being rubbed off some of the surface atoms of the glass on to the silk.

Charge is a scalar quantity, usually denoted by Q or q . The SI unit of charge is the coulomb (C).

A proton's charge is denoted by e . An electron's charge is $-e$. Experiments show that, to four significant figures, $e = 1.602 \times 10^{-19} \text{C}$.

The law of conservation of charge (see **Terms and definitions**) has no exceptions, as far as we know. It applies even when particles are created or destroyed, for example when a neutron (no charge) 'decays' into a proton and an electron (with equal and opposite

Fig. 2.1.2
Charles-Augustin de Coulomb

charges) and an antineutrino (no charge). A more mundane case of charge conservation would be the touching together of two metal spheres, one with a positive charge, the other with a negative. Net charge (total positive charge – total negative charge) remains the same; the ‘neutralisation’ – complete or partial – is simply due to redistribution of free electrons.

2.1.2 Electric current

Charge flows through wires in an electric circuit. The flow is invisible, so how do we know that it takes place at all? One indicative experiment uses the apparatus shown in Fig. 2.1.3.

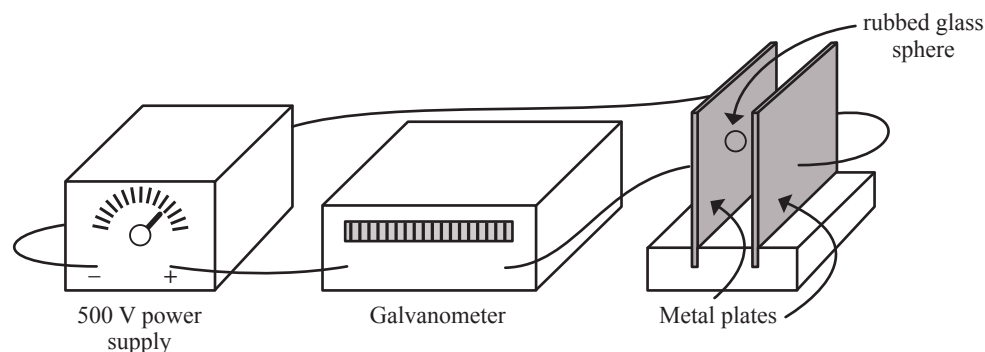


Fig. 2.1.3

When the power supply is turned on, the galvanometer (a sensitive detector of electric current) deflects briefly. If a small glass ball (hollow like a Christmas tree bauble for low mass) is now rubbed with silk and dangled on a long insulating thread in the gap, it accelerates away from one plate (which must therefore be charged positively) and towards the other (charged negatively). [Gravitational effects need to be discounted.]

The galvanometer deflection coincides with the plates acquiring charges – which must be via the connecting wires. The power supply urges electrons in one direction through the wires and its own internal conducting pathway so that some electrons are taken from one plate, and extra electrons are deposited on the other.

Before the discovery of protons and electrons, it wasn't known whether it was positive or negative charge that flowed in conductors. Scientists made a convention (an agreement) to assume it was positive. *In circuit diagrams, arrows denoting currents still show the direction in which positive charge would flow.*

We now know that in metals (by far the most commonly used conductors) it is electrons that flow – in the opposite direction to the conventional current! The positive nucleus of each atom is surrounded by most of the atom's electrons, making a positive ion. The ions vibrate randomly but about fixed positions in a regular crystal *lattice*. In most metals only a small proportion of the electrons are free to flow. For example, in copper, each atom contributes one electron to the ‘pool’ of free electrons.

From now on we shall be dealing with circuits having complete conductive paths. The simplest is shown in Fig. 2.1.5. Charge flows continuously. Note that symbols used in circuit diagrams will be labelled *the first time* they are used. You need to learn any that you don't know.

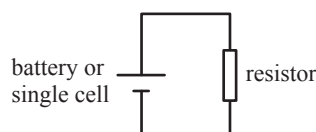


Fig. 2.1.5

Current as a measurable quantity is defined in **Terms and definitions**. In symbols,

$$I = \frac{\Delta Q}{\Delta t}$$

Terms and definitions

The **electric current**, I , through a conductor is rate of flow of charge: the charge passing per unit time through a cross-section of the conductor.

UNIT: ampere (A) = C s⁻¹

Study point

The ampere is an SI base unit (defined in terms of magnetic forces between wires carrying currents). The coulomb is a derived unit: the charge that passes in 1 second when the current is 1 A. So C = A s.

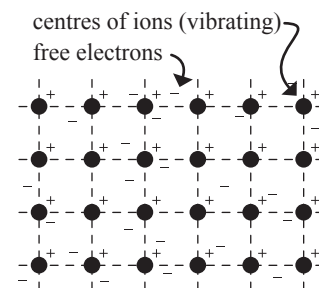


Fig. 2.1.4 Highly stylised metal structure

Study point

The ampere is named after André-Marie Ampère, who discovered (in the 1820s) several magnetic effects of electric currents.

Study point

We all fall into the trap sometimes, but try to avoid writing ‘Current flows...’ If it meant anything it would mean ‘rate of flow of charge flows’.

2.1.3

Self-test

A car battery is rated at 45 Ah. (45 ampere hour). On one 'charge', it can sustain a current of 5 A for 9 hours (or 3 A for 15 hours, etc.). State what physical quantity is given as 45 Ah, and express it in SI.

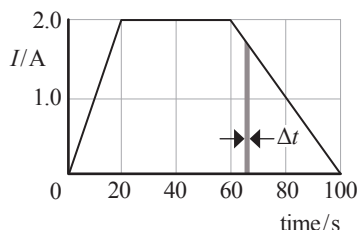
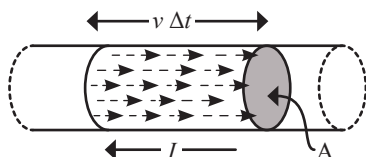


Fig. 2.1.6

Study point

The algebra of the derivation gives us $I = -nAve$, but we've dropped the minus sign. In $I = nAve$, I and v are the *magnitudes* of current and drift velocity. We remember that the conventional current is in the opposite direction to the free electron drift velocity.


 Fig. 2.1.7 To help show $I = nAve$

2.1.4

Self-test

Suppose the copper wire in the example of using $I = nAve$ had only half the diameter. Calculate v for the same current. Try to do this by noting the single factor that needs inserting, rather than by doing the same calculation all over again with $1.25 \times 10^{-3} \text{ m}$ instead of $2.5 \times 10^{-3} \text{ m}$!

Example: using the definition of current

By using a variable power supply, the current through an electric lamp is made to vary as shown in Fig. 2.1.6. Calculate the charge that passes through the lamp during the time shown.

In any small time interval Δt the charge ΔQ flowing is $I\Delta t$. But $I\Delta t$ is the 'area' of the strip under the graph, for that interval. At least it is when Δt is so small that we can forget the sloping top. So the total charge is the sum of the very narrow strip 'areas', that is the 'area' under the graph.

In this case, $Q = \frac{1}{2} \times 2.0 \text{ A} \times 20 \text{ s} + 2.0 \text{ A} \times 40 \text{ s} + \frac{1}{2} \times 2.0 \text{ A} \times 40 \text{ s} = 140 \text{ C}$.

2.1.3 How current depends on drift velocity of free electrons in a metal

(a) Drift velocity

Free electrons share in the random thermal energy of the metal. A typical thermal speed for an electron at room temperature is 10^5 m s^{-1} . Because the directions of motion are random, changing when the free electrons hit the (vibrating) ions, the thermal motion doesn't produce a flow of charge along the wire. We're concerned only with any superimposed *drift velocity* of free electrons in one direction *along* the wire, such as occurs when a battery is connected across the ends of the wire.

(b) The derivation

Suppose the mean drift velocity of free electrons in a wire is v . Then all the free electrons in a length $v\Delta t$ of conductor will pass through the shaded cross-section of the wire in Fig. 2.1.7 in a time Δt .

How many free electrons is that? The volume of a length $v\Delta t$ of wire of cross-sectional area, A , is $Av\Delta t$, so the number of free electrons in it is $nAv\Delta t$, in which n is the *free electron concentration*, the number of free electrons per *unit* volume of the metal. As each electron has charge $-e$...

Charge passing through a cross-section in time Δt is $\Delta Q = -nAv\Delta t e$.

But current, $I = \frac{\Delta Q}{\Delta t}$, so we have (see Study point): $I = nAve$

 Example: using $I = nAve$

An insulated copper wire in a car's headlamp circuit carries a current of 8.0 A. The diameter of the copper conductor is 2.5 mm. Calculate the drift velocity of the free electrons in the wire, and comment on your answer. The free electron concentration in copper is $8.47 \times 10^{28} \text{ m}^{-3}$.

$$I = nAve \text{ so } v = \frac{I}{nAe} = \frac{I}{n\pi\left(\frac{d}{2}\right)^2 e}$$

in which d is the diameter of the wire. So, substituting the numbers,

$$v = \frac{8.0 \text{ A}}{8.47 \times 10^{28} \text{ m}^{-3} \pi \left(\frac{2.5 \times 10^{-3} \text{ m}}{2}\right)^2 1.60 \times 10^{-19} \text{ C}} = 1.2 \times 10^{-4} \text{ m s}^{-1}$$

[Check that the units work out!] The drift velocity is small, despite 8.0 A being a fair-sized current. [It would take a free electron over 2 hours to travel 1 metre through the wire!] However, the free electrons start drifting through the wire within nanoseconds of closing the switch to turn on the headlights. The so-called 'electric field' that sets them drifting travels along the wire at almost the speed of light.

Ammeters

An ammeter (A) is an instrument for measuring current. The current to be measured must be routed through it. Analogue (pointer-and-scale) and digital types work on quite different principles, but both have low resistance conductive pathways for the current, as we don't want their inclusion in a circuit to reduce the current significantly.

Current is the same all around a series circuit

This means, for example, that in Fig. 2.1.8, $I_1 = I_2 = I_3$. This follows from the law of conservation of charge. Take $I_1 = I_2$; it means that the rate of flow of charge into the resistor is the same as the rate of flow of charge out of it; charge can't disappear or be created. But couldn't free electrons pile up in the resistor? No: excess free electrons, all having the same sign of charge (negative) would repel one another.

Experimentally, you would expect to find that all three ammeters read the same. If they didn't, what do you do before claiming that you've disproved $I_1 = I_2 = I_3$? You could swap round the positions of the ammeters and see whether the readings followed the meters. If so, what would you deduce? Or you could just put a single ammeter in different places in turn!

Study point

An ammeter often takes the form of a digital 'multimeter' set to a *current* range. Always start with a range with a high maximum (e.g. 10 A). Then change, if it's safe to do so, to a lower current range (e.g. 200 mA maximum) to gain more significant figures.

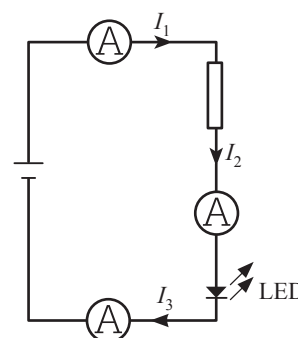
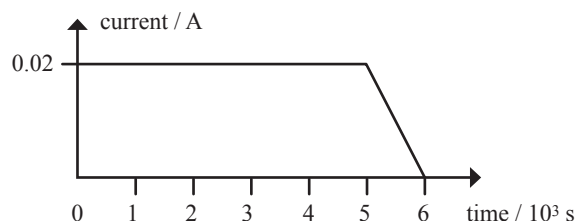


Fig. 2.1.8

Exercise 2.1

- Calculate the charge on a ${}^{235}_{92}\text{U}$ nucleus.
- 69.1% of natural copper atoms are ${}^{63}_{29}\text{Cu}$, each of which has a mass of $1.046 \times 10^{-25} \text{ kg}$, and 30.9% ${}^{65}_{29}\text{Cu}$, with a mass of $1.079 \times 10^{-25} \text{ kg}$.
The density of copper is 8930 kg m^{-3} . Calculate the total charge carried by all the electrons in a 1 cm cube of copper.
- The artificial radioactive material ${}^{99}_{43}\text{Tc}$ is β^- radioactive. A sample of 1.0 mg of ${}^{99}_{43}\text{Tc}$ has an activity of 640 MBq, i.e. it gives out 640 million β^- particles every second.
Calculate the electric current between the ${}^{99}_{43}\text{Tc}$ sample and its support for the sample to remain electrically neutral. State the direction of the current.
- The current supplied by a Ni-Cd rechargeable battery is monitored over its discharge and this graph is produced.



- Calculate the total charge supplied.
- The battery is labelled as having a capacity of 100 mAh. Comment on this.

5. A capacitor is a device for storing separated charge. A student connects a resistor across a charged capacitor, which discharges, i.e. the charge on one of its plates decreases. The student monitored the remaining charge, Q , over time and plotted the graph below.



Use the graph to calculate:

- The mean current between $t = 0$ and 15 s.
 - The initial current, i.e. the current at $t = 0$ s.
6. The student in Q5 notices that the graph looked similar to a radioactive decay graph. This means that the time taken for Q to halve should always be the same. By taking readings from the graph, show that this appears to be true and find the value of this half-life.
7. Aluminium has an ionic radius of **63 pm**. Metallic aluminium has 3 free electrons per aluminium ion.
- By considering the volume of an aluminium ion, estimate the number of free electrons per m^3 of metallic aluminium.
 - Calculate the drift velocity of free electrons in a **1 cm** diameter aluminium conductor carrying a current of **1 kA**.
8. Germanium is a semiconductor. It has many fewer mobile charge carriers than do metallic conductors. In germanium at room temperature there are approximately 18 mobile charge carriers for every million atoms of germanium. A **1 mm** diameter germanium wire carries a current of **10 mA**.

Estimate the drift velocity of the charge carriers from the following data:

Mass of a germanium atom = $1.20 \times 10^{-25} \text{ kg}$;

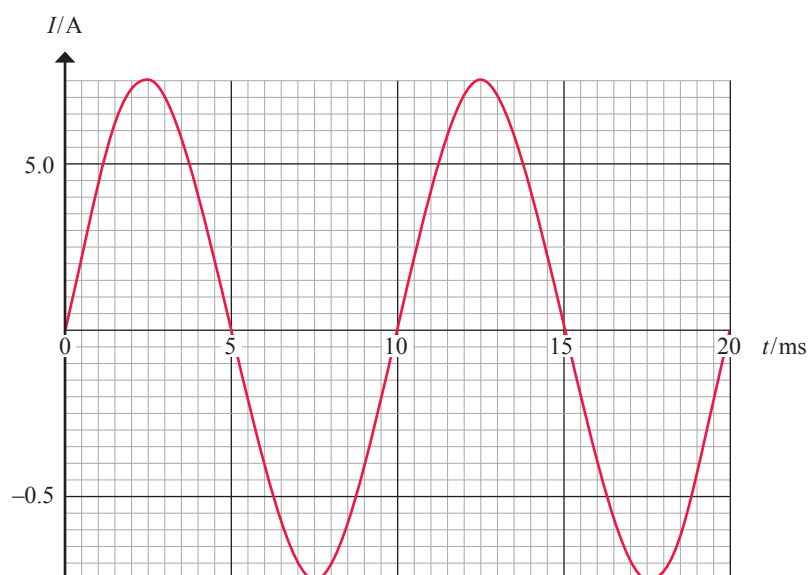
Density of germanium = $5.3 \times 10^3 \text{ kg m}^{-3}$.

9.

A 1 mm diameter copper wire carries an alternating current which varies as shown in the graph. The free electron concentration is $8.5 \times 10^{28} \text{ m}^{-3}$.

- (a) Sketch a graph of how the electron drift velocity varies with time.
 (b) Use your graph to estimate the distance drifted by a free electron in half a cycle.

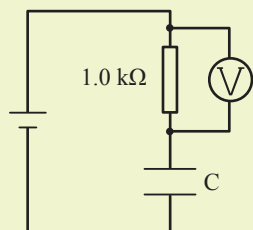
Comment on your answer.



Stretch & Challenge

This question involves some concepts from Section 2.2.

A capacitor, C , (see question 5) is initially uncharged. It is connected into the following circuit:



When the switch is closed, the pd across the resistor varies with time as follows:

time / s	2.0	5.0	7.0	10.0	12.0	15.0	20.0
pd / V	6.53	3.54	2.02	1.19	0.78	0.41	0.14

- (a) By drawing a suitable graph, estimate the total charge transferred around the circuit as the capacitor charges up.
 (b) Determine the half-life of the I - t relationship
 (c) What fraction of the total charge is transferred in the half-life you determined in part (b)?

2.2 Resistance

Terms & definitions

The **potential difference**, V , between two points, X and Y, is the work done, that is the loss of electrical potential energy, per unit charge passing between X and Y,

UNIT: volt (V) = J C⁻¹.

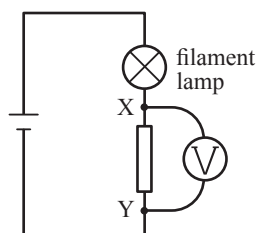


Fig. 2.2.1 The voltmeter: knowing its place

Study point

The symbol for pd is V (or ΔV) and, like all symbols for physical quantities, it is in italics. The symbol for its unit is **V**, an upright or 'Roman' letter, like all symbols for units.

Study point

To help avoid mistakes, connect the voltmeter last when wiring a circuit.

2.2.1

Self-test

A typical flash of lightning has been estimated to carry a mean current of 30 kA for a time of 0.5 ms and to dissipate 450 MJ. Calculate the pd.

Having studied current as rate of flow of charge, we now turn our attention to the accompanying *energy* transfers. The key idea is *potential difference* (*pd*), and we get to grips with this first of all. It enables us to define the very useful concept of *resistance*, and we discuss in some detail what gives a metal wire its resistance. Finally, we look briefly at the extraordinary phenomenon of superconductivity.

2.2.1 Potential difference

A voltmeter (V) reads the *potential difference* (*pd*) between two points, X and Y, in a circuit – as long as one of its leads (wires) is connected to X and the other to Y! So, in Fig. 2.2.1, the voltmeter will tell us the pd across the resistor, *not* across the filament lamp. If we want the pd across a particular component, the voltmeter must be connected across that component.

X is said to be at a higher potential than Y if the voltmeter gives a positive reading when the meter lead that is red or labelled '+' is connected to X.

Suppose the voltmeter reads 6.0 V. This tells us that *for every coulomb passing between X and Y, 6.0 J of work is done, resulting in 6.0 J of energy changing category: from electrical potential energy to another form.*

Voltage is a term often used to mean potential difference. A formal definition of *potential difference* is given in **Terms and definitions**.

If there is a resistor between X and Y, as in Fig. 2.2.1, charge does work as it travels through the resistor (more about this later), thereby losing electrical potential energy (the ability of a charge to do work because of its position), and the resistor acquires random thermal energy, which escapes to the surroundings as heat – so-called *energy dissipation*.

Examples

1. If the pd across a resistor is 6.0 V, and the current through it is 1.5 A, how much heat would it give out every minute once it reaches a constant temperature?

Heat emitted = Electrical PE lost per unit charge $\times$ charge passing

$$= 6.0 \text{ V} \times (1.5 \text{ A} \times 60 \text{ s}) = 540 \text{ J}$$

2. Referring to Fig. 2.1.3, the glass ball loses electrical potential energy as it goes across the gap from the positive plate to the negative. It gains kinetic energy. [This is the principle of the particle accelerator.]

Suppose the ball has a mass of 10 g and a charge of 6.0 nC, and that it starts from rest and reaches a speed of 0.060 m s⁻¹. What would be the pd between the plates?

$$\text{pd} = \frac{\text{Electrical PE lost}}{\text{charge passing}} = \frac{\text{KE gained}}{\text{charge passing}} = \frac{\frac{1}{2} \times 0.010 \times 0.060^2 \text{ J}}{6.0 \times 10^{-9} \text{ C}} = 3000 \text{ V}.$$

2.2.2 Electrical power

Generalising the reasoning in the first example in 2.2.1, when there is a current I for a time Δt through a conductor across which the pd is V (see Fig. 2.2.2) then...

Work done = Elec.PE lost per unit charge $\times$ charge passing

So $\text{Work} = V \times I\Delta t$

The term *power*, P , can be used in connection with any system (mechanical, electrical, thermal...) See **Terms and definitions**. In this electrical case,

$$\text{Power} = \frac{\text{Electrical work}}{\text{time taken to do work}} = \frac{VI\Delta t}{\Delta t} \text{ so } P = VI.$$



Fig. 2.2.2

Terms & definitions

Power means rate of doing work or rate of transfer of energy (for example between categories).

$$\text{So Power} = \frac{\text{Work done}}{\text{time taken}}$$

$$\text{or Power} = \frac{\text{Energy transferred}}{\text{time taken}}$$

UNIT: watt (W) = J s⁻¹

Example

In a simple particle accelerator, a beam of protons passes from a metal plate X to a metal plate Y. The pd between these plates is 150 kV, with plate X at the higher potential. The number of protons leaving X (and arriving at Y) per second is $7.0 \times 10^{16} \text{ s}^{-1}$. Calculate the electrical power.

Current = charge flowing per unit time = $7.0 \times 10^{16} \text{ s}^{-1} \times e$

$$= 7.0 \times 10^{16} \text{ s}^{-1} \times 1.60 \times 10^{-19} \text{ C} = 11.2 \text{ mA}$$

$\therefore$ Power, $P = VI = 150 \text{ kV} \times 11.2 \text{ mA} = 1.7 \text{ kW}$.

Self-test

2.2.2

When the pd across a resistor is 6.0 V, the current is 120 mA. Calculate: (a) the power dissipation (b) the charge that flows in 30 minutes. (c) the energy dissipation in 30 minutes.

Self-test

2.2.3

In time t , a charge Q passes through a resistor across which the pd is V . The power dissipation is P . Express t in terms of Q , V and P .

2.2.3 I against V (I – V) graphs for conductors

The current through a conductor and the pd across it are related. When one is zero, so is the other. When one increases, so (almost always) does the other. We're using 'conductor' to mean anything that conducts, for example a piece of wire, a resistor, a filament lamp, a diode connected in the 'forward' direction, such as the light-emitting diode shown in Fig. 2.1.8.

Water flow analogy

The electric current through a conductor is rather like the rate of flow of water from one tank to another through a pipe (Fig. 2.2.3). The *height difference* between water levels in the tanks is analogous to *potential difference* in an electric circuit. When the height difference is zero the flow rate is zero. The greater the height difference the greater the flow rate. A pump to take water from the bottom tank back the top to keep the water levels constant could be included to act like a battery in an electric circuit.

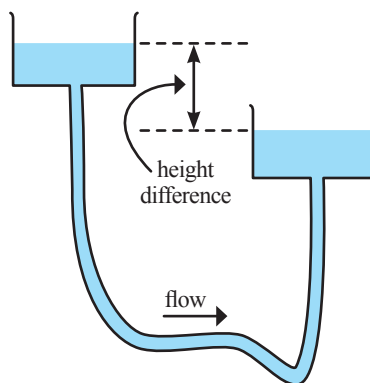


Fig. 2.2.3 Water flow analogy

Terms & definitions

Ohm's Law

The current I through a conductor is proportional to the pd, V , across it.

The law applies to metals (and many other single-substance conductors) at constant temperature.

2.2.4 Resistance

Terms & definitions

The **resistance**, R , of a conductor is defined by

$$R = \frac{\text{pd across conductor}}{\text{current through conductor}}$$

$$\text{In symbols: } R = \frac{V}{I}$$

$$\text{UNIT: ohm } (\Omega) = \text{V A}^{-1}$$

The **conductance**, G , of a conductor is defined by

$$G = \frac{\text{current through conductor}}{\text{pd across conductor}}$$

$$\text{In symbols: } G = \frac{I}{V}$$

$$\text{UNIT: siemens (S) = A V}^{-1}$$

The resistance of a conductor is defined in **Terms & definitions**.

A conductor obeying Ohm's law has a *constant* resistance whatever pd we put across it – as long as it *continues* to obey Ohm's law!

To convince yourself that this claim is true, calculate the resistance at (say) three different voltages for the metal wire whose I – V graph is given in Fig. 2.2.4. If this is too easy, go to the **Stretch and Challenge**.

A conductor that doesn't obey Ohm's law has a resistance that *does* depend on the applied pd. By considering the ratio $\frac{V}{I}$, you should be able to deduce, from looking at the graphs (Fig. 2.2.4), that the LED's resistance falls (dramatically) with increasing pd, but that filament lamp's resistance rises. In fact, up to about **0.2 V**, the filament lamp illustrated *does* obey Ohm's law, because the resistance of the filament (a thin metal wire) hardly changes temperature.

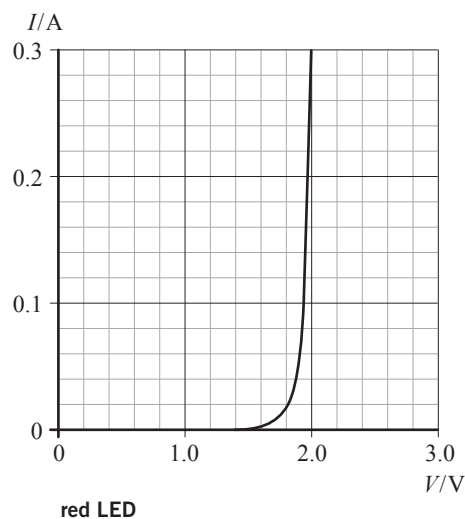
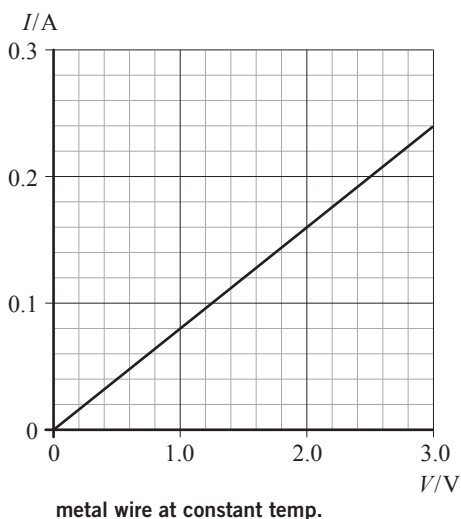
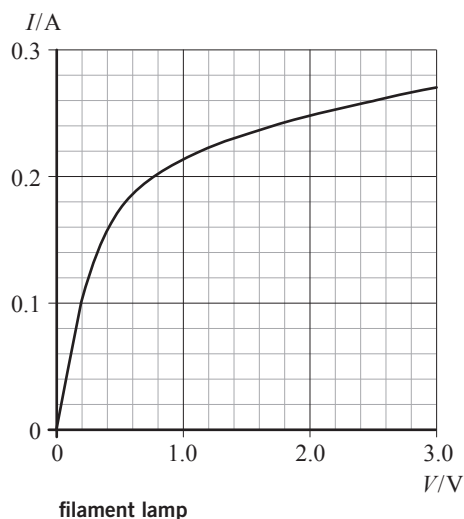


Fig. 2.2.4 I – V graphs

Stretch & Challenge

Use similar triangles on an I – V graph that is a straight line through the origin to show that R must be constant.

The middle graph, a straight line through the origin, demonstrates Ohm's law. If the pd is doubled, the current doubles, and so on.

As you can see, the law is not obeyed by all (indeed by most) conductors: a filament lamp and an LED (a light-emitting diode) are two examples of **non-ohmic conductors**. But, *provided that their temperatures are constant*, some very important types of conductor *are* ohmic, including metal wires and most single-substance devices (such as carbon resistors).

Conductance

This is defined in **Terms & definitions**. Clearly for any conductor,

$$G = \frac{1}{R} \text{ and } R = \frac{1}{G}$$

We shan't use the idea of conductance very often.

(a) How resistance arises in a metal: a simplified picture

We start by recalling (see Section 2.1.2) that if charge flows in a metal, it is by the drift of free electrons. For a drift to occur through a metal wire, each free electron needs a force on it, urging it along the wire. Putting it another way, there must be a pd across the wire. [A pd implies *work* done on the charges, which implies forces acting in on them in the direction of their motion.]

There seems to be a flaw in the argument just presented: a constant force on a free electron will give it not a constant velocity but a constant acceleration. Yes – until the electron collides with one of the vibrating ions. [The high thermal speeds of the free electrons will make such collisions very frequent.] The collision wipes out the electron's acquired drift velocity: on average it has to accelerate again from rest. And so on. The result (see Fig. 2.2.5 for crude representation) is a particular mean drift velocity, v , and, since $I = nAve$, a particular current, for a given pd placed across the wire. In other words the wire will have a finite *resistance* – due, essentially to collisions between free electrons and vibrating ions.

(b) Power dissipation in a conductor

Suppose we know that the current through a $5.0\ \Omega$ resistor is $0.30\ \text{A}$. How much power is dissipated in it? We could do this in two stages:

pd across resistor is $V = IR = 0.30\ \text{A} \times 5.0\ \Omega = 1.5\ \text{V}$,

so $P = VI = 1.5\ \text{V} \times 0.30\ \text{A} = 0.45\ \text{W}$.

We don't really need, though, to *work out* the pd, because, sticking a little longer with symbols,

$$P = VI = (IR)I, \text{ that is } P = I^2R$$

Similarly, if we know the resistance R of a conductor and the pd, V , across it we can calculate the power directly because, *as you should be able to show*:

$$P = \frac{V^2}{R}. \text{ Summarising, } P = IV = I^2R = \frac{V^2}{R}.$$

Example

A resistor of resistance $47\ \Omega$ has a maximum power rating of $5.0\ \text{W}$. Calculate the maximum pd that can safely be applied across it.

$$\text{So, } P = \frac{V^2}{R} \text{ So, } V = \sqrt{PR} = \sqrt{5.0\ \text{W} \times 47\ \Omega} = 15\ \text{V}$$

(c) Power dissipation in a metal: free electron picture

The power dissipation we've just been dealing with is the rate at which energy changes category from electrical potential energy to random vibration energy. In a metal the change occurs because the free electrons, with extra energy due to the applied pd, keep colliding with the ions, as discussed earlier. The amplitude of vibration of the ions increases because of the harder hits; in other words the wire gets hotter! This electrical heating is sometimes called *Joule* heating (See Fig. 2.2.6) after James Prescott Joule, who did much to establish the idea of conservation of energy. It is the principle of the electric heater and the filament lamp.

Self-test

2.2.4

Calculate the resistance of the filament lamp from the graph in Fig. 2.2.4, when the applied pd is (a) $0.20\ \text{V}$, (b) $3.0\ \text{V}$.

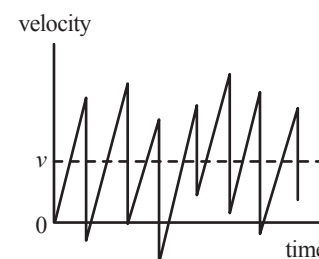


Fig. 2.2.5 Mean drift velocity, v (stylised)

Self-test

2.2.5

Calculate the current needed for a $2.0\ \Omega$ heating coil to produce $50\ \text{W}$ of power.

Fig. 2.2.6 James Prescott Joule

2.2.5 Resistivity

What are the factors that determine the resistance of a piece of wire?

- Suppose there is a current I in a wire of length l when a pd V is placed across it. Energy is changing category (from electrical potential energy to thermal energy) uniformly along its length. The pd across half of it (Fig. 2.2.7) must therefore be $V/2$.



Fig. 2.2.7 Halves of a wire

So the resistance of length $\frac{l}{2}$ must be:

$$\frac{V/2}{I} = \frac{1}{2} \frac{V}{I} = \frac{1}{2} \times \text{resistance of length } l.$$

Generalising, a wire's resistance, R , is proportional to its length, l .

- But we can also regard the piece of wire as made up of two 'length-ways halves' each with half the cross-sectional area of the original (Fig. 2.2.8). Each will carry current (half that of the whole), so

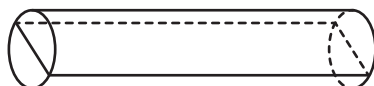


Fig. 2.2.8 Wire split lengthways

Resistance of wire area $\frac{A}{2}$ must be $\frac{V}{I/2} = 2 \frac{V}{I} = 2 \times \text{resistance of area } A$.

Generalising, a wire's resistance is inversely proportional to its cross-sectional area, A . [The shape of cross-section doesn't matter, as the distribution of current is determined by free electron concentration, which is the same throughout the metal whatever its shape.]

Terms & definitions

The resistance, R , of a wire of length l and cross-sectional area A is given by

$$R = \frac{\rho l}{A}$$

in which ρ is a constant for the material of the wire at a given temperature, called its **resistivity**.

UNIT of ρ : $\Omega \text{ m}$

We can incorporate the dependencies on l and A in the single equation given in **Terms & definitions**, which also includes a factor ρ (resistivity) taking care of the effect of the wire's material (and temperature) on its resistance. In fact the equation serves to *define* resistivity – as long as the meanings of the other letters are stated! Check that the unit given for ρ must be correct.

The lower the resistivity, the better the material conducts electricity. The best conventional conductor is silver, which has a resistivity of $1.59 \times 10^{-8} \Omega \text{ m}$, at 20°C , followed by copper ($1.68 \times 10^{-8} \Omega \text{ m}$), gold ($2.44 \times 10^{-8} \Omega \text{ m}$) and aluminium ($2.82 \times 10^{-8} \Omega \text{ m}$). Iron ($9.72 \times 10^{-8} \Omega \text{ m}$) is unimpressive. Contrast metals with insulators: the resistivity of sulphur is about $1 \times 10^{15} \Omega \text{ m}$.

Measuring the resistivity of a metal in the form of a wire is in Section 2.2.8.

2.2.6

Self-test

A metal wire is stretched so that its length increases by 1.0%. If its volume and resistivity are unchanged, by what percentage does its resistance increase?

2.2.7

Self-test

Inside the pvc sheath of a flexible connecting wire there are 19 strands of copper wire, each of diameter 0.30 mm and length 0.50 m. Calculate the resistance of the connecting wire. What would be its ideal resistance?

Example

Constantan is a nickel-copper alloy with a low dependence of resistivity on temperature and – for a metal – a high resistivity: $4.9 \times 10^{-7} \Omega \text{ m}$. It is ideal for making 'wire-wound' resistors. Calculate the length of constantan wire of diameter 0.19 mm needed to make a 15Ω resistor.

Re-arranging the resistivity equation and putting in data:

$$l = \frac{RA}{\rho} = \frac{R\pi\left(\frac{d}{2}\right)^2}{\rho} = \frac{15 \Omega \times \pi \times (0.095 \times 10^{-3} \text{ m})^2}{4.9 \times 10^{-7} \Omega \text{ m}} = 0.87 \text{ m}.$$

Conductivity

The *conductivity*, σ , of a material (at a particular temperature) is the reciprocal of its resistivity.

$$\text{Thus } \sigma = \frac{1}{\rho} \text{ and } \rho = \frac{1}{\sigma}$$

The *conductance*, G (see Section 2.2.4), of a wire is

$$G = \frac{1}{R} = \frac{A}{\rho l}, \text{ which is perhaps more naturally expressed as } G = \frac{\sigma A}{l}.$$

2.2.6 How the resistance of a metal depends on temperature

A metal wire's resistance increases with temperature. This can be investigated using simple apparatus over the temperature range 0°C to 100°C as described in Section 2.2.8. The effect is almost entirely due to the metal's change in resistivity, ρ , since thermal expansion of the wire makes only very small fractional changes to l and A .

The sketch-graph shows a typical relationship, as found by experiment over a wider range of temperatures than obtainable in a non-specialist laboratory.

- Note that we have used Celsius temperature, θ .
- On the vertical axis is the resistivity, ρ , divided by a constant, ρ_0 , the resistivity at 0°C . [It's not essential to divide ρ by ρ_0 ; the benefit of so doing is that the graph will fit most pure metals at least roughly.]

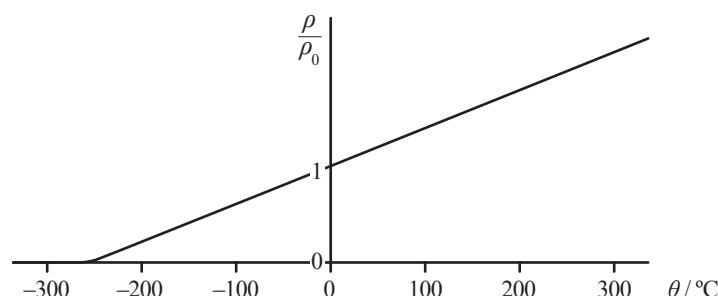


Fig. 2.2.9 Resistivity of a pure metal wire against temperature

The graph is nearly straight over quite a range of temperatures (at least from -100°C to $+200^{\circ}\text{C}$). Here the gradient is different for different metals, but is somewhere around $0.004^{\circ}\text{C}^{-1}$ for most pure metals. For alloys it is lower.

The resistivity of metals at very low temperatures is considered later.

Resistance and temperature for a metal: free electron explanation

For a given pd applied across a wire, the mean drift velocity of the free electrons is limited by their collisions with the vibrating ions. (See earlier section: How resistance arises in a metal.) The higher the temperature, the greater the vibration amplitude and the shorter the mean time between collisions. This reduces the mean drift velocity, v , and, because

$I = nAve$, the current (for a given pd). So, because $R = \frac{V}{I}$, the resistance increases!

Note that in a metal, the free electron concentration, n , doesn't depend on temperature.

Example: the shape of the I - V graph for a filament lamp.

Applying a large enough pd causes Joule heating (by harder collisions between free electrons and ions). The temperature rise increases the filament's resistance (because of the increased vibration amplitude of the ions)! So when we double the pd, the current goes up – but less than double. Note that the resistance of a filament at its operating temperature of perhaps 2500°C may be more than 10 times its resistance at room temperature; see Self-test 2.2.8.

Self-test

2.2.8

A mains tungsten filament lamp (rightly condemned as an energy waster!) is rated at '240 V, 60 W'. At 0°C its resistance is measured to be $67.0\ \Omega$. Calculate:

- How many times its resistance increases between 0°C and its working temperature.
- Its working temperature, assuming that for every degree Celsius rise in temperature its resistance increases by 0.0045 of its resistance at 0°C .

Stretch & Challenge

A wire made of copper alloyed with another metal has a higher R_0 than the same length and thickness of copper wire. This is because ions of the other metal spoil the regularity of the crystalline arrangement of copper ions, and make collisions by free electrons more likely. However, the straight portion of the R/R_0 against temperature graph is less steep than for pure copper. Suggest why.

2.2.7 Superconductivity

Terms & definitions

A **superconductor** is a material that, below a certain temperature, the *superconducting transition* (or *superconducting critical temperature*), loses all its electrical resistance.

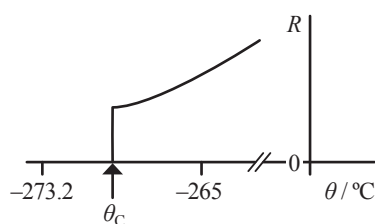


Fig. 2.2.10 Superconducting transition

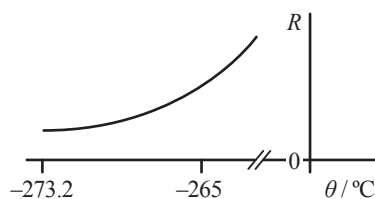


Fig. 2.2.11 Non-superconducting metal

In 1911 the Dutch physicist Heike Kamerlingh Onnes cooled a wire made of (frozen) mercury to lower and lower temperatures, and made the astonishing discovery that, at $-269.0\text{ }^{\circ}\text{C}$, its resistance suddenly dropped to zero, or at least became too low to measure. It had become what we now call a *superconductor*. Its *superconducting critical temperature* or *superconducting transition temperature*, θ_c , is $-269.0\text{ }^{\circ}\text{C}$. Bear in mind that *absolute zero*, the lowest temperature possible, is $-273.15\text{ }^{\circ}\text{C}$.

Physicists have now observed superconductivity in many metals. The transition temperatures are all within a few degrees of absolute zero. Note the sketch-graph of resistance against temperature in Fig. 2.2.10. Among metals which *haven't* been made to superconduct, despite cooling to within a minute fraction of a degree above absolute zero, are copper, silver and gold – the best conductors at ordinary temperatures! See Fig. 2.2.11.

In 1986 it was discovered that certain special ceramic materials could be made to superconduct, with transition temperatures much higher than those of metals, and mostly somewhat above $-196\text{ }^{\circ}\text{C}$. [$-196\text{ }^{\circ}\text{C}$ is the boiling point of liquid nitrogen, which can therefore be used as a (relatively cheap) coolant to keep these so-called *high temperature superconductors* superconducting.]

Is the resistance of a superconductor really zero? A current once started in a ring of superconducting metal has been found not to diminish noticeably over periods of years, even with no applied potential difference!

Uses of superconductors

Superconducting wires will carry currents without dissipating any energy at all. As well as the *energy saving*, there is no unwanted heat to get rid of.

There is a limit to the current that a superconducting wire can carry. This is not because it gets hot (there's no Joule heating!) but because it gives rise to a magnetic field, and too great a magnetic field makes a superconductor 'go normal' *even at temperatures below θ_c* .

Several prototype electrical power transmission cables have been set up using 'high temperature' (ceramic) superconductors. Keeping the whole length of the cable very cold is seriously expensive, but the energy savings could make such systems economic.

Electromagnets producing large magnetic fields over quite large volumes of space are needed in MRI (magnetic resonance imaging) machines for medical diagnosis, for most types of particle accelerator, and for magnetically levitating vehicles. Superconducting wires are routinely used for the coils of these electromagnets. Whereas the coils of conventional electromagnets need iron cores, superconducting coils don't. In an MRI machine, this leaves room for the patient – quite an advantage. [To prevent the superconductor going normal because of the magnetic field, it has to be cooled *well below* its transition temperature.]

Fig. 2.2.12 MRI magnet coil

Study point

There's more to superconductors than zero resistance. For example, when cooled through the transition temperature, one class of superconductor expels all magnetic field lines from its interior.

We don't attempt to *explain* superconductivity here. There is an accepted (but fiendishly difficult) explanation of the effect in metals, but none generally agreed for high temperature superconductors.

2.2.8 Experimental investigations

(a) Investigation of the I – V relationship for a conductor

We place the conductor under test in the circuit shown in Fig. 2.2.13. Note the ammeter in series with it and the voltmeter across it.

- Precautions for using a multimeter as an ammeter are explained in a previous study point.
- The variable voltage supply could be a purpose-made mains-powered unit or the ‘potential divider’ circuit shown in Fig. 2.2.14. How this works will be explained in Section 2.3.3.
- *Starting from zero*, we increase the pd in steps from zero up to the maximum allowed for the conductor under test (e.g. a filament lamp might be labelled ‘3 V, 0.45 A’), taking readings of I and V each time.
- It’s usual to present the results on an I – V graph: I (vertical) against V . We take extra readings where the local shape of the graph is not clear, for example where it bends. For a curved graph we’d expect to take about ten pairs of readings, for a straight graph, perhaps fewer.

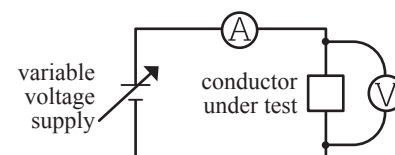


Fig. 2.2.13 Circuit for obtaining V and I readings

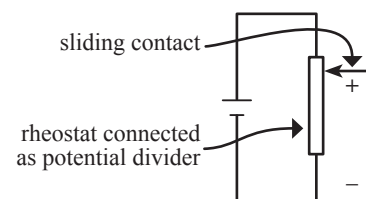


Fig. 2.2.14 A variable voltage supply

(b) Determining the resistivity of the metal of a wire

It’s easiest to investigate an uninsulated (bare) wire made of an alloy (e.g. constantan) with a relatively high resistivity. Essentially, we need to measure the wire’s resistance, R , length, l and diameter, d , since

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi \left(\frac{d}{2}\right)^2} \text{ that is } R = \frac{4\rho l}{\pi d^2} \text{ so } \rho = \frac{\pi d^2 R}{4l}.$$

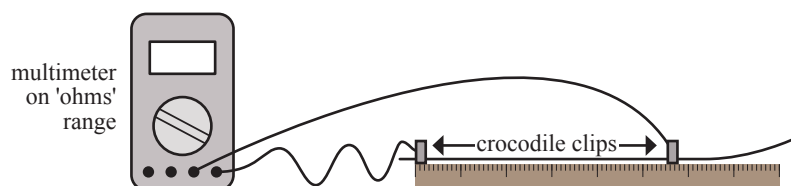


Fig. 2.2.16 Measuring resistance and length of thin wire

R We can use a digital meter on its *ohms* range. Its zero error (what it reads when the crocodile clips at the ends of its leads are held together) must be subtracted from any resistance reading. Alternatively we could use a battery, ammeter and voltmeter, connected as suggested in Fig. 2.2.15.

l The length of wire between the crocodile clips is measured with a metre rule. We need to minimise uncertainties due to parallax, to wire not being straight and to the unknown point of contact between wire and crocodile clips.

d The wire is likely to have a diameter of less than 0.3 mm, so an absolute uncertainty of 0.01 mm would constitute an uncertainty of more than 3% in d and of more than 6% in d^2 , since it is d^2 that we see in the equation for ρ . We certainly need an instrument with a resolution no coarser than 0.01 mm: either electronic calipers or micrometer screw gauge will serve. We take the mean of five or six measurements spaced along the length of the wire, and across different diameters.

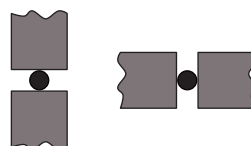


Fig. 2.2.17

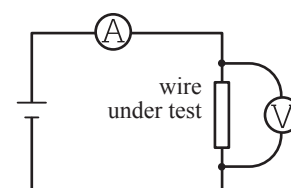


Fig. 2.2.15 Alternative circuit for measuring resistance

Study point

The mean value of d , the diameter of the wire for which the **top** $R-l$ graph in Fig. 2.2.18 was plotted, was 0.215 mm ($\pm 0.01 \text{ mm}$). The gradient of the graph is

$$\frac{(9.60 - 0.00) \Omega}{(0.700 - 0.000) \text{ m}} = 13.7 \Omega \text{ m}^{-1}$$

This gives a resistivity of

$$\rho = \frac{\pi d^2}{4} \times \text{gradient}$$

$$= \frac{\pi(0.000215 \text{ m})^2}{4} \times 13.7 \Omega \text{ m}^{-1}$$

$$= 5.0 \times 10^{-7} \Omega \text{ m}$$

We could determine ρ by putting one set of mean measurements into the equation. However, it's instructive to measure the resistance of progressively longer lengths of the same wire, and to plot a graph of R against l as below.

Since $R = \frac{4\rho l}{\pi d^2}$, we expect the graph to be a straight line through the origin, with a gradient of $\frac{4\rho}{\pi d^2}$. So $\rho = \frac{\pi d^2}{4} \times \text{gradient}$. See Study point.

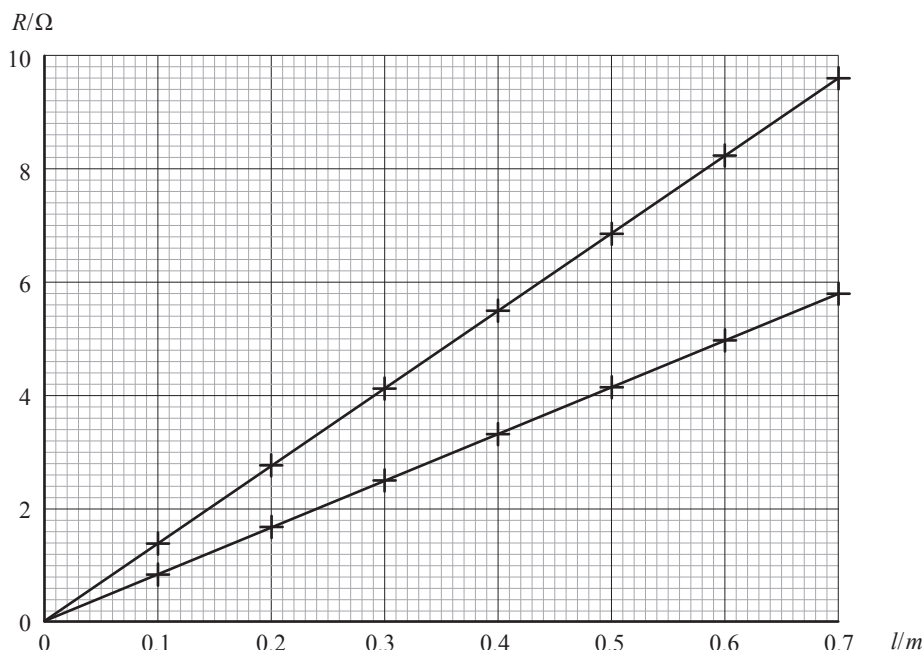


Fig. 2.2.18 Resistance against length for constantan wires

2.2.9

Self-test

The **bottom** graph in Fig. 2.2.18 is plotted from R and l measurements taken from a thicker piece of constantan wire. Determine the gradient of the graph, and hence the diameter of the wire, taking the resistivity of constantan as $5.0 \times 10^{-7} \Omega \text{ m}$.



Safety alerts

- Hot water: scalding hazard. Do not overfill kettle. Stir water only with long stirrer.
- Mains electricity. Keep clear of apparatus when kettle turned on. Mop up spilt water – with the kettle turned off at the mains.

(c) How the resistance of a metal wire depends on temperature

We could use the apparatus shown in Fig. 2.2.19, though this is only one possibility. A beaker of water heated by a Bunsen burner could be used instead of the kettle. Instead of using the multimeter on its *ohms* range, we could insert the coil of wire in the circuit shown in Fig. 2.2.19 and calculate the resistance from $V = IR$.

- We start with the kettle switched off at the mains, and a mixture of crushed ice and water surrounding the coil of wire.
- After stirring, we read the temperature and the resistance; we calculate the resistance by subtracting the zero error of the meter (its reading when its probes are touched together).
- We switch on the kettle for long enough for the ice to melt.
- With the kettle off, we stir the water gently with a long stick until the temperature stabilises and take another pair of readings.
- We keep repeating the process aiming for temperature rises of between 10° and 15° each time, until $\sim 100^\circ\text{C}$.
- A graph is plotted of resistance against temperature. The graph is expected to be a straight line of positive gradient. It is worth calculating the temperature at which the resistance would be zero, if the straight line relationship continued down to low temperatures.

Stretch & Challenge

To achieve the same rises in temperature as the water gets hotter, longer and longer heating times will be needed. Suggest why.

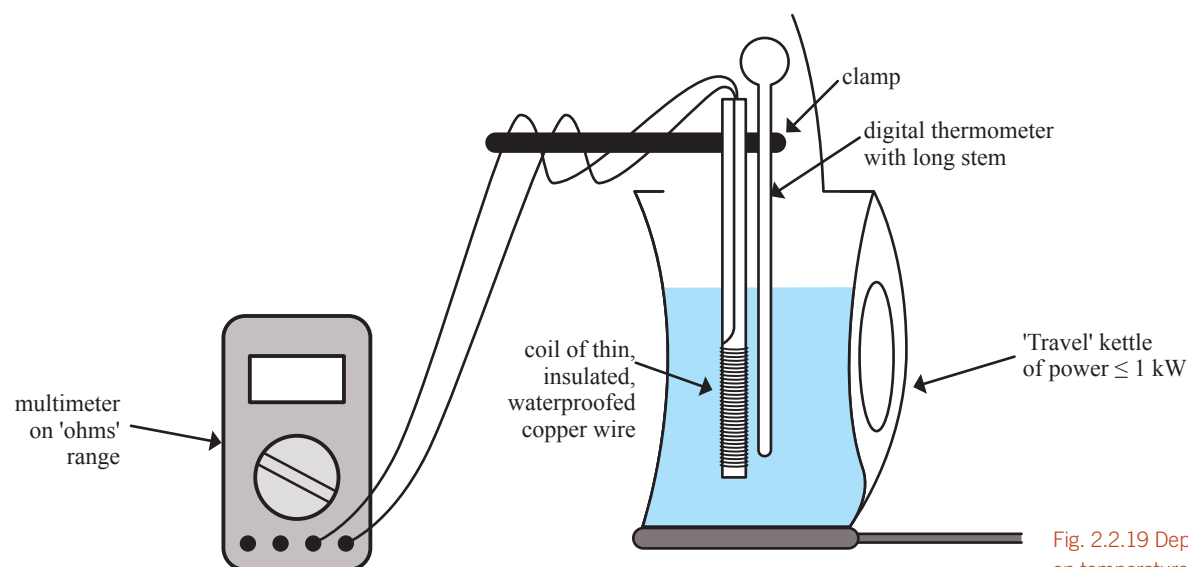


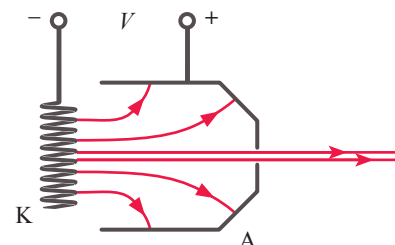
Fig. 2.2.19 Dependence of resistance on temperature: one method

Exercise 2.2

- An electric kettle is labelled 230 V, 2.5 kW. Calculate:
 - The current taken by the kettle.
 - The resistance of the heating element.
- The nominal mains pd in the EU is 230 V, with an allowed tolerance of +10%/−6% in the UK and +6%/−10% in much of mainland Europe, e.g. Germany. However for historical reasons, the transmitted voltage is ~240 V in the UK and ~220 V in Germany. Calculate the actual powers used by the kettle in Q1 when used in (a) the UK and (b) Germany. Comment on the time taken for the kettle to boil.
- An old incandescent light bulb is labelled 240 V, 60 W. A torch bulb is labelled 6 V, 0.25 A. Show that these two devices should be able to operate at rating if connected in series in a suitable circuit.
- The diagram shows an electron gun. It consists of a heated coil, the cathode K, which gives off electrons and a thimble-shaped anode, A. The electrons are accelerated by the pd between the anode and cathode. Most of the accelerated electrons hit the anode but a small fraction emerges in a narrow beam from the hole. ($m_e = 9.11 \times 10^{-31} \text{ kg}$)

The electron gun is set up with a pd of 1 kV. The cathode gives off 5.0×10^{15} electrons per second, 95% of which hit the anode. Calculate:

 - The power supplied by the voltage source.
 - The energy transfer when an electron travels from K to A.
 - The speed reached by the electrons.
 - The current in the wire between A and the + terminal.
- Calculate the momentum attained by the electrons in Q4.
 - The anode is observed to glow bright red. Explain why this should be and state the power which it radiates away.
 - Estimate the force which the electrons exert on the anode, explaining your answer in terms of N2 and N3.



6. A beam of protons ($m_p = 1.66 \times 10^{-27} \text{ kg}$) is produced in a similar piece of apparatus to the electron gun in Q4, operating at the same 10 kV. Calculate the speed of the protons in the beam.

7. Newton's and Einstein's relativistic formulae for momentum are compared in the box.

Newtonian momentum: $p_N = mv$

Relativistic momentum: $p_E = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$

They are effectively the same for very low speeds, $v \ll c$.

- (a) Show that for speeds less than 14% of the speed of light, the two values of momentum differ by less than 1%.
- (b) Use your answer to (a) to suggest a maximum voltage through which we can accelerate (i) electrons and (ii) protons without needing to use Einstein's relativity theory.

8. A student measured the resistance, R , of a length of iron wire at temperatures, θ , between 0 °C and 100 °C.

$\theta/^\circ\text{C}$	9	20	34	49	69	85	99
R/Ω	12.0	12.7	13.6	14.6	16.5	17.2	18.4

The results are shown in the table. The room temperature (20 °C) and 'boiling' temperatures are thought to be correct to within $\pm 0.5 \text{ }^\circ\text{C}$. The other temperatures have uncertainties of $\pm 2 \text{ }^\circ\text{C}$. The uncertainty in the resistance values is $\pm 0.1 \text{ }^\circ\text{C}$.

- (a) Plot the R , θ readings together with their error bars and draw extreme (max/min) graphs.
- (b) Use the answer to (a) to determine the gradient, m , and intercept, R_0 , on the R -axis, together with their estimated uncertainties.
- (c) The quantity $\frac{1}{R_0} \frac{\Delta R}{\Delta \theta}$ is known as the *temperature coefficient of resistance*, with the symbol α .
- (i) Use the definition to find the unit of α .
- (ii) The value of α for iron is given in a web page as 6.41×10^{-3} units. Show whether this is in agreement with your answer to (b).
- (d) Pure iron does not become superconducting, i.e. it has a non-zero electrical resistance at all temperatures. Use the results to show that the above linear relationship between resistance and temperature does not hold at very low temperatures.

9. Two students investigate the current–voltage (I – V) relationship for a filament car headlamp bulb which is labelled 12 V, 24 W. They set up a suitable circuit to explore both the low and high voltage variation. They expected the filament to obey Ohm's law for low voltages but not for high voltages.

V/V	I/A	V/V	I/A
0.00	0.000	2.00	0.741
0.25	0.117	4.00	1.078
0.50	0.234	6.00	1.342
0.75	0.352	8.00	1.568
1.00	0.469	10.00	1.768
1.50	0.634	12.00	1.951

- (a) Explain briefly why they expected two different behaviours.
- (b) Their power supply only gave outputs in 2.0 V steps. Draw a circuit they could use to investigate the current for lower voltages and explain how it works.
- (c) Their results are given in the table. Plot a graph of I against V and estimate the value of V at which the behaviour changes.
- (d) Calculate the resistance of the filament at low voltages and at the operating voltage.
- (e) The students read that, in the high voltage region, the relationship between I and V is $I = kV^n$ and that the value of n is approximately 0.6 . Plot a graph of I against $V^{0.6}$ to investigate this and discuss the extent to which the line agrees with $I = kV^{0.6}$ for voltages above the transition. Refine your answer to the transition voltage between the Ohmic and non-ohmic behaviours.
- (f) [For A level candidates] Plot a suitable log graph to determine a more accurate value for n . Use the results to obtain a value for k .

Stretch & Challenge

This question involves some concepts from Section 2.3.

The specific heat capacity, c , of a substance is the heat energy needed to raise the temperature of 1 kg of the substance by 1°C .

To measure the value of c for aluminium, some students use an electrical immersion heater to heat a 1.00 kg block of aluminium for 20 minutes. They monitor the temperature and note down the highest temperature reached. Their results were:

pd = 12.0 V ; $I = 2.50 \text{ A}$; initial (room) temperature = 22.0°C , highest temperature = 53.6°C

- (a) Use the data to estimate the specific heat capacity of aluminium.
- (b) In an attempt to allow for the effects of heat loss, the students monitor the temperature after the electrical heater is switched off and notice that the temperature drops to 50.1°C after 5 minutes.

Making the assumption that the rate of heat loss is proportional to the difference between the temperature of the block and that of the room, estimate:

- (i) The highest temperature that would have been reached in the absence of heat loss and
- (ii) an improved value for the specific heat capacity.

2.3 Direct current circuits

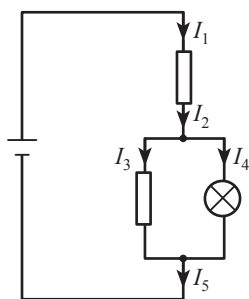


Fig. 2.3.1 Currents

Study point

Kirchhoff's first law

The sum of currents coming into a point in a circuit equals the sum of currents going out from it.

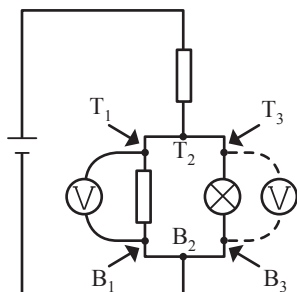


Fig. 2.3.2 Voltmeter choice

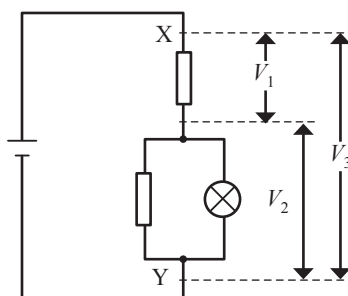


Fig. 2.3.3 Pds in series

Many circuits can be resolved into series and/or parallel combinations of conducting elements (for example, resistors, lamps) and a power supply. We show how to calculate currents and pds in these circuits, and finish by considering two important cases, the potential divider and the power supply with internal resistance.

2.3.1 Currents and pds in series and parallel circuits

(a) Currents

We've already seen (in 2.1.2) how the law of conservation of charge shows that current must be the same all round a simple *series* circuit (that is all components in a ring with no branches). But suppose there *are* branches, so we have components *in parallel* as in the lower resistor and the filament lamp in Fig. 2.3.1. Because charge is conserved, the currents (rates of flow of charge) through these two components must add up to the current in the circuit 'before' and 'after' it branched. In other words:

$$I_1 = I_2 = (I_3 + I_4) = I_5$$

This illustrates the simple rule known as *Kirchhoff's first law* (See Study point).

(b) Potential differences

We start by looking at the parallel combination in Fig. 2.3.2. Whether the voltmeter is in the left-hand position or the right-hand position it must read the same, because T_1 and T_3 are joined by a wire (of negligible resistance), and so are effectively the same point. Similarly with B_1 and B_3 .

In other words, when components are in parallel, the pd is the same across both of them: there is only *one* pd!

[More fundamentally: pds arise from forces on free electrons due to distributions of charge brought about by the battery. The forces do work on free electrons going from one point to another (e.g. from B_2 to T_2). The amount of work is independent of the route taken between the points, just like the work done on us by the pull of the Earth when we change levels by using a staircase instead of a sloping ramp.]

Finally, consider pds across components in series, as shown in Fig. 2.3.3. For an electron going from Y to X, the work done on it as it goes through either the lower resistor or the filament lamp is eV_2 and as it goes through the top resistor is eV_1 . So the total amount of work as it goes from Y to X is $(eV_1 + eV_2)$, but it is also eV_3 . So $eV_1 + eV_2 = eV_3$. Dividing through by e :

$$V_1 + V_2 = V_3.$$

Thus pds in series add together.

The rules for pds in circuits with series and parallel components are, then, a consequence of the conservation of *energy*.

2.3.1

Self-test

The filament lamp whose I - V graph is given in Fig. 2.2.5 is designed for a pd of 2.5 V across it.

(a) Calculate the value of the series resistor needed to run the lamp correctly from a 12 V battery.

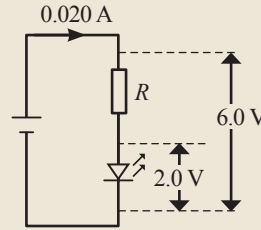
(b) Calculate the power dissipated (i) in the lamp and (ii) in the resistor, and comment on these values.

Example

The red LED whose I - V graph is given in Fig. 2.2.5 has the desired brightness with a current of 20 mA . Calculate the resistance of the resistor that must be placed in series with the LED to run it at this current from a 6.0 V supply.

We first put the information on a circuit diagram (Fig. 2.3.4). But Fig. 2.2.5 shows that for a current of 20 mA , a pd of 2.0 V (to 2 s.f.) is needed across the LED. Since pds in series add, we need to ‘drop’ ($6.0\text{ V} - 2.0\text{ V}$) across the resistor, so its value, R , must be...

$$R = \frac{4.0\text{ V}}{0.020\text{ A}} = 200\ \Omega$$

Fig. 2.3.4 Data to find R

(c) A note on the resistance of ammeters and voltmeters

Look at the circuit in Fig. 2.3.5. The quantities alongside the meters in Fig. 2.3.5 are their readings. It wouldn't be silly to ask: are these the pd and currents *before the meters were connected*? The answer is: yes, but only if:

- The voltmeter's resistance is so high that the current through it is negligible.
- Each ammeter's resistance is so low (compared with those of the resistors) that inserting it in series doesn't lower the current.

With modern meters, these are usually safe assumptions. For example, a typical auto-ranging digital multimeter on its 'dc volts' range has a resistance of $10\text{ M}\Omega$ or more. On its 10 A range its resistance is usually a small fraction of an ohm. See Exam tip.

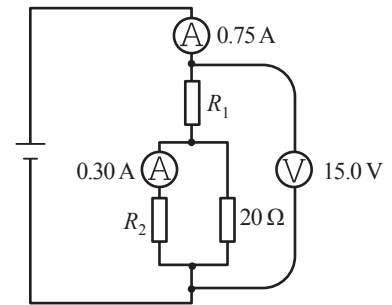


Fig. 2.3.5 Unknown resistors

Multi-step example

Determine the resistances R_1 and R_2 in Fig. 2.3.5.

This example, unlike the last, is rather contrived, but it does demonstrate the use of just about all the basic circuit rules.

We know the currents through R_1 and R_2 , but we don't know the pds across either of them – yet. We start by calculating the only thing we can immediately calculate. This gives us another, and so on.

- For the $20\ \Omega$ resistor, current = $0.75\text{ A} - 0.30\text{ A} = 0.45\text{ A}$.
- So the pd across the $20\ \Omega$ resistor is $V = IR = 0.45\text{ A} \times 20\ \Omega = 9.0\text{ V}$.
- This is also the pd across R_2 .
- So $R_2 = \frac{9.0\text{ V}}{0.30\text{ A}} = 30\ \Omega$.
- But R_1 is in series with the parallel combination, across which the pd is 9.0 V . So pd across R_1 is $V = 15.0\text{ V} - 9.0\text{ V} = 6.0\text{ V}$.
- So $R_1 = \frac{6.0\text{ V}}{0.75\text{ A}} = 8.0\ \Omega$.

This is straightforward as long as one is very careful to associate each pd with the circuit component(s) to which it belongs, i.e. make your working clear.

Exam tip

Unless otherwise instructed, assume that ammeters and voltmeters have 0 and infinite resistances respectively.

Self-test**2.3.2**

Calculate R in Fig. 2.3.6.

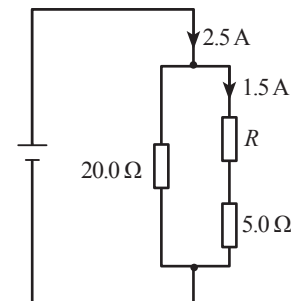


Fig. 2.3.6 Practice circuit

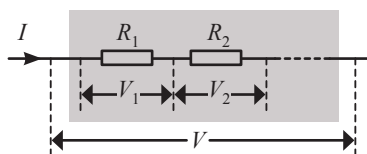


Fig. 2.3.7 Resistances in series

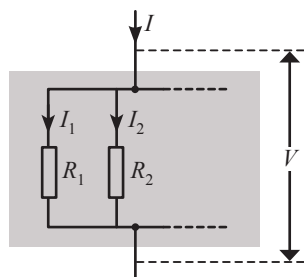


Fig. 2.3.8 Resistances in parallel

2.3.3

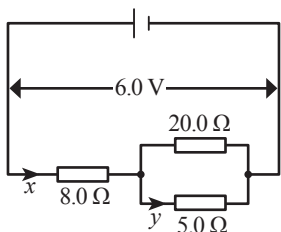
Self-test

Three $12\ \Omega$ resistors can be connected together into combinations with two terminals: all in series, all in parallel and in two other ways. Calculate the resistances of these four combinations.

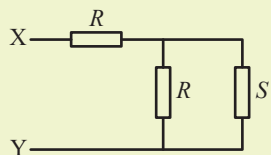
2.3.4

Self-test

Calculate currents x and y .



Stretch & Challenge



Let g be the ratio $\frac{S}{R}$. Determine the value of g for which the resistance measured between X and Y is simply S . You should find yourself solving a quadratic equation in g . [Then, look up *golden ratio* for interest.]

2.3.2 Formulae for resistances in series and in parallel

(a) Resistances in series

If meters were placed to measure the pd V and the current I shown in Fig. 2.3.7 then $\frac{V}{I}$ would give us the combined or equivalent resistance, R , of everything in the grey box, that is of the resistors in series.

But we have shown that pds in series add up, that is

$$V = V_1 + V_2 + \dots$$

The current, I is the same all through, so the equation can be rewritten:

$$IR = IR_1 + IR_2 + \dots$$

Dividing through by I : $R = R_1 + R_2 + \dots$

This result may seem too obvious to need a derivation, but note that resistances don't *always* add...

(b) Resistances in parallel

The resistors in Fig. 2.3.8 are in parallel. This time it is the currents through the individual resistors which add to give the current, I , entering and leaving the combination.

$$I = I_1 + I_2 + \dots$$

There is only one pd, V , so $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \dots$

Dividing through by V gives $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$

Here, R is the *equivalent* or combined *resistance* of the resistors, as would be found by measuring V and I and dividing V by I . [We could write the final equation in terms of conductance simply as $G = G_1 + G_2 + \dots$]

Example

Determine the resistance of resistors of $3.0\ \Omega$ and $4.0\ \Omega$ in parallel.

$$\text{Leaving out units, } \frac{1}{R} = \frac{1}{3} + \frac{1}{4}$$

We could find R very quickly with a calculator, and that's good if the answer is all we want, but for our purposes it's more instructive to add the fractions by putting each over the *common denominator* of 3×4 .

$$\text{Doing so, } \frac{1}{R} = \frac{4}{3 \times 4} + \frac{3}{4 \times 3} = \frac{3+4}{3 \times 4} = \frac{7}{12}$$

$$\text{Thus } R = \frac{12}{7} = 1.71\ \Omega.$$

Notice that the combined resistance is less than either of the individual resistances. That is as it should be because, with two resistors connected between the same two points, the current will be greater (for the same pd) than if there were only one.

Product divided by sum for two resistances in parallel

It's useful to re-do the above example in algebra for two resistances.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \text{ so } R = \frac{R_2 R_1}{R_1 + R_2} = \frac{\text{product of } R_1 \text{ and } R_2}{\text{sum of } R_1 \text{ and } R_2}.$$

This is easy to remember (the *units* tell you that the product must be on the top), *and* easy to use *and* without a final reciprocal to forget to take! The sole caveat is that it works only for two resistances at a time.

n equal resistances in parallel

You should show for yourself that the equivalent resistance is given by

$$R = \frac{1}{n} \times \text{one individual resistance}$$

Resistor networks can't always be resolved into combinations of resistors in series and parallel. An example is the network shown in Fig 2.3.9. Its resistance can be calculated, all the same. See Stretch and challenge, for a technique a little beyond A-level.

2.3.3 The potential divider

This name is given to resistances connected in series in order to 'divide up' the pd, V_{total} , placed across the combination. In Fig. 2.3.10 we have...

$$V_1 = IR_1, V_2 = IR_2 \dots V_{\text{total}} = I(R_1 + R_2 \dots) \text{ that is } V_{\text{total}} = IR_{\text{total}}.$$

By division, $\frac{V_1}{V_2} = \frac{R_1}{R_2}$ and so on, $\frac{V_1}{V_{\text{total}}} = \frac{R_1}{R_{\text{total}}}$, $\frac{V_2}{V_{\text{total}}} = \frac{R_2}{R_{\text{total}}}$ and so on.

The ratio of pds is simply equal to the ratio of the resistances across which the pds would be measured!

(a) Using a potential divider to give a desired output pd

If we place an *input* pd, V_{in} , across two resistors, as in Fig. 2.3.11, we can obtain an *output* pd across either of the resistors (we've picked R_2). By choosing the resistors correctly we can have any V_{out} we like, (as long as $V_{\text{out}} \leq V_{\text{in}}$) since

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1 + R_2}, \text{ that is } V_{\text{out}} = \frac{R_2}{R_1 + R_2} V_{\text{in}}.$$

As an example, we'll select R_1 and R_2 in order that $V_{\text{out}} = 3.0 \text{ V}$ when $V_{\text{in}} = 9.0 \text{ V}$.

$$\frac{R_1}{R_2} = \frac{\text{pd across } R_1}{\text{pd across } R_2} = \frac{6.0 \text{ V}}{3.0 \text{ V}} = 2.0.$$

So it looks as though we could have $R_1 = 2.0 \Omega$, $R_2 = 1.0 \Omega$, or $R_1 = 30 \Omega$, $R_2 = 15 \Omega$, or $R_1 = 2000 \Omega$, $R_2 = 1000 \Omega$ and so on.

In practice we'd avoid very *low* resistances, so as not to tax the power supply and overheat R_1 and R_2 . Consider the combination $R_1 = 2.0 \Omega$, $R_2 = 1.0 \Omega$. In this case the

total power dissipation in the resistors would be $\frac{V^2}{R} = \frac{(9.0 \text{ V})^2}{3.0 \Omega} = 27 \text{ W}$: equal to the power of a small soldering iron!

Stretch & Challenge

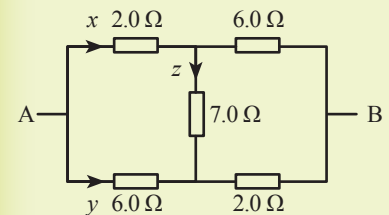


Fig. 2.3.9 Challenge

Find the resistance between A and B in Fig. 2.3.9 by supposing a pd (8.0 V is neat) to be placed between A and B. Use Kirchhoff's first law to express currents through right-hand resistors in terms of z and x or y . Between any two points the pd is independent of route. From A to B via the top resistors we have $2x + 6(x - z) = 8$.

Write two more such equations. Solve for x and y , hence calculate $x + y$ which will let you calculate the resistance.

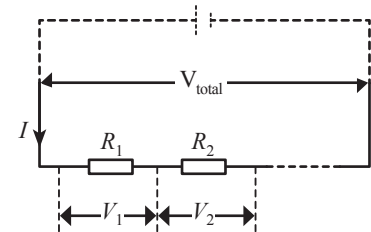


Fig. 2.3.10 Potential divider (general)

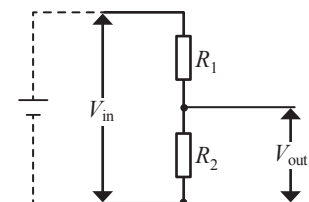


Fig. 2.3.11 Potential divider

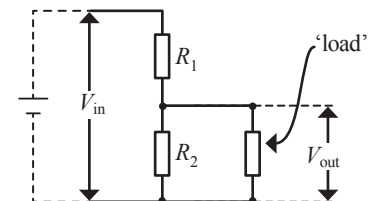


Fig. 2.3.12 Loaded potential divider

2.3.5

Self-test

If $R_1 = 30\ \Omega$, $R_2 = 15\ \Omega$, and $V_{\text{in}} = 9.0\ \text{V}$ in the potential divider of Fig. 2.3.12, calculate the percentage by which V_{out} falls when R_L , a load of $1000\ \Omega$, is connected across R_2 .

R_1 and R_2 and could also be too *high*... The $3.0\ \text{V}$ is no doubt needed because some device, shown as R_L in Fig. 2.3.12, requires $3.0\ \text{V}$ across it. We say that R_L is a *load* connected across the potential divider output. Suppose $V_{\text{in}} = 9.0\ \text{V}$, $R_1 = 2000\ \Omega$, $R_2 = 1000\ \Omega$, $R_L = 1000\ \Omega$. This will give us a V_{out} of less than $3.0\ \text{V}$, because the load has made the ‘lower’ resistance in the potential divider $500\ \Omega$ (two $1000\ \Omega$ resistances in parallel) so

$$V_{\text{out}} = \frac{500\ \Omega}{2000\ \Omega + 500\ \Omega} 9.0\ \text{V} = 1.8\ \text{V}$$

Loading a potential divider reduces its output pd. The reduction is, however, very small if R_1 and R_2 are much lower than the load resistance, R_L . See Self-test 2.3.5.

(b) A variable potential divider

The idea is simple: we make the ratio of R_1 to R_2 variable. This can be done using an ordinary laboratory rheostat (Fig. 2.3.13). Note that it has three terminals. The lower two – call them ‘A’ and ‘B’ – connect to the ends of a single-layer coil of bare wire of high resistivity. So there’s a fixed resistance (often about $15\ \Omega$) between A and B. The top terminal, ‘S’, connects to a *sliding contact* which can press against the coil anywhere along its length, so ‘dividing’ it into two portions, AS and SB, our friends R_2 and R_1 , but whose ratio we can vary.

Fig. 2.3.13 A rheostat

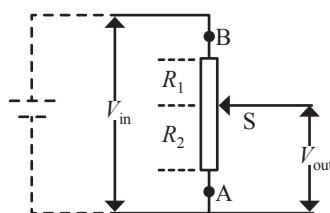


Fig. 2.3.14 Variable potential divider

Fig. 2.3.14 shows the circuit symbol for a variable potential divider. Note carefully how the connections to it correspond to the terminals on the rheostat, and how V_{in} is connected across AB. V_{out} will be approximately $\frac{3}{4}V_{\text{in}}$ if the sliding contact is $\frac{3}{4}$ of the way between A and B, and so on.

Variable potential dividers have many uses in electronics. The usual form consists of a carbon ‘track’ forming a major arc of a circle, and a sliding contact which can be moved by turning a spindle.

(c) Potential dividers incorporating resistive sensors

Sensors are devices that ‘respond’ to changes in their surroundings. We shall consider two that respond by changing their resistance. *Thermistors* do so when the temperature changes, and *light-dependent resistors* (LDRs) do so when the light level changes. By making a thermistor or an LDR one of the resistances in a potential divider, the change will result in a change in output pd – useful for ‘triggering’ digital systems, alarms and so on.

Thermistor circuit

We shall consider only ‘ntc’ (negative temperature coefficient) thermistors, whose resistance goes down as the temperature goes up, typically as shown in Fig. 2.3.15. These thermistors are made of a semiconductor material, usually a metal oxide with deliberately added ‘impurity’ atoms. When the temperature is increased the number of mobile charge carriers (mainly free electrons or mainly ‘positive holes’) increases, making the resistivity less.

Fig. 2.3.16 shows a thermistor used as one of the resistances in a potential divider circuit – note the (not very intuitive) thermistor symbol. When the temperature rises, its resistance, R_1 , will decrease, but R_2 will stay (almost) the same, so V_{out} will increase.

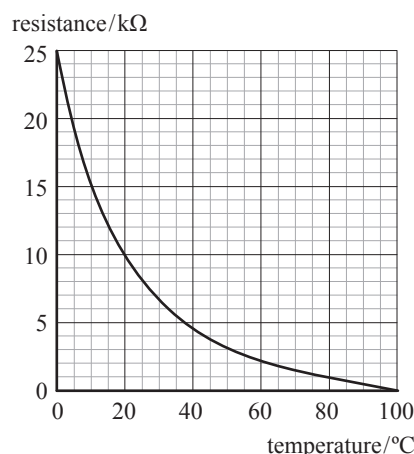


Fig. 2.3.15 Resistance against temperature for ntc thermistor

Example

In Fig. 2.3.16, V_{in} is 9.0 V. V_{out} is connected to the input of an alarm that will be set off when V_{out} reaches 2.5 V. The resistance-temperature relationship of the thermistors is as in Fig. 2.3.15. Calculate the value of the fixed resistor, R_2 , needed in for the alarm to be set off when the temperature reaches 40 °C.

We first note that at 40 °C the thermistor resistance, R_1 , is about 4.3 k Ω .

Since $\frac{R_2}{R_1} = \frac{\text{pd across } R_2}{\text{pd across } R_1}$ in this case $\frac{R_2}{4300 \Omega} = \frac{2.5 \text{ V}}{9.0 \text{ V} - 2.5 \text{ V}} = 0.385$.

This gives $R_2 = 1.7 \text{ k}\Omega$.

Note: We have assumed that the *alarm* has a very high input resistance, so it doesn't *load* the potential divider significantly.

LDR circuit

Certain semiconductors, such as cadmium sulphide, have very high resistivities in the dark, but conduct better and better as the light level increases. This is because photons can supply enough energy to knock a small proportion of the electrons out of the bonds between atoms, creating free electrons (and positive holes). An LDR (or *photoresistor*) is made by depositing a layer of such a semiconductor as a zig-zag 'track' on an insulating 'substrate' enclosed in a transparent case.

Fig. 2.3.17 shows an LDR used as one of the resistors in a potential divider circuit. The LDR simply replaces the thermistor in Fig. 2.3.16, so the brighter the light falling on the LDR, the higher V_{out} . The circle is often omitted from the LDR symbol, but officially it should be there!

Stretch & Challenge

for A level candidates in the second year

We take a closer look at the graph of resistance against temperature for the ntc thermistor. Let us assume that, for a given pd, the current is proportional to the free electron concentration, n , so that the resistance will be inversely proportional to n .

We might expect n to be proportional to a Boltzmann factor, $e^{-\frac{E}{kT}}$, in which k is the Boltzmann constant, T is the kelvin temperature and E is the energy needed to excite an electron from some ground state to a state in which it is mobile. (See Stretch and Challenge at the end of 2.8.1.) Thus, ignoring relatively minor effects, such as increasing frequency of collisions between free electrons and vibrating ions, we might expect the thermistor's resistance to vary as $R = R_1 e^{\frac{E}{kT}}$, in which R_1 is some constant with the unit of Ω .

Take the values of R at 0 °C and 20 °C from the graph and hence find E . Now repeat for 10 °C and 40 °C, and see if you get the same value for E .

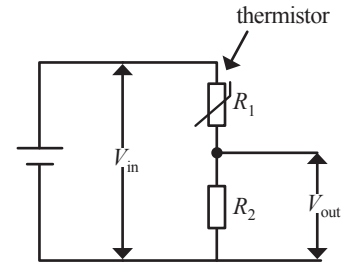


Fig. 2.3.16 Thermistor as part of potential divider

Self-test

2.3.6

Following on from the example, if R_2 were changed to 2.4 k Ω , determine the temperature at which the alarm would go off.

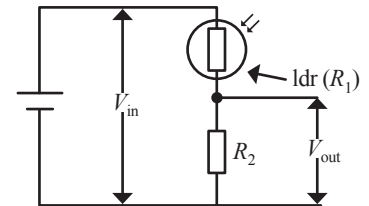


Fig. 2.3.17 LDR as part of potential divider

Self-test

2.3.7

State how the V_{out} would behave if the LDR and the fixed resistor were swapped round in Fig. 2.3.17.

Fig. 2.3.18

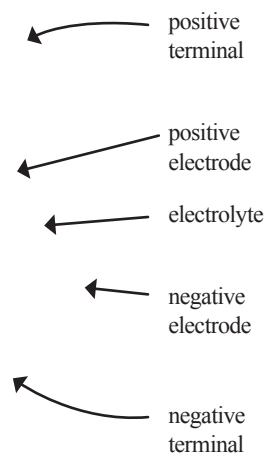


Fig. 2.3.19 A cell (simplified)

Terms & definitions

The **emf** of a cell or battery is the energy that changes category from chemical to electrical potential per unit charge passing through the cell. UNIT: $\text{J C}^{-1} = \text{V}$.

Study point

Emf stands for *electromotive force*, a silly name for a quantity with the unit **V**, so it is better abbreviated than used in full. *Electromotance* is a sensible proposed alternative.

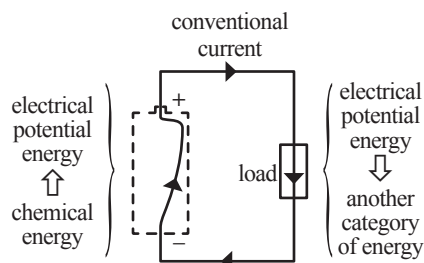


Fig. 2.3.20 Energy transfers inside and outside a cell (simplified)

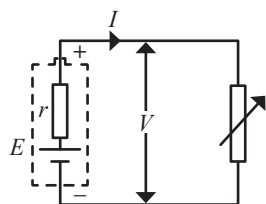


Fig. 2.3.21 Showing a cell with internal resistance included

2.3.4 Power supplies

Here we consider the role of a battery, or other power supply, in a circuit. Strictly, a *battery* is a series combination of cells, but a single cell is often, loosely, called a battery. A cell consists of two *electrodes* made of different conducting materials separated by a conducting liquid or paste *electrolyte* in what we hope is a leak-proof case. Fig. 2.3.19 is a simplified diagram of the popular *alkaline* cell. The symbol for a cell is $\text{---}|$, though this can now also be used for a battery or power supply. A battery of two cells is sometimes shown as $\text{---}|$, and so on.

(a) The emf of a cell

Chemical reactions in a cell involve the transfer of charge (by means of ions) from one electrode through the electrolyte to the other. So one electrode acquires a positive charge (a small electron deficit) and the other a negative charge (a small electron surplus).

A specific amount, E , of work per unit charge is done on the charge as it is transferred. E is called the emf of the cell. See **Terms and definitions**. E depends on the chemical make-up of the cell, and is often about 1.5 J C^{-1} , that is 1.5 V .

When the cell is *open circuit*, meaning that nothing is connected to it, the charges that have been transferred to the electrodes prevent (through repulsive forces) the further transfer of charge – and the associated chemical reaction. The condition for no further transfer of charge is that the pd, V , across the cell terminals is equal to E . [It may help to think of V (open circuit) as like the height of a heap built by throwing material on to it, and E as the maximum height the thrower can throw.]

So, on open circuit, $V = E$.

We measure a cell's emf by connecting a voltmeter, but nothing else, across the cell's terminals. Recall that a voltmeter's resistance is very high indeed, so the cell is effectively open circuit.

(b) Internal resistance

If we connect a *load* (such as a resistor, led, buzzer, electric motor) between the cell terminals we provide an *external* conductive path, so charge can flow continuously in a complete closed loop, and energy transfers occur as in Fig. 2.3.20. [The symbols are unofficial.]

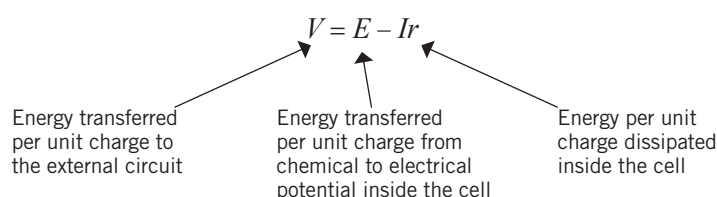
In fact Fig. 2.3.20 is rather simplified. We find that the pd, V , across the cell terminals drops when we connect a load across the terminals so that there is a current. The lower the load's resistance the more the current, I , and the more the reduction in V . Usually, to a fair approximation,

$$V = E - Ir$$

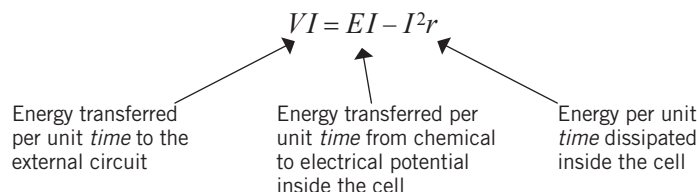
in which r is a constant with the unit of Ω , called the *internal resistance* of the cell.

It often helps to include r in circuit diagrams, as in Fig. 2.3.21. Although r arises mainly from collisions of ions drifting through the electrolyte, it can't be measured directly (even if we could delve inside a cell with the probes of a multimeter on its ohms range) since it is inseparable from the charge-pumping role of the cell, as demonstrated by its emf.

The interpretation of the terms on the equation is...



Multiplying through the previous 'voltage' equation by I , we have an equation for power:



Example 1

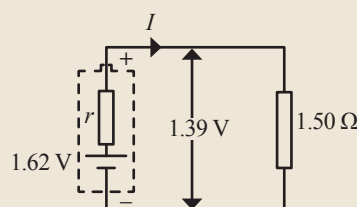
A cell has an emf of 1.62 V . When a 1.50Ω resistor is connected across its terminals, the pd falls to 1.39 V . Calculate: (a) the cell's internal resistance, (b) the fraction of the total power dissipated in the internal resistance.

(a) We first put the data on a diagram, as shown.
Considering the 1.50Ω load:

$$I = \frac{1.39 \text{ V}}{1.50 \Omega} = 0.927 \text{ A.}$$

So (see diagram)

$$r = \frac{1.62 \text{ V} - 1.39 \text{ V}}{0.927 \text{ A}} = 0.25 \Omega.$$



$$(b) \quad \frac{\text{power in } r}{\text{power generated}} = \frac{rI^2}{EI} = \frac{rI}{E} = \frac{1.62 \text{ V} - 1.39 \text{ V}}{1.62 \text{ V}} = 0.14.$$

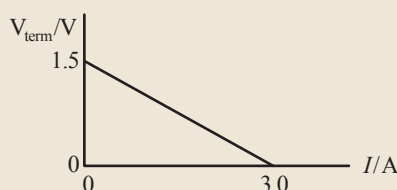
Example 2

Sketch a graph of V against I for a cell of emf 1.50 V and internal resistance 0.50Ω .

We have $V = 1.50 \text{ V} - 0.50 \Omega \times I$.

V falls linearly with I . When $I = 0$, $V = 1.50 \text{ V}$; when $I = 0.50 \text{ A}$, $V = 1.00 \text{ V}$ and so on, hence the graph sketched.

Note that $I_{\text{max}} = \frac{E}{r}$, corresponding to zero external resistance; we say that the cell is *short-circuited*. Cells get hot and run out of energy quickly when short-circuited. They don't like it.



(c) Batteries

Cells are often connected in series to produce a larger emf. In practice there is little point in using anything but identical cells.

Emf of battery = sum of emfs of cells in series

The positive of one cell must be connected to the negative of the next, and so on. The emf of any cell connected the wrong way round counts as a negative emf. See example in the Study point.

Self-test

2.3.8

Calculate (a) the current and (b) the total power generated by the cell, when a 5.0Ω resistor is connected across a cell of emf 1.50 V and internal resistance 0.50Ω .

Self-test

2.3.9

When two 2.5Ω resistors are connected in parallel across a cell the current through the cell is 0.88 A . When the two resistors are connected in series across the cell, the current is 0.28 A . Write down two equations, each involving E and r , and solve them simultaneously to determine the values of E and r .

Stretch & Challenge

For a cell of emf E and internal resistance r ,

(a) Sketch a graph of I against the resistance, R of the external load.

(b) Show that $\frac{1}{I}$ against R should be a straight line of gradient $\frac{1}{E}$ and intercept $\frac{r}{E}$.

Study point

Suppose a battery is built from 4 cells each with $E = 1.60 \text{ V}$, $r = 0.25 \Omega$.

Then for battery $\text{---}|||||\text{---}$

$E = 6.4 \text{ V}$, $r = 1.0 \Omega$.

But for misconnected battery $\text{---}|||||\text{---}$

$E = 3.2 \text{ V}$, $r = 1.0 \Omega$.

2.3.10

Self-test

A battery consists of 3 cells, each with $E = 1.60 \text{ V}$, $r = 0.25 \Omega$.

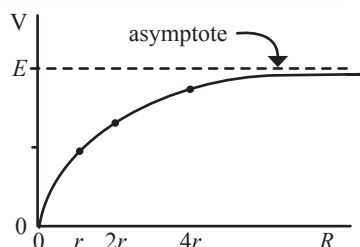
Calculate the current it drives through a load of 2.0Ω and the pd across the load.

Repeat for the case when one cell is the wrong way round.

2.3.11

Self-test

Sketch a graph of I against R for a power supply of emf E and internal resistance r . What are the values of I when $R = 0$, $R = r$, $R = \infty$?



Study point

The equation $V = \frac{ER}{R+r}$ can be interpreted as the pd across the output resistor, R , for a potential divider with input voltage E .

Stretch & Challenge

- Derive the above equation for the power output of a power supply.
- State the value(s) of P when R is zero and R is infinite.
- Sketch a graph of P against R .
- [For mathematicians] Determine the value of R (in terms of r) for which P is a maximum, and give the value of this maximum P [hint: calculus].

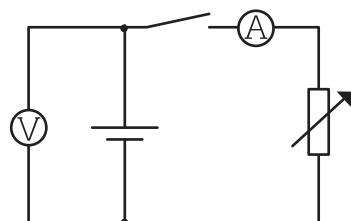


Fig. 2.3.22 Determining r by measuring V and I

Note also that:

Internal resistance of battery = sum of internal resistances of cells

Here there are no minus signs even when a cell is the wrong way round.

If n identical cells are connected in *parallel* (positive to positive and negative to negative) we'd have a battery with the same emf as one cell, but an internal resistance of

$$\frac{1}{n} \times \text{internal resistance of one cell.}$$

In practice, though, cells are never quite identical,

and, even without any load, there would be insidious currents in the cells and the wires joining them. The usual rule is: *never connect cells in parallel*.

(d) Current, pd and power outputs of a power supply

We can use the power supply relationship $V = E - Ir$, together with the usual equations for power, pd, current and resistance, to investigate how the various quantities vary with external load resistance.

- $V = Ir$: substituting for V in $V = E - Ir$ gives $IR = E - Ir$, which can be rearranged to give:

$$I = \frac{E}{R+r}$$

Thus the current in the circuit is the emf (which we can loosely think of as the 'total voltage') divided by the total resistance.

- $I = \frac{V}{R}$: this is slightly trickier, but substituting for I in $V = E - Ir$ gives

$$V = E - \frac{V}{R}r.$$

Multiplying by R and rearranging, which is left as an exercise, gives $V(R+r) = ER$, which leads to

$$V = \frac{ER}{R+r}$$

- Similarly we can show that the power output, $P = \frac{E^2R}{(R+r)^2}$

2.3.5 Determination of the internal resistance of a power supply

Section 2.3.4 provides the material for determining the internal resistance of a battery or other power supply.

(a) By measuring V and I

We set up the circuit in Fig. 2.3.22. The variable resistance could be a laboratory rheostat. [We are not, this time, using it as a potential divider.] It could also be several resistors. For this technique the actual values of the variable resistor are not important. They need to be of the same order as the internal resistance.

The idea is to take several pairs of readings of V and I (at least 7 is good), and to plot a graph of V against I . Because the relationship between V and I is

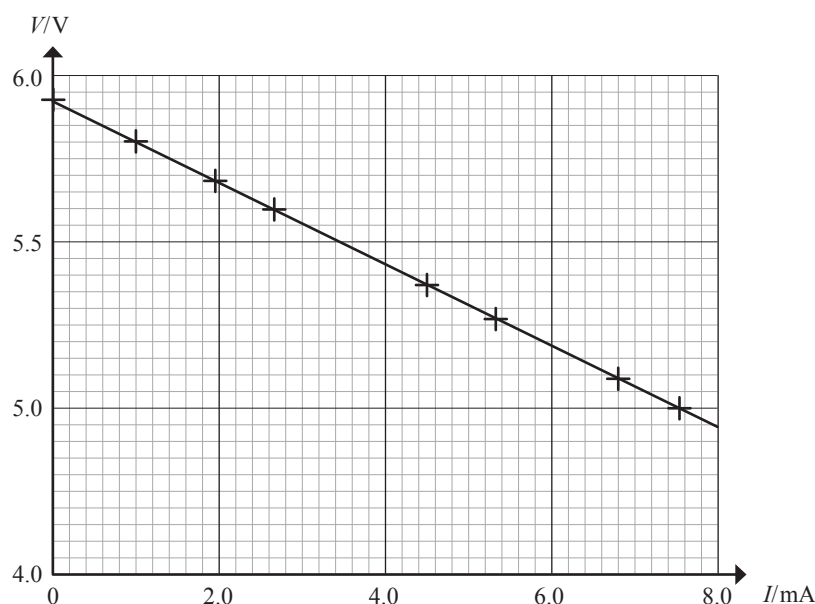
$$V = E - Ir$$

a linear graph is expected with a negative gradient, which is equal to $-r$, and an intercept of E on the V -axis.

The first readings should be with the switch open, so that $I = 0$. If an approximate value of the internal resistance is not known, trial readings need to be taken to establish a value of current which significantly reduces V and then a set of readings taken with roughly equally spaced currents up to several times this value.

The graph in Fig. 2.3.23 shows a typical set of results for a power supply with a high internal resistance.

Fig. 2.3.23
Typical V - I
graph results
for a power
supply



(b) By measuring I and the external resistance

If we measure (or know) the value of the external resistance, R , in the circuit of Fig. 2.3.22, we do not need to measure both the current and the terminal pd. If you have done the Stretch & Challenge in Section 2.3.4 (b) you will have shown that eliminating V from $V = E - Ir$ and $V = IR$ leads to the relationship:

$$\frac{1}{I} = \frac{1}{E} R + \frac{r}{E}$$

so a graph of $\frac{1}{I}$ against R has a gradient $\frac{1}{E}$ and intercept $\frac{r}{E}$.

(c) By measuring V and the external resistance

As we saw in Section 2.3.4(d), the terminal pd, V , depends upon the value of the external resistance, R , by:

$$V = \frac{ER}{R+r}.$$

Flipping this equation gives $\frac{1}{V} = \frac{R+r}{ER}$. We can separate the right-hand side into two terms to give

$$\frac{1}{V} = \frac{r}{E} \frac{1}{R} + \frac{1}{E},$$

so a graph of $\frac{1}{V}$ against $\frac{1}{R}$ is straight line of gradient $\frac{r}{E}$ and intercept $\frac{1}{E}$.

Practical tip

The switch should be open when not taking readings or adjusting the variable resistor; real cells suffer from a downward drift of emf under load – a nuisance we try to minimise.

Practical tip

In this experiment, there is little value in taking repeat readings. It is better to take a large number of pairs of values.

Self-test

2.3.12

For the power supply with the V - I graph in Fig. 2.3.23 determine: (a) the emf and internal resistance and (b) the current and terminal for an external resistance of 200Ω .

Self-test

2.3.13

State how the emf and internal resistance of a power supply can be found from the gradient and intercept of a graph of $\frac{1}{I}$ against R .

Self-test

2.3.14

State how the emf and internal resistance of a power supply can be found from the gradient and intercept of a graph of $\frac{1}{V}$ against $\frac{1}{R}$.

Exercise 2.3

1. In this question, subscripts refer to resistors, so V_{10} , I_{10} and P_{10} are respectively the pd across, current in and power dissipated by a $10\ \Omega$ resistor.

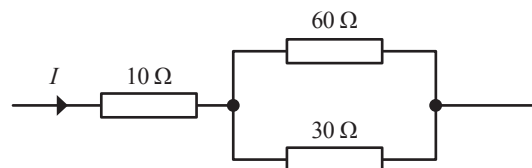
(a) Find I_{10} , I_{60} and I_{30} in terms of I .

(b) Find the ratio $\frac{V_{60}}{V_{10}}$

(c) Find the ratio $\frac{P_{10}}{P_{30}}$

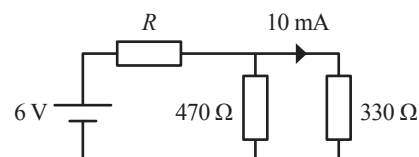
(d) If $I = 0.15\text{ A}$,

- (i) calculate the pd between the ends of the network and
(ii) the total power dissipated.



2. In the circuit shown, the power supply has negligible internal resistance. Determine the unknown resistance, R to 2 s.f.

3. Calculate the length of nichrome wire of diameter 0.2 mm which has a resistance equal to R in Q2. [Resistivity = $4.7 \times 10^{-7}\ \Omega\text{ m}$]



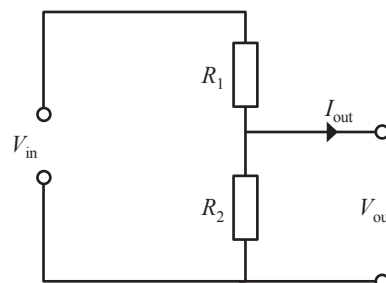
4. A 24 W , 12 V fish-tank heater element is made from a length, L , of resistance wire. Calculate the length of:

- (a) Wire of the same material and diameter which would be needed to make a 12 W , 24 V heater.
(b) Wire of the same material and double the diameter necessary to make a 12 W , 12 V heater.
(c) Wire of the double the diameter and half the resistivity needed to make a 12 W , 6 V heater.

Questions 5, 6 and 7 relate to the potential divider circuit.

5. $V_{\text{in}} = 12\text{ V}$, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$.

- (a) Calculate V_{out} if $I_{\text{out}} = 0$.
(b) Calculate V_{out} when a second $20\ \Omega$ resistor is connected in parallel to the first.
(c) Calculate V_{out} if $I_{\text{out}} = 0.15\text{ A}$.



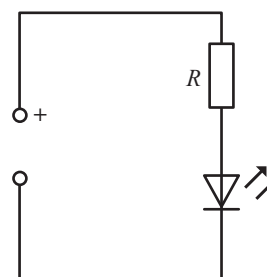
6. With the same components as in Q5, by calculating suitable values draw a graph of V_{out} against I_{out} .

7. Show algebraically that the potential divider circuit behaves as a power supply of emf $\frac{V_{\text{in}}R_2}{R_1+R_2}$ and internal resistance $\frac{R_1R_2}{R_1+R_2}$. [Hint: use the same working as in Q6 to find an expression for V_{out} in terms of I_{out} and write it in the form $V_{\text{out}} = a + bI_{\text{out}}$.]

8. The voltage across a light-emitting diode (LED) is roughly constant over a wide range of currents when it is in the conducting state and emitting light. This voltage is approximately 2 V for many LEDs.

An electronics application requires an on-indicator.

- (a) The engineer decides to use a red LED, with a current of 20 mA , powered by a 5.0 V supply, as in the diagram. Calculate the value, R , of resistor required.
(b) A batch of red, yellow, green and blue LEDs have turn-on voltages of 1.9 V , 2.1 V , 2.5 V and 2.8 V respectively. Select resistors to drive them from a 5.0 V supply if the current is to be $20 \pm 5\text{ mA}$. The resistors available are $100\ \Omega$, $120\ \Omega$, $150\ \Omega$, $180\ \Omega$, $220\ \Omega$.



9. In an experiment to determine the resistivity of nichrome, a student used an analogue micrometer to measure the diameter of the wire at various points and obtained the following results: **0.311 mm**, **0.316 mm**, **0.314 mm** and **0.317 mm**. The student used crocodile clips to attach an ohm-meter (multimeter on resistance scale) to the wire which was stretched along a metre rule. The resistance readings, R , for various lengths, l , were:

l / cm	20	30	40	50	60	70	80	90	100
R / Ω	2.15	3.15	4.00	5.08	5.99	7.10	7.95	8.85	9.87

The uncertainty in the l measurements was estimated as $\pm 1.0 \text{ cm}$.

Use the results to determine a value for the resistivity of nichrome. Comment on the intercept on the R axis.

Questions 10–13 relate to circuits containing a **6.0 V**, **60 mA** indicator bulb, with the characteristic shown, and a **100 Ω** resistor.

10. Use the graph to determine the resistance of the bulb when (a) it is operating at its rated voltage and (b) when the pd across it is **2.0 V**.

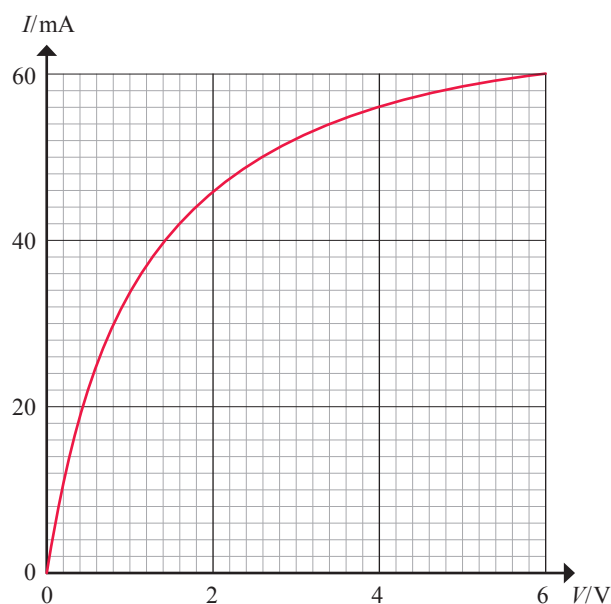
11. Calculate (a) the total pd across the bulb and resistor in series with a current of **45 mA**, and (b) the total current in a parallel combination of the two with a pd of **4.0 V** across them.

12. The bulb and resistor are connected in parallel and a total current of **80 mA** is in the pair. Determine the current in each and the pd across the pair. [Hint: first draw a current-voltage graph for the resistor on the same axes as for the bulb.]

13. A pd of **10 V** is applied across the bulb and resistor in series. Determine the pd across each and the current.

14. A student uses a set of three resistors, each of marked resistance **10.0 $\pm$ 0.2 Ω** , and a voltmeter to measure the internal resistance of a power supply. She assumed the resistors to be identical. She measured the pd across the terminals of the supply with one resistor and all possible series and parallel combinations of two and three resistors connected. She forgot to measure the pd with no resistor and she also failed to write down the combinations she used. Her pd readings (in V) were as follows: **7.92**, **8.58**, **8.82**, **6.88**, **6.07**, **8.34**, **7.36**

By identifying the possible combinations of resistors and calculating their resistances, decide which external resistance leads to each pd reading. Plot a suitable graph and determine the emf and internal resistance of the supply.



2.4 The nature of waves

How can energy be transferred from one place to another? Here are some examples.

- 1 A bullet fired from a gun carries kinetic energy.
- 2 Oil or gas flowing in a pipeline carries some kinetic energy, but, more importantly, chemical energy.
- 3 Seismic waves travelling through the Earth's crust, and tsunamis travelling through the sea, transport terrifying amounts of energy spread over short time intervals.
- 4 Sound waves travelling through air carry energy which can activate neurons in our inner ears, or generate electric 'signals' in microphones.

In 1 and 2, matter (stuff) travels from one place to another taking the energy with it; 3 and 4 are quite different: a 'disturbance', carrying energy, travels (or *propagates*) through a so-called *medium* (solid, liquid or gas). The particles of the medium are merely displaced temporarily from their usual positions as the disturbance, or *wave* passes through.

Even in a rock concert, except close to the loudspeakers, the maximum displacement of air molecules due to the sound waves is less than 0.1 mm. When a major tsunami reaches the shore the water particles may be displaced by many metres, but the wave itself may have travelled for tens or hundreds of kilometres.

Fig. 2.4.1 A tsunami

2.4.1

Self-test

Give two more examples of energy transfer: one enabling life on Earth, and one developed by human beings.

Terms & definitions

A (**progressive**) **wave** is a disturbance, or sequence of disturbances, travelling through a medium, taking energy with it, but not taking the particles of the medium with it.

2.4.2

Self-test

The speed of transverse waves on a stretched rope is given by

$$v = \left(\frac{T}{\mu}\right)^n$$

in which μ is the mass per unit length of rope, and T is the tension in it. By considering *units*, find the value of n .

2.4.3

Self-test

The speed of sound in air is given by

$$v = \sqrt{\frac{1.40p}{\rho}}$$

in which p is the air pressure. ρ is another property of the air. Suggest which, and confirm your suggestion by checking units.

2.4.1 How waves propagate

We'll look at a wave travelling in a taut rope. This is, arguably, the simplest example of a wave.

Suppose someone sharply displaces one end of the rope by a small amount in a direction at right angles to the rope itself. Fig. 2.4.2 (a) shows the rope shortly afterwards and (b) is a free body diagram for the small portion P of the rope. The resultant force on P is (almost) at right angles to, or *transverse* to the line of the rope, and so P will be accelerating (from rest) in that direction, and will itself shortly be displaced. In this way the transverse displacement is passed along the rope: (c).

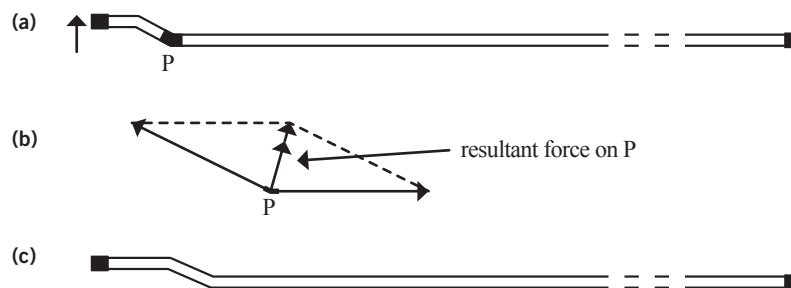


Fig. 2.4.2 Propagation of transverse waves on a rope

All mechanical waves (seismic waves, sound waves, water waves...) propagate in the same general way. A displaced part of the medium exerts an (unbalanced) force on a neighbouring part and so on. Usually the *speed* at which the wave travels depends on the properties of the *medium*. See Self-test 2.4.1 and 2.4.2 for examples.

2.4.2 Transverse waves

The waves in a rope (see 2.4.1) are *transverse*: each particle's displacement is at right angles to the direction of wave travel.

Another case of a transverse wave is the *secondary* (S) wave or *shear wave* (Fig. 2.4.3) that can travel through the Earth's crust from an underground event (such as the sudden slipping of one rock mass against another).

Light and other *electromagnetic* waves are also transverse. They are special as they can travel through a vacuum, where their speed, c , is $3.00 \times 10^8 \text{ m s}^{-1}$. [Their speed in air is the same to 3 significant figures.] Even though they don't need a medium, in many ways e-m waves behave just like other transverse waves. We'll present evidence for this later. Now read **Stretch and Challenge** – if you dare...

Fig. 2.4.4 (a) is a snapshot of a rope that is being wiggled at one end in a vibrating or *oscillating* motion. The transverse waves are *polarised* (strictly speaking, *linearly polarised*), meaning that the particle displacements are confined to just one of the possible directions at right angles to the direction of wave travel. Fig. 2.4.4 (b) also shows a polarised wave, but with a different direction of polarisation.

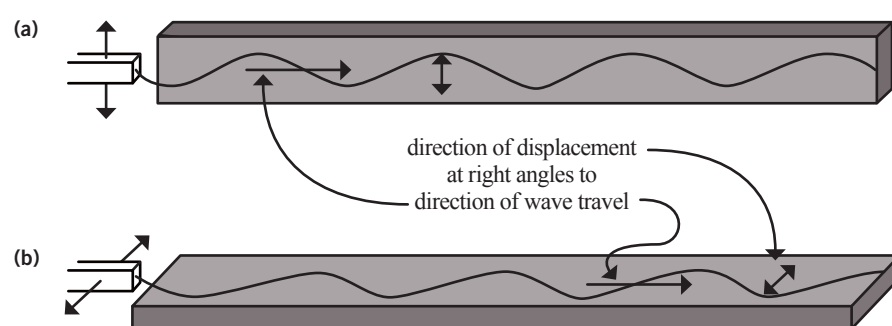


Fig. 2.4.4 Polarised transverse waves on a rope

If we wiggled one end of the rope in a random succession of different directions, each at right angles to the line of the rope, then the waves travelling in the rope would be called *unpolarised*.

2.4.3 Longitudinal waves

Shear waves can't travel through a gas or a liquid (because the successive layers (Fig. 2.4.3) of a gas or liquid can't exert tangential forces on each other without slippage and energy dissipation, that is transferred to random thermal energy).

Longitudinal waves (see **Terms and definitions**) *can* travel through a gas or liquid, as well as through a solid. The *primary* (P) waves travelling through the crust, mantle and core of the Earth from an underground event are longitudinal. So, too, are *sound* waves: in Fig. 2.4.5 the paper cone of the loudspeaker is oscillating backwards and forwards sending waves of rarefactions (R) and compressions (C) through the air. Longitudinal waves can also be sent through a *Slinky* spring (Fig. 2.4.6).

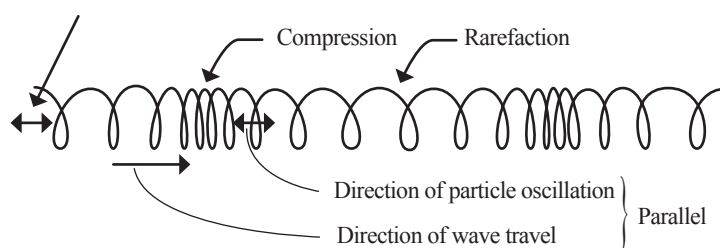


Fig. 2.4.6 Longitudinal wave on slinky

Terms & definitions

In a **transverse wave** the particles of the medium are displaced at right angles to the direction of wave travel.

In a **linearly polarised** transverse wave the displacements are in just one of the directions at right angles to the direction of wave travel.

In a **longitudinal wave** the particles of the medium are displaced parallel to the direction of wave travel.

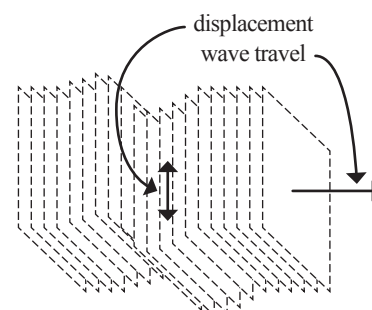


Fig. 2.4.3 Shear wave

Stretch & Challenge

Instead of *particles* oscillating as an **e-m wave** travels, it is vector quantities called *electric field strength* and *magnetic field strength* that oscillate at each point. Both vectors oscillate at right angles to the direction of travel of the wave (and to each other). When we talk of *displacement* for an e-m wave, by convention we mean the *electric field strength*. We shall come back to this. Until then, thinking of displacements in an e-m wave as like particle displacements will do.

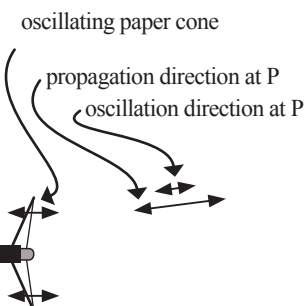


Fig. 2.4.5 Sound waves from a loudspeaker

Terms & definitions

A **cycle** is the smallest portion of an oscillation, starting at any point, which repeats exactly.

The **amplitude**, A , of an oscillation is the maximum value of the displacement. UNIT: m

The **period** (or **periodic time**), T , is the time for one cycle of oscillation. UNIT: s

The **frequency**, f , of an oscillation is the number of cycles per unit time. UNIT: hertz (Hz). [The unit 'hertz' means 'per second' (s^{-1}) but we use it only for oscillations.]

2.4.4

Self-test

A point on a taut string performs 56 cycles of oscillation in 35 s. Calculate:

- The shortest time for it to go from maximum to zero displacement.
- The frequency.

Terms & definitions

Oscillations of the same frequency are **in phase** if they are the same point in their cycles at the same time.

The **wavelength**, λ , is the distance between consecutive particles that are oscillating in phase. UNIT: m

2.4.5

Self-test

A student calls Fig. 2.4.7 a 'graph of a wave', he calls the distance from one peak to the next in Fig. 2.4.7 'the wave-length' and A 'the maximum amplitude'. Explain to him briefly and with only minimal scorn why he *must* be wrong in each case, and what he should have said.

2.4.4 Waves from oscillating sources

We shall now assume that *all* our wave sources are *oscillating* (vibrating) *sinusoidally*, as shown in the graph of displacement against time (Fig. 2.4.7). *Sinusoidal* describes the graph shape. Note that it is *periodic* – repeating the same cycle over and over again. The displacements of the particles in the medium will also vary sinusoidally.

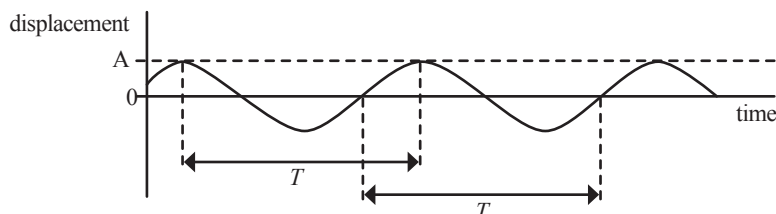


Fig. 2.4.7 Displacement–time graph for oscillating particle

Referring to the graph, we define the basic terms in the **Terms and definitions** box.

Example

What is the period of oscillations of frequency 10 Hz?

10 cycles will fit into 1 second, so the period is $\frac{1}{10} \text{ s} = 0.10 \text{ s}$.

Generalising, $T = \frac{1}{f}$, which can be re-arranged as $f = \frac{1}{T}$.

Note these points:

- All particles of the medium oscillate with the frequency of the wave source; cycles of oscillation can't be lost or gained as a wave travels!
- Why does the graph start at that particular point in its cycle? Where in the cycle it is at the time we choose to call $t = 0$ depends on how far away the particle is from the wave source.

2.4.5 Snapshot of a wave

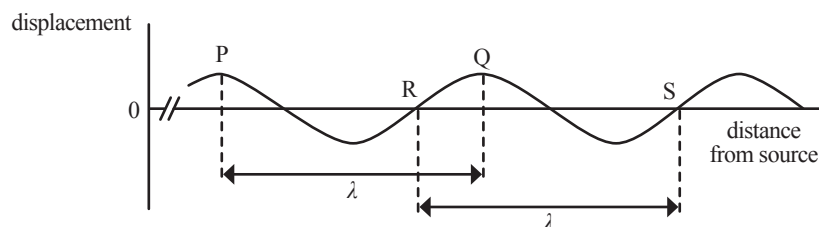


Fig. 2.4.8 Displacement – distance from source at some particular time

The graph above (Fig. 2.4.8) looks like Fig. 2.4.7 but note that, now, *distance from the source* at one particular time is along the horizontal axis! We'll call this sort of graph a *snapshot*. For a wave in a stretched rope it could be a single flash photograph of the rope, with axes added.

[For waves that spread out as they travel further from the source, the more distant particles will oscillate with smaller amplitudes, as the energy is more thinly spread. Our graphs won't usually show this.]

Particle Q is oscillating in *phase* with particle P. Another example of in-phase particles is the pair R and S. Make sure you know the definitions of *in phase* and *wavelength* from the **Terms and definitions** box.

2.4.6 Speed of a wave

The whole pattern of displacements in the medium is moving to the right, away from the source. That's what we mean by the wave *travelling*! In Fig. 2.4.9 the full line is a snapshot of the wave at time, $t = 0$, and the dashed line is a snapshot of the wave at time $\frac{T}{4}$.

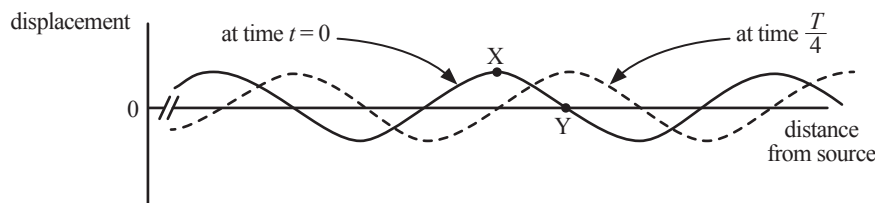


Fig. 2.4.9 Snapshots of wave at time 0 and time $T/4$

Suppose we let a whole period, T , elapse between one snapshot and the next. Each particle will oscillate through one extra cycle, bringing it back to where it started. So the snapshot will be unchanged. Yet the pattern will have moved – away from the source. It must, then, have moved on by a whole wavelength, λ .

$$\text{So: wave speed, } v = \frac{\text{distance gone}}{\text{time taken}} = \frac{\lambda}{T} = \frac{1}{T}\lambda = f\lambda$$

We have, then, the well-known equation $v = f\lambda$.

Example

Sketch displacement–time graphs starting at $t = 0$ for particles X and Y in Fig. 2.4.9, and comment on the phase difference.

Extrapolating from the positions of X and Y at time t and time $\frac{T}{4}$ we have

Note that each peak (or any other feature) of Y's graph occurs time later than the corresponding feature on X's graph. We say Y has a phase lag of a quarter of a cycle behind X. This phase lag is just what we'd expect: the wave reaches Y *after* it reaches X!

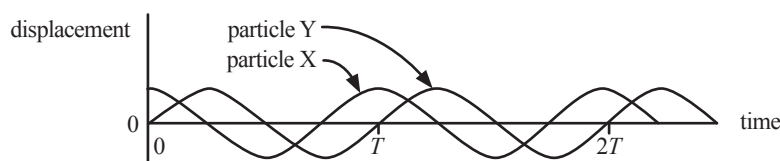


Fig. 2.4.10 Displacement – time for particles X and Y in Fig 2.4.9

2.4.7 Wavefront diagrams

The moving peaks and troughs we see when water waves are travelling are examples of *wavefronts*. Water waves, like waves on a drum-skin, travel in 2 dimensions, and their wavefronts are essentially lines, usually curved lines. In **Terms and definitions**, we've referred to wavefronts as 'surfaces', because we're thinking especially of waves like sound and light which travel out from a source in all directions, that is in 3 dimensions.

- We usually draw wavefronts at intervals of one wavelength, like the peaks (crests) of a water wave.
- The direction of travel of a wavefront at any point is at right angles to the wavefront through that point. (See Fig. 2.4.11).
- Wavefronts from a small source are spherical. A long way from the source they are therefore almost plane (flat) over any small region. For example, the wavefronts of light reaching the Earth from a star will be, to all intents and purposes, plane.

Self-test

2.4.6

Water waves from a source of frequency 3.0 Hz travel across a pond at a speed of 3.6 m s^{-1} . Calculate the shortest distance between water particles oscillating in antiphase (exactly half a cycle out of phase with each other).

Stretch & Challenge

Water waves are neither transverse nor longitudinal. The particles move in vertical circles with one diameter parallel to the propagation direction. The circles get smaller with increasing depth.

Self-test

2.4.7

Mobile phones transmit and receive e-m waves of frequency in the order of 900 MHz , using a built-in aerial – a metal rod. Ideally this ought to be at least a quarter of a wavelength long. Investigate whether such an aerial could fit in a mobile phone.

Terms & definitions

A **wavefront** is a surface at all points on which the oscillations are in phase.

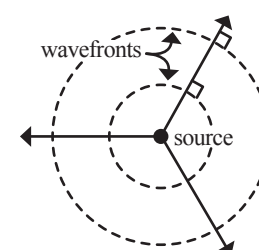


Fig. 2.4.11 Wavefronts from a point source

Study point

(This Study Point relates to Section 2.4.8 on page 128.)

The interaction of an e-m wave with matter is mainly by the electric field, so this is normally taken to define the plane of polarisation.

Study point

The effectiveness of the polarising filter normally varies across the visible spectrum. If the filters are most effective in the middle of the range (yellow), the filters will transmit low intensity magenta (red + blue) even when they are crossed.

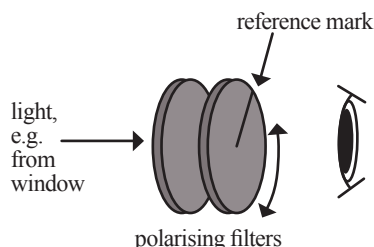


Fig. 2.4.12 Investigating polarisation

2.4.8 Investigation of the intensity variations for polarisation

A qualitative investigation of the effect of a polarising filter on a polarised beam of light requires nothing more than two filters and a light source, which could be one of the laboratory room lights or from the window.

The light emerging from the first filter is plane polarised, i.e. the oscillations of the electric field are all in the same plane, which is at right angles to the direction of propagation.

It is useful to put a reference mark on one of the filters, so that you can keep track of the angle through which it is rotated.

As the second filter is rotated, the intensity of the transmitted light varies smoothly; two maxima and two minima are observed per rotation. These are equally spaced, i.e. there is 90° between each maximum and adjacent minima.

The effect can also be investigated quantitatively using the setup shown in Figs 2.4.13 and 2.4.14.

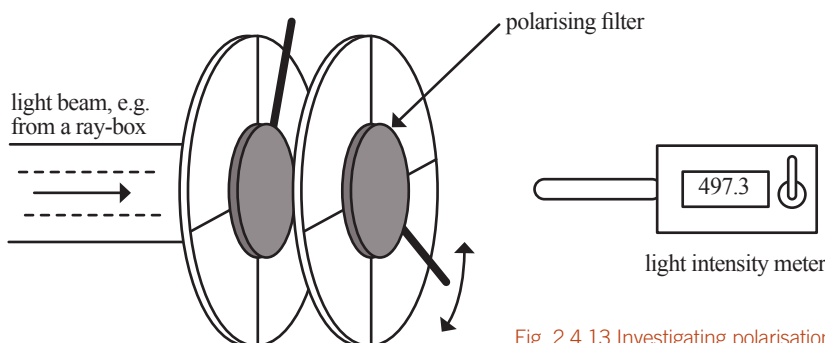


Fig. 2.4.13 Investigating polarisation

Fig. 2.4.14 Rotatable polarising filter

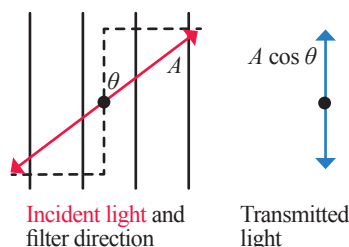


Fig. 2.4.15 Effect of 2nd polarising filter

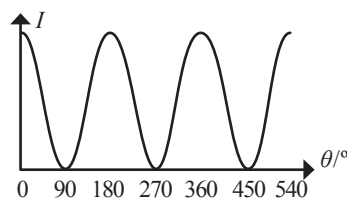


Fig. 2.4.16 Variation of intensity with angle

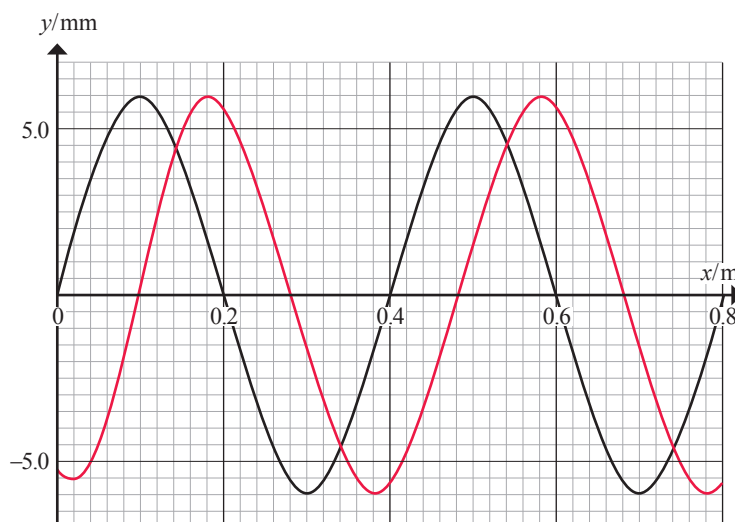
The light transmitted through a 100% polarising filter will all be polarised with all its oscillations in the same plane, called the plane of polarisation (see Study point). If a second such filter is placed in the path of the light, any emergent light will be 100% polarised in the direction defined by this filter. The plane of polarisation of the light is **not rotated** by the second filter: only the component of the oscillating vector parallel to the transmission plane of the second filter is transmitted; the component at right angles is absorbed. In Fig. 2.4.15, light of amplitude A (red arrow) is oscillating at θ to the transmission plane of a polarising filter (vertical lines). The transmitted light (blue arrow) has a smaller amplitude and is polarised in the direction defined by the transmission plane.

Using simple trig, if the angle between the polarisation directions of the two filters is θ , the amplitude of the light which is transmitted through the second filter is $A \cos \theta$, where A is the amplitude of the light between the filters. As with all wave motions, the energy carried by the light is proportional to the square of the amplitude, so the intensity, I , of the transmitted light varies as: $I \propto \cos^2 \theta$ assuming a 100% effective filter. This relationship is shown in Fig. 2.4.16.

Exercise 2.4

- The diagram shows two snapshots of the same transverse wave, travelling in the positive x -direction, the red snapshot being 0.25 ms later than the black.
 - Give the wavelength and amplitude of the wave.
 - Explain why it is not possible to calculate the speed of the wave just from the information given.

- (c) Calculate the lowest possible wave speed which is consistent with this information.
- (d) Calculate the period and frequency of the wave corresponding to the speed in (c).
2. Consider the wave in Q1 at the instant of the black diagram.
- (a) State the values of x for which the vertical velocity of the particles in the medium is (i) zero, (ii) maximum positive and (iii) maximum negative.
- (b) [Tricky] Use the speed calculated in 1(c) and the graph to estimate the maximum vertical velocity of the particles.
3. For the wave in Q1, give the x co-ordinates of all points in between $x = 0$ and $x = 2.0$ m which are oscillating (i) in phase with and (ii) in antiphase with the particle at $x = 0.15$ m.
4. Diagram (a) is an **actual size** drawing of a longitudinal wave moving from left to right at 5.0 m s^{-1} .



The vertical lines represent planes of the material which are equally spaced in the absence of a wave.

(a)

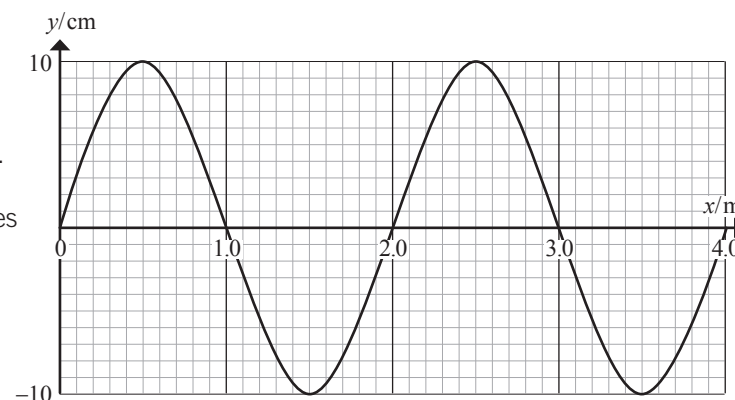
Diagram (b) shows an enlarged wavelength of the same wave.

(b)

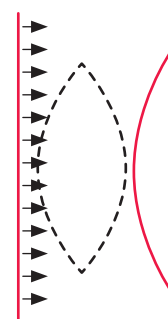
Use a ruler to take suitable measurements of the diagrams and hence determine:

(a) the wavelength, (b) the frequency, (c) the period and [tricky] (d) the amplitude of the wave.

5. The diagram shows a snapshot of a wave at time $t = 0$. The wave is moving to the right with a speed of 5.00 m s^{-1} .
- (a) Calculate the frequency, f , and period, T , of the wave.
- (b) Find the gradient of the wave at $x = 2.0$ m and use this to find the maximum vertical speed of the particles of the wave.
- (c) Sketch a graph to show the vertical movement of the point with $x = 2.0$ m between $t = 0$ and $t = 1.00$ s.



6. A beam consists of a mixture of 70% polarised and 30% unpolarised light. A perfect polarising filter (i.e. it transmits 100% of the light vibrating in one plane of polarisation and 0% of the light vibrating at right angles) is put into the beam and rotated about its axis. Sketch a graph to show how the intensity of the transmitted radiation varies with the angle of the filter.
7. The diagram shows a wavefront in a ripple tank approaching (from the left) and crossing a submerged barrier, which has the effect of slowing the wave and deforming its wavefront.
- Copy the diagram. Add arrows to the second wavefront to show the direction(s) of propagation and show the position(s) of the wavefront at two successive instants – the time intervals between the four wavefronts should be approximately equal.
8. [For A level students]. The equation $y = 10 \cos 6.28x$ (with y in cm and x in m) represents a wave at time $t = 0$. The wave propagates with a speed of 800 m s^{-1} . A student suggests that the equation of motion of a particle at $x = 2.5$ m is given by $y = A \cos(\omega t + \phi)$. Find the values of A , ω and ϕ that make this statement correct.



2.5 Wave properties

2.5.1 Diffraction

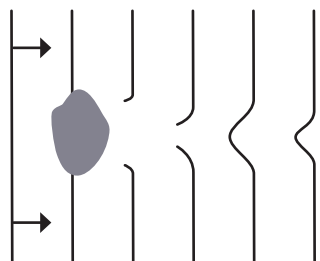


Fig 2.5.1 Water waves diffracting round a rock

Diffraction is the spreading of waves round obstacles in their way. For example, when there is a rock in the path of waves at sea, after passing the rock to either side, the wavefronts spread back into the 'shadow' of the rock, as sketched in Fig. 2.5.1.

We shall concentrate on diffraction around the edges of a slit. Fig. 2.5.2 shows straight wavefronts in a 'ripple tank' approaching a slit, and having gone through, spreading out beyond its edges.

When the slit's width is equal to, or less than, the wavelength, the diffracted wavefronts at some distance from the slit are more or less semi-circular as in Fig. 2.5.3 (a), though the amplitude is greater in the middle than at the edges.

When the slit width is several times greater than the wavelength as in Fig. 2.5.3 (b), there is a main or central beam of diffracted waves that spreads through only a small angle either side of the 'straight-through' direction. There are also 'side' beams of much smaller amplitude than the main beam. These side beams can also be seen in the ripple tank image. We'll see later how, in principle, this rather complicated behaviour arises.

Fig. 2.5.2 Single slit diffraction

2.5.1

Self-test

Using information from Fig. 2.5.2, calculate a rough value for the angle of spread of the main beam either side of the straight-through direction.

2.5.2

Self-test

You can hear the news blaring out from a neighbour's radio when the windows are open, but you can't always make out the words. Explain this.

Hint: consonants contain higher frequencies of sound (> 3000 Hz).

Speed of sound = 330 ms^{-1} .

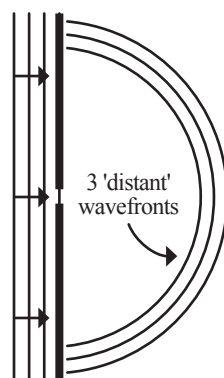


Fig. 2.5.3 (a) Diffraction: slit width $\leq$ wavelength

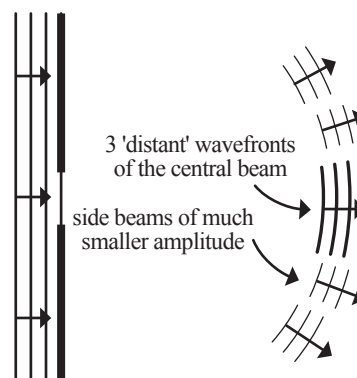


Fig. 2.5.3 (b) Diffraction: slit width $\gg$ wavelength

We don't usually observe *light* diffracting around obstacles such as the sides of holes or slits. Indeed a pinhole camera relies on light not spreading much, and so not changing its direction noticeably, as it passes through the pinhole. Close examination of the picture might reveal some blurring due to diffraction, but the set-up in Fig. 2.5.4 demonstrates diffraction of light unmistakably.

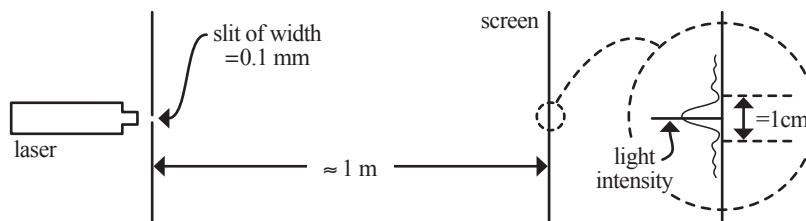


Fig. 2.5.4 Demonstrating diffraction of light at a slit

The very small angular spread of the main beam through this narrow slit, compared to that in Fig. 2.5.2, suggests that the wavelength of light is very small indeed. We shall soon describe how to confirm this.

2.5.2 Interference

Interference is probably the most distinctive wave effect: if you can demonstrate it taking place with some unknown ‘radiation’, you know that waves are involved!

Interference is what happens when waves from more than one source, or waves travelling by different paths from the same source, *superpose* or ‘overlap’ in the same region. The behaviour is governed by the **principle of superposition** as stated in **Terms & definitions**.

As an example of this, suppose two identical but inverted pulses are travelling at 1.0 m s^{-1} in opposite directions along a taut string, as shown at time $t = 0$. This is shown in Fig. 2.5.5(a).

What does the string look like at times $t = 1.0 \text{ s}$ and $t = 2.0 \text{ s}$?

Applying the principle of superposition, we get the situations shown in Fig. 2.5.5(b)

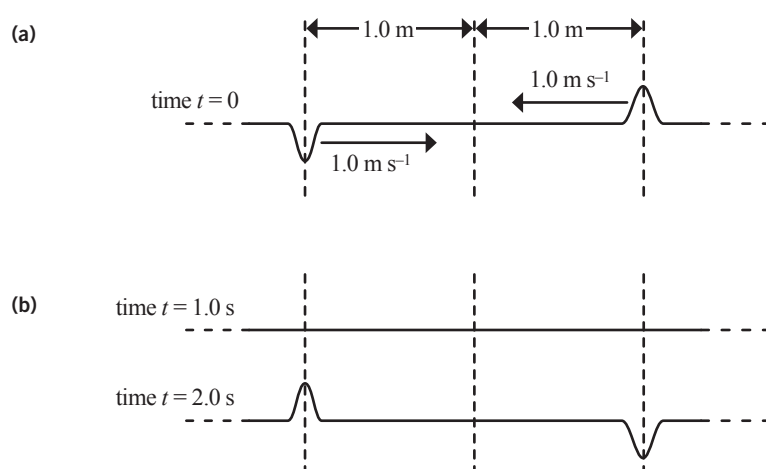


Fig. 2.5.5 Superposition in progress

(a) The two-source interference pattern

Fig. 2.5.6 is a snapshot of a ripple tank when two small rods, S_1 and S_2 , vibrating *in phase*, move up and down in contact with the water surface. These are the wave sources. Note the ‘beams’ of waves (dark blobs: troughs; light blobs: peaks) separated by ‘channels’ of very low amplitude.

The crosses marked P and R are examples of points where the wave amplitude is (locally) highest, because waves from S_1 and S_2 arrive **in phase** there, and interfere **constructively**. This is shown graphically in Fig. 2.5.7. [Note that the *displacement* isn’t necessarily a maximum at P and R at the instant of the snapshot.]

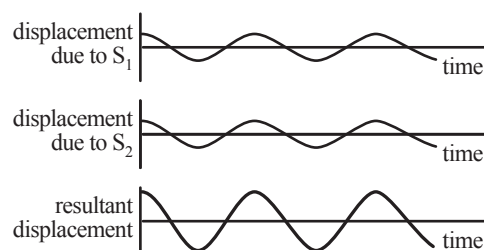


Fig. 2.5.7 Constructive interference

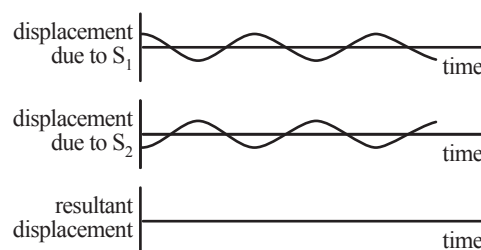


Fig. 2.5.8 Destructive interference

Terms & definitions

The **principle of superposition**:

The resultant displacement at each point is the vector sum of the displacements that each wave passing through the point would produce by itself.

Self-test

2.5.3

In the example, all traces of the pulses seem to have vanished at $t = 1.0 \text{ s}$. How can they re-appear? Where has the energy gone?

Hint: it’s the *displacement* that’s zero at $t = 1.0 \text{ s}$.

Self-test

2.5.4

Suppose two beams of light cross over: how, if at all, will they have been affected by each other *beyond* the region of overlap?

S_1	+ R
	+ Q
	+ P
S_2	
	+ U
	+ V
	+ W

Fig. 2.5.6 Two-source interference

Study point

Waves from different sources pass through one another without themselves being changed. This allows them to combine by superposition at any point.

2.5.5

Self-test

In Fig. 2.5.6 determine λ by measuring the distance along a beam between centres of dark blobs separated by a number of wavelengths, and dividing by that number. Now measure S_1R and S_2R and comment on whether your results agree with the path difference rules.

2.5.6

Self-test

In Fig. 2.5.6 determine the path differences for points U and V in terms of wavelength.

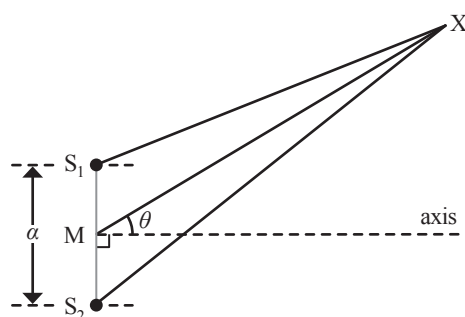


Fig. 2.5.9 Calculating path difference

2.5.7

Self-test

Two transmitters, 120 m apart, are sending out radio waves of wavelength 24 m, in phase and polarised vertically. One is sited 60 m North of a point M, and the other is 60 m South of it. Determine what type of interference there will be at a distant point, P, at 60° East of North from M.

Where the *amplitude* is lowest in the two source pattern, at Q, for example, the waves from S_1 and S_2 are arriving in antiphase (half a cycle out of phase) and are interfering *destructively*: Fig. 2.5.8.

Note that cancellation can never take place if the component waves are vibrating at right angles to each other. To produce an interference pattern, transverse waves cannot be polarised at right angles to each other.

(b) Path difference

Even without seeing the pattern, we can predict where, in relation to the sources, S_1 and S_2 , there will be constructive or destructive interference. For example, we know that there must be constructive interference at points such as P in Fig. 2.5.6, which are equal distances from S_1 and S_2 . This is because S_1 and S_2 are oscillating in phase, so the waves from the two sources, having travelled by *paths*, S_1P and S_2P , will *arrive* in phase at P. The same applies for all points on the central axis (the central line through the point midway between S_1 and S_2 and perpendicular to S_1S_2).

For point R, the paths are S_1R and S_2R . The *path difference*, $S_2R - S_1R$ is 1 wavelength, so waves from S_2 arrive at R a whole cycle later than waves from S_1 . This means that they arrive at R *in phase* with the waves from S_1 , so there will be constructive interference at R.

For point Q, the path difference $S_1Q - S_2Q$ is half a wavelength, so waves arrive at Q in antiphase and will destructively interfere.

The general rules for waves from in-phase sources are:

For constructive interference at a point X,

$$\text{path difference, } |S_1X - S_2X| = 0, \lambda, 2\lambda, 3\lambda \dots$$

That is: path difference, $|S_1X - S_2X| = n\lambda$, where $n = 0, 1, 2, 3 \dots$

For destructive interference at a point X,

$$\text{path difference, } |S_1X - S_2X| = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \dots$$

That is: path difference, $|S_1X - S_2X| = (n + \frac{1}{2})\lambda$, where $n = 0, 1, 2, 3 \dots$

Example

Referring to point W in Fig. 2.5.6, determine the path difference, $|S_1W - S_2W|$ in terms of λ .

W is on the mid-line of a beam of waves, so there is constructive interference here. So $|S_1W - S_2W| = 0, \lambda, 2\lambda$ or $3\lambda \dots$. The point P is on the perpendicular bisector of S_1S_2 so W is equidistant from the two points, i.e. the path difference at P is zero. So counting the centre beam as 'beam zero', we find W on the third beam 'out', so $|S_1W - S_2W| = 3\lambda$.

There's a simple **equation** for *calculating* the path difference, in terms of where X is situated (if it's far enough away) in relation to S_1 and S_2 .

$$S_2X - S_1X \approx a \sin \theta$$

in which a and θ are shown in Fig. 2.5.9. M is the point midway between S_1 and S_2 . The *axis* is the line through M perpendicular to the line S_1S_2 .

The equation is an approximation, but the error is less than 1% if MX is greater than $4a$. The equation becomes exact when X is so far away that S_2X and S_1X can be taken as parallel. For this 'ideal' case we shall derive the equation – in one line – when we deal with the *diffraction grating*.

2.5.3. Young's fringes experiment

In the early 1800s, Thomas Young investigated light passing through two parallel slits (or, sometimes two pinholes) close together. He observed a pattern of light and dark *fringes* (stripes) on a screen placed some distance from the slits. He recognised this pattern as a section through an interference pattern, and deduced that light was wave-like. He ran various checks, such as moving the screen nearer the slits and finding that the fringes became closer together. [Compare horizontal sections through Fig. 2.5.6, at different distances from the sources.] Young was also able to determine the wavelengths of different colours of light.

Here is a modern version of Young's experiment.

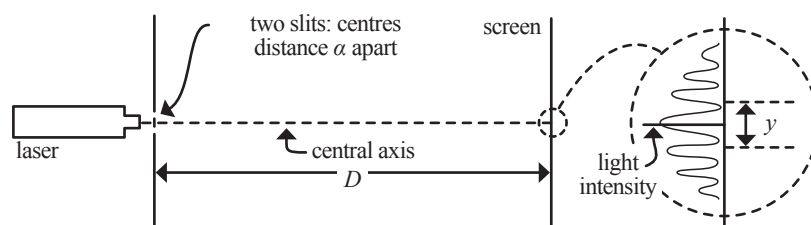


Fig. 2.5.10 Modern Young's *fringes* experiment

With $a = 0.5 \text{ mm}$ and $D = 1.5 \text{ m}$, and using red light, the fringe separation, y (between the centres of adjacent bright fringes or adjacent dark fringes) is found to be about 2 mm .

How does it work? The slits act as sources. Light passing through them spreads out slightly by *diffraction* so, if the slits are about 0.1 mm wide, there will be a few millimetres on the screen 'above' and 'below' the axis, where light from both slits overlaps and interferes. [This would still happen without the screen!]

The wavelength of light can be found from the equation

$$\lambda = \frac{ay}{D}$$

This is an approximation based on the *path difference* rules. As long as $a \ll D$ and $y \ll D$, as in the set-up described, the equation is almost exact. To see where it comes from, go to Stretch & Challenge.

Coherence

If we illuminate each slit with light from a different source, we find we can never produce fringes. Even using a single source such as an LED we can obtain fringes only by taking special precautions (such as putting a single narrow slit between the LED and the double slits). Using a laser, though, the simple arrangement shown above gives beautiful fringes. This is because a laser produces *coherent light*: see **Terms & definitions**.

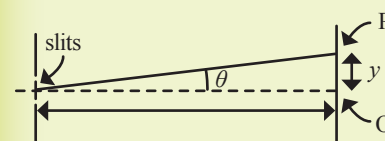
If we could illuminate both slits with a laser beam accurately normal to the plane of the slits, these would act as in-phase sources because each wavefront in the beam would hit both at the same time. This is not usually achieved, but the slits will still act as *coherent sources*, meaning that there will be a *constant phase relationship* between them. A fringe

Stretch & Challenge

Suppose the slits act as in-phase sources. Then there will be a bright fringe at O (see diagram). Let the first off-axis bright fringe be at P.

Then $S_2P - S_1P = \lambda$. Since P is far from the slits, we can write this as a $\sin \theta = \lambda$. Use the small angle approximation to show that

$$\lambda = \frac{ay}{D}$$



Self-test

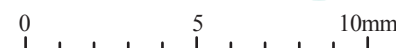
2.5.8

Why does fringe intensity fall off with distance from the central axis?

Hint: at still greater distances from the axis, fringes re-appear, but faintly.

Self-test

2.5.9



Determine the fringe spacing from the enlarged picture of the fringes with millimetre scale, and hence determine the wavelength of the light, if $a = 0.40 \text{ mm}$ and $D = 0.80 \text{ m}$.

Terms & definitions

A beam of **coherent light**

- is nearly **monochromatic**, that is a continuous stream of oscillations of a single frequency,
- has wavefronts extending across its width, as if it came from a point source.

Two or more sources are called **coherent** if there is a *constant phase relationship* between their oscillations.

pattern will still be seen with the same fringe separation, but there probably won't be a bright fringe on the axis.

The light from an 'ordinary' source such as an LED is far from coherent. Even that from a 'coloured' LED contains a range of frequencies. This in itself wouldn't prevent us seeing a fringe pattern (actually many super-imposed patterns), but there is no fixed phase relationship between light emitted from points all over an emitting area in the order of 1 mm^2 , so the LED doesn't meet the *point source* condition for producing coherent light (at least not without the single narrow slit mentioned above).

Study point

In a real diffraction grating, often made by a photographic process, there isn't always a sharp transition between opaque and clear.

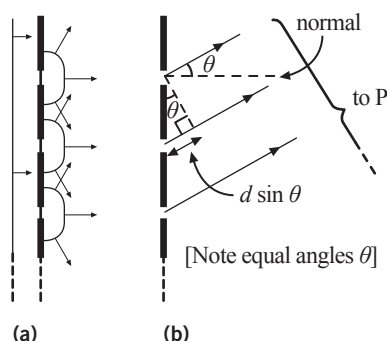


Fig. 2.5.11 Diffraction grating

2.5.4 Accurate measurements of wavelength

These are needed in many areas of science. For example, astronomers measure the wavelengths of light emitted and absorbed by a distant star, and from these wavelengths can identify elements present in the star's outer layers, determine the star's line-of-sight velocity, and perhaps infer the existence of orbiting planets.

For this application, and for most others, the two slit set-up won't do. It is impossible to measure the fringe separation accurately enough, because:

- The fringes aren't sharp: bright fringes fade gradually into dark (Fig. 2.5.10).
- The brightest parts of the pattern are not as bright as if the light were concentrated on them. This matters if the light source is faint.
- The fringe separation is small.

The diffraction grating deals successfully with all three of these issues.

(a) The diffraction grating

At its simplest, a diffraction grating is a flat plate, which is opaque except for thousands of straight, parallel, equally-spaced slits.

For a grating to be used with visible light, the distance, d , between the centres of adjacent slits is typically 2 or $3 \text{ }\mu\text{m}$. The makers supply the value of d .

We shall assume that light is shone normally (at right angles) on to the grating, so that the slits act as in-phase sources. Each slit is very narrow (in the order of a wavelength wide), so the diffracted wavefronts are almost semi-circular, spreading right round. See Fig. 2.5.11(a).

Now consider the light arriving at a distant point, P, from each slit. The light paths from the slits to P will be almost parallel, so the *path difference* between light from adjacent slits will be $d \sin \theta$, as we see from the small right-angled triangle constructed in Fig. 2.5.11(b). The condition for constructive interference at P is therefore

$$d \sin \theta_n = n\lambda \text{ for } n = 0, 1, 2 \dots$$

We can see beams emerging from the grating at these angles θ_n if we sprinkle dust in the air around the grating, or let the beams brush against a piece of paper. Note how the beams are called *zeroth order*, *first order*, *second order* and so on, according to the value of n .

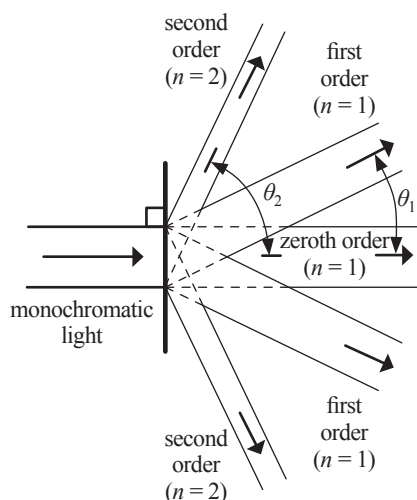


Fig. 2.5.12 Diffraction grating orders



Example

A laser beam is shone normally on to a grating with 5.00×10^5 slits per metre. The angle to the normal of the second order beams is 35.1° .

(a) Calculate the wavelength of the light.

(b) Calculate the number of beams emerging from the grating.

(a) Since there are slits per metre the distance, d , between centres of slits must be

$$d = \frac{1.00 \text{ m}}{5.00 \times 10^5} = 2.00 \times 10^{-6} \text{ m}$$

$$\text{So } \lambda = \frac{d \sin \theta_2}{2} = \frac{2.00 \times 10^{-6} \text{ m} \sin 35.1^\circ}{2} = 532 \times 10^{-9} \text{ m} = 532 \text{ nm}.$$

(b) When $\theta = 90^\circ$, $\sin \theta = 1$, so the path difference between light going to a distant point from adjacent slits is simply d , its greatest possible value – as should be clear without trigonometry! The number of wavelengths contained in distance d is simply

$$\frac{d}{\lambda} = \frac{2.00 \times 10^{-6} \text{ m}}{532 \times 10^{-9} \text{ m}} = 3.76.$$

At certain lower values of θ , $d \sin \theta$ must equal 3λ , 2λ , λ and 0 .

So 7 beams emerge: the zeroth order and 3 orders either side.

If the beams from the grating fall on a screen, as shown in Fig. 2.5.13, they produce bright spots on the screen. This is easily demonstrated using a laser shone normally at the grating. See **Stretch and Challenge**.

These dots correspond to the bright fringes in Young's experiment, but they are much further apart, because the slits are much closer together in the grating than in Young's double slit experiment ($d \ll a$).

There is another major difference between the performance of the grating and the double slit. In the grating, if $d \sin \theta = n\lambda$, light from each slit will reach the distant point in phase with light from its neighbour. Therefore light from *all* slits will arrive there in phase. So the same *constructive* interference condition applies however many slits there are. Now take an angle, θ , which is slightly larger than θ_1 , e.g. such that $d \sin \theta = 1.1\lambda$. In the double slits case, the waves from the two slits arrive almost in phase, so the bright fringe would still be almost as bright at this angular displacement. But for the grating there would be a path difference of 0.5λ between light from the 1st slit and the 6th slit, the 2nd slit and the 7th slit ... the 5th and the 10th, the 11th and the 16th and so on. Thus light from *all* the slits except, maybe, a few left over, will destructively interfere at this angle. So the bright fringe will *not* extend this far. This illustrates the essential point that the fringes or *lines* are sharper for the grating than the double slit because more slits give more possibilities for destructive interference (but no more possibilities for constructive).

2.5.5 Making sense of diffraction

We introduced *diffraction* in 2.4.8 as a *phenomenon*: something that happens! Having now discussed *interference*, we can begin to understand why diffraction happens in the way it does.

We start with a suggestion, made by Christiaan Huygens in the 1670s, as to how wavefronts advance. Each point on a wavefront behaves like a point source, sending out low amplitude 'wavelets' in all 'forward' directions, so that, in section, their wavefronts are semicircles. This is shown in Fig. 2.5.14 for waves diffracting through a slit (less than λ wide). A later position

Self-test**2.5.10**

A diffraction grating has lost its label. The slit separation, d , is determined by shining a beam of light of wavelength **650 nm** normally at it and noting that the second order beam emerges at 48° . Calculate the wavelength which gives a third order beam at 54° with this grating.

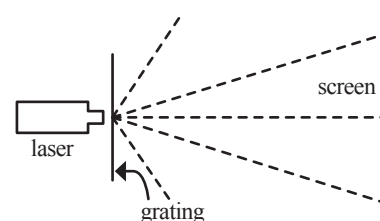


Fig. 2.5.13 Bright spot production

Stretch & Challenge

In more advanced work, the light source for a diffraction grating is an illuminated slit, and a lens is used to form a parallel beam incident normally on the grating. The beams from the grating are focused by another lens to form images of the slit – bright lines if the slit is illuminated with monochromatic light. This is why single wavelengths are often called spectral lines.

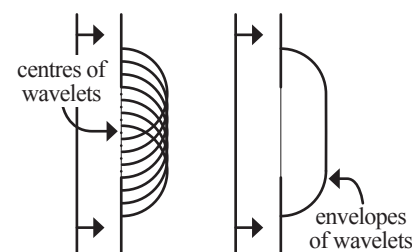


Fig. 2.5.14 Wavelets and envelope

2.5.11

Self-test

Summarise in two words what limits the spread of the main diffracted beam for a slit wider than λ .

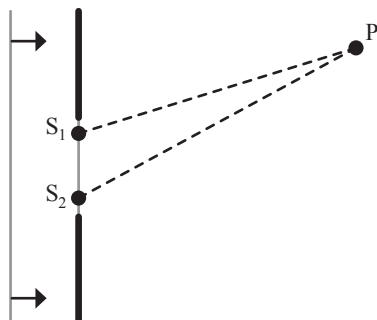


Fig. 2.5.15 Path difference

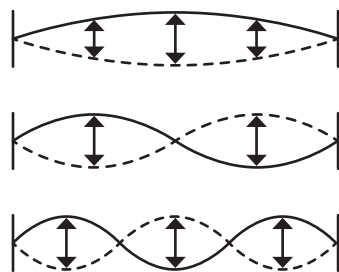


Fig. 2.5.16 Stationary waves on a string or wire

Terms & definitions

In a stationary wave, a **node** is a point of minimum and an **antinode** a point of maximum amplitude.

2.5.12

Self-test

Draw the next stationary wave in the sequence shown in Fig. 2.5.16, i.e. one with five nodes.

Terms & definitions

The **internodal distance** is the distance between the nodes of a stationary wave. It is $\frac{1}{2}\lambda$.

of the *observed* wavefront is found by drawing the *envelope* of the wavelets. The result, seen in section, is a sort of flat-fronted semicircle. The flat bit ceases to show as the wavefront advances and expands. This is just what we observe in a ripple tank.

Huygens's construction needs modifying to account for what happens when the slit width is not small compared with λ . We need to think of each point in the slit as an oscillating source of wavelets of wavelength λ . We find the displacement at any point, P, 'ahead' of the slit by applying the principle of superposition to the wavelets arriving there.

For a slit of width much less than λ there is no point P at which the path difference, $S_2P - S_1P$ amounts to $\frac{\lambda}{2}$, for *any* two points, S_1 and S_2 , in the gap (Fig. 2.5.15), which is why the wave spreads right round. It turns out that even for a slit as wide as λ , there is destructive interference for wavelets from *all* the points in the gap added together only for P at 90° to 'straight ahead'. So the waves just manage to spread right round.

For slit widths between λ and 2λ , larger path differences are possible, and overall destructive interference occurs at an angle of less than 90° to the straight ahead direction (Fig. 2.5.12). At larger angles there is net constructive interference, but the resultant amplitude is small. For greater and greater slit widths (compared with λ) destructive interference occurs for the 'first' time at smaller and smaller angles, so there is less and less spreading of the main beam. There are also more and more weak 'side beams': constructive interference separated by 'channels' of destructive interference.

2.5.6 Stationary waves

(a) The nature of stationary waves

The waves we have been considering so far are **progressive waves**: they progress through space (or a medium) transferring energy as they do so. Within a confined space, a second kind of oscillation can exist, which is known as a **stationary** (or **standing**) **wave**. Three possible modes of transverse vibrations for a stretched string or wire, such as a violin or harp string, are shown in Fig. 2.5.16. In each case the string vibrates up and down: the full line shows one extreme position of the string and the pecked line shows the position half a cycle later. Fig. 2.5.17 illustrates the motion in more detail.

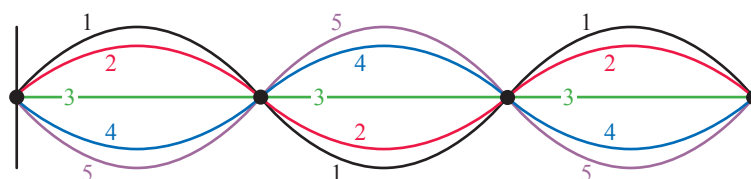


Fig. 2.5.17 Half a cycle of a stationary wire

It shows $\frac{1}{2}$ a cycle of oscillation, starting with the string in one extreme position (1), and then moving through roughly equal steps of $\frac{1}{10}$ cycle (2, 3, 4 and 5). Following this half cycle, the string tracks back: 4 $\rightarrow$ 3 $\rightarrow$ 2 $\rightarrow$ 1 and so on. The points which are always stationary, marked with dots, are called **nodes** and the points with maximum motion, midway between the nodes are **antinodes**. Notice the following differences between stationary and progressive waves:

1. In a stationary wave, all points between a pair of neighbouring nodes oscillate in phase; points either side of a node oscillate in antiphase. In a progressive wave there is a gradual change of phase along the wave.
2. In a progressive wave, all points oscillate with the same amplitude (apart from a smooth decrease in amplitude with increasing distance from the source). In a stationary wave, the amplitude of vibration varies smoothly from zero, at the nodes, to a maximum, at the antinodes.

(b) The relationship between progressive and stationary waves

Why are stationary waves so named? Although a snapshot image looks the same as a wave, a stationary wave looks more like an oscillation than a wave. The wave (or oscillation) arises when two waves of the same frequency travel in opposite directions. Superposition of the two waves produces the stationary wave pattern. The clue is in Fig. 2.5.5, especially the 1.0 s snapshot.

As you'll discover from Self-test 2.5.13, the **internodal distance** is $\frac{1}{2}\lambda$, where λ is the wavelength of the progressive waves that combine to produce the stationary wave.

So how do stationary waves form on instrument strings?

1. Energy is put into the string by plucking (guitar), bowing (violin) or striking (piano).
2. Progressive waves are produced which travel in opposite directions along the string.
3. The waves reflect at each end.
4. Most waves die out quickly leaving those with a node at each end, i.e. those for which the length of the string is an integer multiple of $\frac{1}{2}\lambda$.

(c) Harmonics

The different modes of vibration shown in Fig. 2.5.16 are known as **harmonics**. The top graph shows the first harmonic (also known as the fundamental); the second and third harmonics are also shown. In the case of stationary waves on a string, the frequencies of the harmonics have a simple mathematical relationship to one another as is shown in the example.

Example

The vibrating section of a violin's A string has a length of 33 cm. Its fundamental frequency is 440 Hz. Calculate:

- (a) The speed of transverse waves on the string.
- (b) The frequencies of the second and third harmonics.

(a) The string length = $\frac{1}{2}\lambda$, $\therefore \lambda = 66$ cm.
The wave speed $v = \lambda f = 0.66 \text{ m} \times 440 \text{ Hz} = 290 \text{ m s}^{-1}$

(b) For the second harmonic, the nodes are 16.5 cm apart, so λ is 33 cm. This is half that of the first harmonic, so the frequency, f_2 , is double, i.e 880 Hz.

Similarly $f_3 = 3 \times 440 \text{ Hz} = 1320 \text{ Hz}$.

(d) Investigating stationary waves

A common way of investigating transverse stationary waves on strings and their harmonics is shown below.

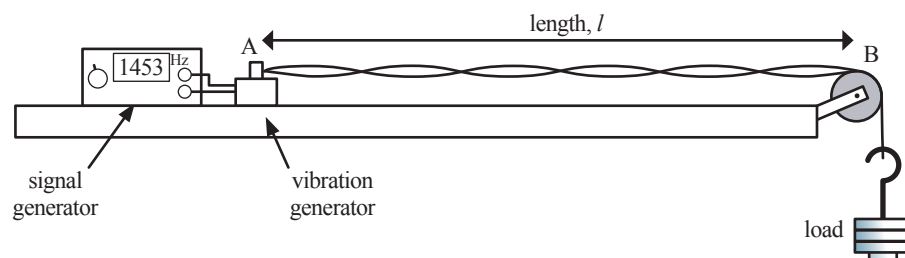
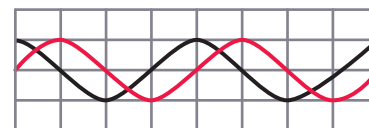


Fig. 2.5.18 Investigating stationary waves on a string

Self-test

2.5.13

The diagram shows a snapshot of two progressive waves of the same wavelength and speed (and thus of the same frequency) travelling in opposite directions.

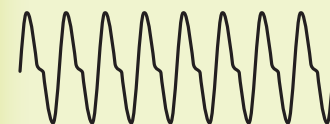


Draw a series of diagrams to show the sum of the waves (by superposition) as the waves move in $\frac{1}{8}$ wavelength steps. Identify the positions of the nodes and antinodes.

Stretch & Challenge

The sound of instruments is produced from stationary waves in their strings or air columns. Different instruments give out sound with characteristic mixtures of harmonics. This is one of the ways we can tell, say, a violin sound from that of a trumpet.

This oscillation has the function,
 $2 \sin \omega t + \sin 2\omega t$



Use a spread sheet program to investigate different combinations of harmonics.

Study point

Point A, in the apparatus in Fig. 2.5.18, cannot be a node, because the peg at A is vibrating up and down. However, the amplitude of the oscillation at A is very much smaller than the amplitude at the antinodes, so it may be taken as a node to a good approximation.

Stretch & Challenge

Theory suggests that the speed, v , of transverse waves on a string is given by

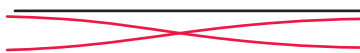
$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension and μ the mass per unit length. How could you verify this using the apparatus in Fig. 2.5.18?

2.5.14

Self-test

'Singing tubes' are open at both ends. The diagram shows the first harmonic.



- Draw the 2nd and 3rd harmonics.
- Write the frequency of the n th harmonic, f_n , in terms of f_1 .

2.5.15

Self-test

An organ pipe of length 50 cm is closed at one end.

- Calculate the frequencies, f_1 and f_2 , of first two harmonics. Ignore the end correction.
- What would be the effect of closing the open end of the pipe? [Speed of sound = 340 m s⁻¹]

The signal generator produces alternating current of a variable frequency. The vibration generator converts these into low amplitude (typically ~1 mm) of the small peg at **A**. The load provides the required tension in the string. For a particular load and length, the frequency, f , is increased from a low value and the values of f is noted at which stable harmonics are produced. This setup may be used to verify that the frequencies of the harmonics have the mathematical relationship shown above.

(e) Stationary waves in air columns

Stationary waves can be longitudinal, too. If you blow across the open top of a tube or pipe, such as a pen top, longitudinal stationary waves are set up in the air column. Unlike waves on a string, which is fastened at both ends, the stationary wave has an antinode at the open end. It is difficult to draw a longitudinal wave, so diagrams are usually drawn as if the wave were transverse.

The first three harmonics for a pipe, which is open at one end, are illustrated in Fig. 2.5.19. Notice that the antinodes do not exactly occur at the open end but slightly beyond it. This 'end correction' can usually be ignored but needs to be considered in accurate work (see Section 2.5.8).

$n = 1$ 2 3

Fig. 2.5.19 Harmonics in a pipe with one open end

Blowing across the mouthpiece (in a flute) or into the mouthpiece (in a trumpet) produces a wide range of frequencies, but only the ones which produce these stationary waves are selected and amplified by the instrument. This is an example of resonance, which you will meet in your year 13 course.

Example

The speed of sound in air is proportional to the square root of the kelvin temperature. A flute produces a note of frequency 440.0 Hz at 20°C. What frequency would it produce if taken outside into a temperature of 0°C?

$$\text{Speed of sound at } 0^\circ\text{C}, v_0 = v_{20} \sqrt{\frac{273}{293}} = 0.965v_{20}$$

Assuming that the change in the length of the flute is not significant, the wavelength of the sound waves is unchanged.

$$\text{But } f = \frac{v}{\lambda}, \therefore f_0 = 0.965f_{20} = 0.965 \times 440 = 425 \text{ Hz.}$$

This frequency is more than halfway down from A to G# (415 Hz), so the flute is seriously out of tune!

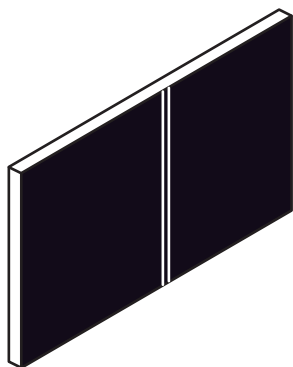


Fig. 2.5.20 Young's slits using a coated microscope slide

2.5.7 Determination of wavelength of light

(a) Using Young's slits

A microscope slide is prepared by coating it with a colloidal suspension of graphite, which is allowed to dry. Two parallel slits are scratched into the graphite using a scribe. Typically these slits are 0.2–0.3 mm wide and 0.4–0.5 mm apart.

To measure the wavelength of a laser, which is a monochromatic light source, the slits are set up in a darkened room, with the laser beam incident on the slits at right angles to the plane of the slits. The resulting pattern is viewed on a screen $\sim 2 \text{ m}$ away. The distance is not critical, but the further the screen is from the slits, the more spread out the fringes will be.

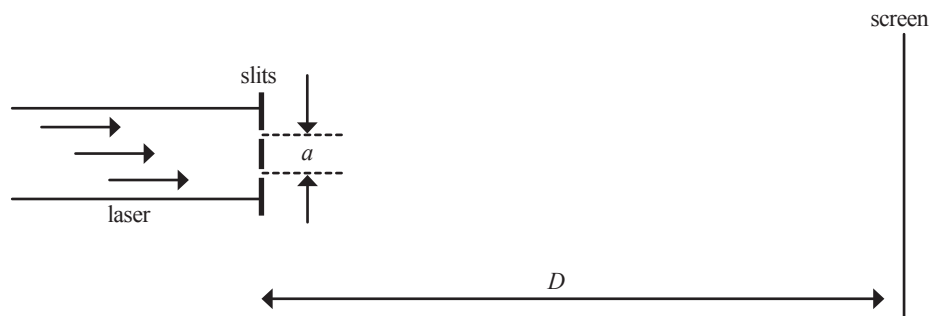


Fig 2.5.22 Young fringes experimental arrangement

The wavelength, λ , of the light is given by $\lambda = \frac{a\Delta y}{D}$, which requires a , D and Δy to be measured.

- D is measured using a tape measure or metre rules with a percentage uncertainty of $\sim 0.5\%$ (1 cm in 2 m) or less.
- a is measured using a travelling microscope (see Fig. 2.6.16), with an uncertainty of $\sim 0.01 \text{ mm}$, i.e. $\sim 2\%$.
- Δy , the fringe separation, is measured using a mm scale. The spread of (say) 10 fringes is measured and divided by the number of fringes. Typically the spread is $\sim 1.5 \text{ cm}$ with an uncertainty of $\sim 1 \text{ mm}$.

Notice that, even with a rather crude measurement of D it still makes the smallest contribution to the overall uncertainty in λ .

(b) Using a diffraction grating

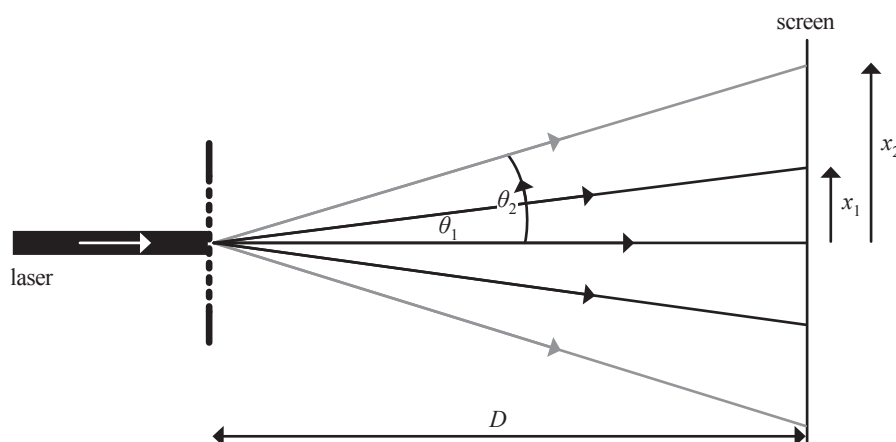


Fig 2.5.23 Measuring wavelength using a diffraction grating

The experiment is set up in the same way as for the Young slits experiment except that a diffraction grating is used instead. With a laser, there is no need for blackout conditions as much more light is transmitted by the grating than the slits and the spectral lines are much more sharply defined.



Fig 2.5.21 Typical fringe appearance

Exam tip

In the expression for λ , $\frac{a\Delta y}{D}$, the very small quantities, a and Δy are on the top and the large quantity, D , is on the bottom. The values of a and Δy are both $< 1 \text{ mm}$ and $D > 1 \text{ m}$, giving $\lambda < 1 \mu\text{m}$, as expected.

Self-test

2.5.16

With the given uncertainties in D , a and Δy estimate:

- The percentage uncertainty in λ .
- The absolute uncertainty if $\lambda \sim 590 \text{ nm}$.

Terms & definitions

The **diffraction grating constant** is the number of lines (or slits) per metre. The reciprocal of this is the separation, d , of the slits.

Self-test

2.5.17

A diffraction grating is marked '2 000 lines cm^{-1} '. Calculate the separation, d , of the slits.

Study point

The slit separation, d , of a diffraction grating can be determined by measuring the angle, θ , of a spectral line of known wavelength and applying the diffraction grating formula.

2.5.18

Self-test

Using a diffraction grating, θ for the first order sodium D line, (wavelength, 589 nm) was found to be 12.36° . Calculate: (a) the slit separation and (b) the diffraction grating constant.

The diffraction grating formula is

$$d \sin \theta_n = n \lambda$$

where θ_n is the angle of the n^{th} order spectrum and d the diffraction grating slit separation. To calculate the wavelength, d and θ_n need to be determined. The value of d is usually calculated from the by the **diffraction grating constant**, which is provided by the manufacturer (but see the Study point). If projecting the spectra on to a screen it is convenient determine the θ_n values by measuring D (as with the Young slits experiment) and x^n using a metre rule, and then using

$$\theta = \tan^{-1} \left(\frac{x}{D} \right).$$

Typically, $D \sim 2 \text{ m}$ and the displacements, x , are of the order of 50 cm. To reduce the uncertainty in the values of x it is sensible to measure the distance between the two first-order spectra and dividing by 2.

(c) Using a spectrometer

More accurate values for λ , using Young slits or a diffraction grating, can be obtained using a spectrometer instead of projecting the spectra onto a distant screen. The angular resolution of a typical school spectrometer is $0.5'$ of arc, i.e. $1/120^\circ$, giving a percentage uncertainty in θ of $\sim 0.06\%$, for an angle of 15° .

A detailed description is not given here but, very briefly, a light source, L, illuminates a slit, S, at the focus in a collimator, C, producing a parallel light beam which is incident on the grating, G (or slits). The spectra (fringes) emerging are examined in the telescope and the angle of deviation measured using the angle scale. Advantages of the spectrometer are:

- High angular resolution.
- Non-coherent light can be used, e.g. gas discharge tubes.
- The spectral lines are very narrow (they are images of the narrow collimator slit).

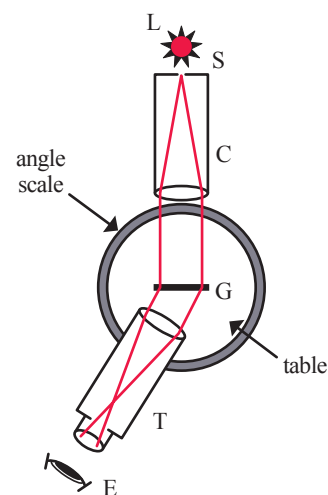


Fig. 2.5.24 Spectrometer

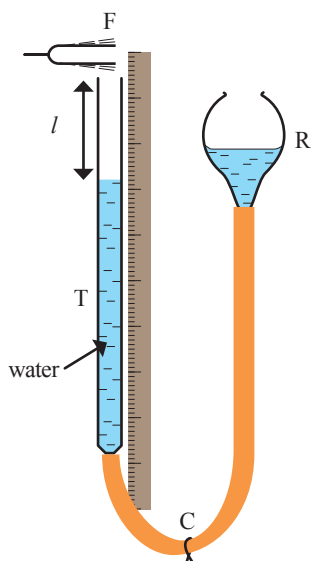


Fig. 2.5.25 Resonance tube

2.5.8 Determination of the speed of sound in air using stationary waves

A vibrating tuning fork, F, is used to set up stationary waves in the column of air above the water surface in a open-ended glass tube (a resonance tube). The water level in the tube is adjusted by raising and lowering the water reservoir (R) until sound of a maximum intensity is heard. The reservoir and resonance tube are connected by a flexible rubber tube, which allows water to flow between the two. The clip, C, is used to restrict or stop the flow of the water between the reservoir and resonance tube.

The smallest distance, l , for a stationary wave, is such that there is a node, N, at the surface of the water and an antinode, A, close to (but just above) the open end of the



Fig. 2.5.26 1st harmonic

tube, as shown in Fig. 2.5.26. The distance l is the measured distance of the water surface below the open end of the tube and the unknown distance c is called the *end correction*. The distance between a node and the adjacent antinode is $\lambda/4$, so we can write

$$\frac{\lambda}{4} = l + c.$$

The simplest method to determine the speed of sound is as follows:

- Starting from a water level close to the open end of the tube, lower the water level (by lowering the reservoir) until the first resonance is detected with a vibrating tuning fork of known frequency, e.g. 256 Hz (middle C).
- By repeated raising and lowering the water level, locate the resonance position accurately.
- Measure the resonance length, l .
- Repeat with a series of tuning forks up to, e.g. 512 Hz (top C).

The analysis

For each frequency, f , $\lambda = \frac{v}{f}$, where v is the speed of sound.

Substituting for λ and rearranging: $l = \frac{v}{4f} - c$. So graph of l against $\frac{1}{f}$ is a straight line of gradient $\frac{v}{4}$ and intercept $-c$ on the l axis.

Variations

1. A small loudspeaker, connected to a calibrated signal generator can be used instead of the set of tuning forks.
2. The second (and third) harmonics can be located for a single frequency. This is illustrated in Fig. 2.5.28. If the length of air for the n^{th} harmonic is l_n , then $l_2 - l_1 = \frac{\lambda}{2}$ and $l_3 - l_2 = \frac{3\lambda}{4}$. If the frequency is known, the wave speed can be calculated.
3. A larger bore glass tube with a wooden piston can be used as the resonance tube together with the signal generator and loudspeaker. The resonance length is measured by withdrawing the piston.

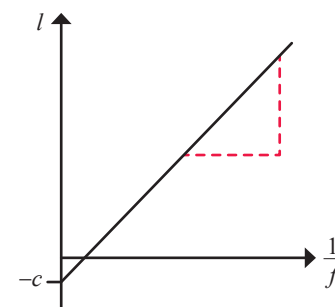


Fig. 2.5.27 Graph of l against $\frac{1}{f}$

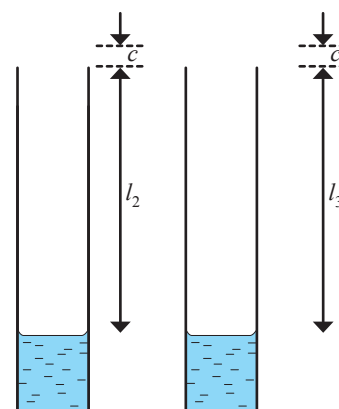
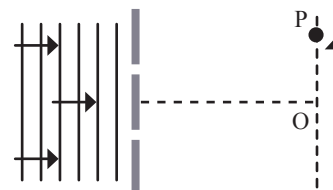


Fig. 2.5.28 2nd and 3rd harmonics

Exercise 2.5

1. The diagram shows a set of plane microwaves approach a pair of slits, whose centres are 8.0 cm apart, made using three aluminium plates. A probe, P, is moved along the dotted line which is 50 cm from the plane of the slits.
 - (a) When one of the slits is covered up, a signal is received at O. Name the effect which is responsible for this.
 - (b) When both slits are uncovered the signal received is much stronger. Explain this.
 - (c) As the probe is moved from O in the direction shown by the arrow, the signal initially decreases to (nearly) zero and then rises to a maximum. Explain this observation and name the effect observed.
 - (d) The distance between the first maxima on either side of the centre line is found to be 36.8 cm.
 - (i) Use the Young slits formula to calculate a value for the wavelength of the microwaves.
 - (ii) Calculate a value of λ using Pythagoras' theorem. Comment on your answers.



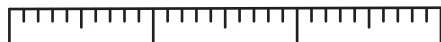
2. A physics student was given the task of identifying a mystery line in the spectrum of a sodium discharge tube. She used a spectrometer and a diffraction grating as follows:
- I. She measured the angles, ϕ , between the two second order spectra for the lines in the hydrogen spectrum.
 - II. She measured the angles between the two second order spectra of the mystery line in the sodium spectrum

Her results are in the table.

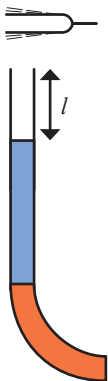
	Hydrogen lines				Sodium	
λ / nm	410.2	434.0	486.1	656.3	589.3	mystery
$\phi / ^\circ$	47.92	50.90	57.54	80.71	71.39	65.46

Is the value of ϕ consistent with the green line of mercury which has a wavelength of 546.1 nm?

3. A student illuminates a single slit with a laser light of wavelength 650 nm and observes this pattern on a screen which is placed 3.0 m from the slit.



- (a) Sketch a graph of intensity against position for this light.
 - (b) He replaces the laser with another of wavelength 450 nm. Sketch a graph of intensity against position for this graph.
 - (c) The angular position, in radians, of the dark fringes from the centre is $\frac{n\lambda}{d}$, where d is the width of the slit and n takes values 1, 2, 3... By taking readings from the cm/mm scale find the width of the slit.
4. In an experiment to measure the speed of sound a student uses a tuning fork of frequency 440.0 Hz. She held the vibrating tuning fork over the top of a resonance tube and allowed the water level to drop until a loud resonance was heard. This was due to stationary sound waves being formed with a node at the water surface and an antinode at the open end of the tube, so the length l represented $\frac{1}{4}$ of a wavelength.
- Repeating the procedure several times, she obtained the following values of l using a mm scale: 185 mm, 189 mm, 190 mm, 187 mm, 189 mm.
- Use the readings to determine a value for, v , the speed of sound together with its uncertainty, Δv .
5. The student in Q4 reads Section 2.5.8 and finds out that the antinode is not exactly at the end of the tube but a short distance (called the end correction), ϵ , above it.
- So $l = \frac{\lambda}{4} - \epsilon$. She repeated the experiment and found a second resonance with $l = \frac{3\lambda}{4} - \epsilon$. The readings were: 579 mm, 576 mm, 577 mm, 573 mm, 575 mm.
- Use the results of both experiments to find values for v and ϵ together with their uncertainties.
6. The speed, v , of transverse waves on a wire is given by $v = \sqrt{\frac{T}{\mu}}$, where T is the tension and μ is the mass per unit length. A vertical wire of mass M and length l is put under tension by hanging a series of masses, m , on it. The wire is plucked so that it vibrates. Ignoring any increase in length in the wire owing to the varying tension, derive a formula relating the frequency of vibration, f , to the suspended mass, m .



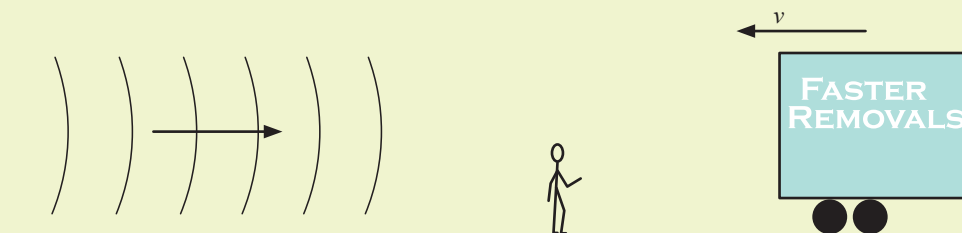
7.

A narrow slit is illuminated by a red laser ($\lambda = 640 \text{ nm}$) and the emergent light observed on a wall 2.00 m away. The central maximum of the diffraction pattern is measured as 2.0 cm wide. The slit is replaced by a pair of parallel slits of the same width as the first. The interference fringes are 2.0 mm apart.

- Calculate the slit separation.
- Estimate the number of interference fringes you would expect to see. Explain your answer.
- Describe and explain what you would expect to see if:
 - The red laser were replaced by a green laser ($\lambda = 500 \text{ nm}$).
 - The slits are now illuminated by both lasers but one slit is covered with a filter which lets only red light through and the other with a green filter.
 - The slits are covered with polaroid filters with the directions of the two polaroids parallel.
 - The slits are covered with polaroid filters with the directions of the two polaroids at right angles.

Stretch & Challenge

- A removals van is reversing from the right and the foghorn sends sound of frequency 150 Hz from the left as shown in the diagram.



The student notices that the combination of sound from the foghorn and the reflection from the back of the van gets louder and quieter with a period of 2.0 s .

Estimate the speed of the van. [Take the speed of sound to be $\sim 300 \text{ m s}^{-1}$.]

- Both the Sun and the Moon have tidal effects. The effects are most easily observed on the oceans.

Sun

The diagram shows, highly exaggerated, the Sun's tidal effect on the oceans of the Earth. The effects of the land masses are ignored. As the Earth rotates, two solar tides are experienced per day. The period of the solar tides is thus 12.00 hours .

Even though the moon is much less massive than the Sun, it exerts tides which are about 2.2 times as large as the Sun, because it is so much nearer. Because of the Moon's orbit (29.3 days relative to the Sun) the period of the lunar tides is slightly longer at 12.42 hours .

The tide experienced on any day is the superposition of the two tidal waves, so the amplitude varies. Sometimes the tides are in phase, giving so-called spring tides and sometimes they are exactly out of phase and we get neaps.

On Thursday 4 June 2015, the solar and lunar tides are in phase (spring tide) and the amplitude of the tide at Aberystwyth is a maximum at 4.8 m .

When will the lowest amplitude tides (neaps) be and what will be their amplitude?

2.6 Refraction of light

Terms & definitions

Refraction is the change of direction of travel of light (or other wave) when its speed of travel changes, e.g. when it passes from one material into another.

All waves, such as sound, light, sea waves and seismic waves, exhibit a change of direction if they move from one material into another in which the speed of propagation is different (unless the direction of propagation is at right angles to the boundary). This effect is called **refraction**. The causes of the speed change depend upon the kind of waves. Some examples:

- Seismic wave speed depends upon stiffness and density of rocks.
- The speed of radio waves through the ionosphere (upper atmosphere) is affected by free-electron concentration.
- Surface water wave speed is affected by the frequency and by the depth of water – the ripple tank waves in Fig. 2.6.1 are travelling from left to right in deep water at B; they slow down at the boundary with shallower water at A, which causes them to bunch up and change direction.
- The speed of sound waves through the atmosphere depends upon the temperature.

In the case of light, we use this effect to control light in useful ways, such as making lenses for correcting defects of vision or for constructing telescopes and microscopes, or in optical fibres to transmit information. Geophysicists use the refraction of natural seismic waves to probe the structure of the Earth or artificially generated ones to locate oil and gas reservoirs.

Fig. 2.6.1. Refraction of water waves in a ripple tank

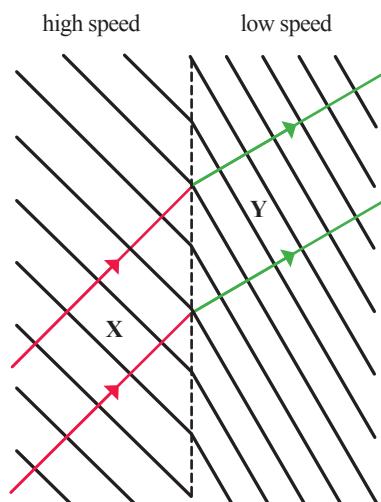


Fig. 2.6.2 Wave refraction

Study point

We usually use the concept of **rays** to illustrate the refraction of light. Light is a wave phenomenon but when the wavefronts are many orders of magnitude larger than the wavelength we can consider it to move in narrow straight line beams at right angles to the wavefronts: these are light rays.

2.6.1 Refraction and wave properties

The waves (the black lines show the crests) in Fig. 2.6.2 are crossing a boundary (pecked line) obliquely from left to right into a region in which they travel more slowly. The red and green arrows show the directions of travel of the waves. These are at right angles to the line of the wave crests and, in the case of light, the red and green lines would be light rays.

As each wave crosses the boundary it slows down: the lower end of the wave slows down first so the line of the wave becomes more vertical (more nearly parallel to the boundary). Hence the direction of travel of the waves becomes more nearly horizontal, i.e. closer to right angles to the boundary.

As these lines represent the wave crests, the separation of the crests along the (coloured) direction lines, is the wavelength, which clearly decreases as it crosses the boundary. What about the frequency? Every wave that passes **X** also passes **Y**. Therefore the frequency of the waves must be unchanged even though the speed changes.

Fig. 2.6.3(a) illustrates the case of a light ray being refracted at a boundary. Some terms associated with refraction are introduced in the diagram and part (b) shows the relationship between the light rays and the wave model of light. Fig. 2.6.3(b) is a magnified extract from the central part of Fig. 2.6.2. We can use these diagrams to derive the relationship between the angles, θ_1 and θ_2 , and the wave speeds v_1 and v_2 .

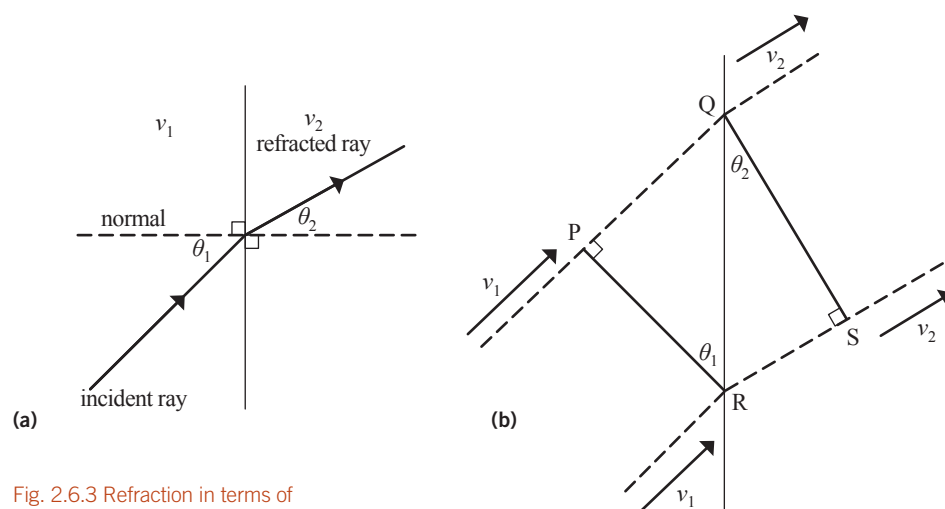


Fig. 2.6.3 Refraction in terms of (a) rays and (b) waves

PR represents a wavefront arriving at the boundary with an angle of incidence θ_1 . QS represents the position of the same wavefront, a time Δt later, at the instant in which point P on the incident wavefront arrives at the boundary.

$$\text{In } \triangle PQR: QR = \frac{PQ}{\sin \theta_1}. \quad \text{In } \triangle SRQ: QR = \frac{RS}{\sin \theta_2}.$$

$$\therefore \text{Equating the expressions for } QR: \frac{PQ}{\sin \theta_1} = \frac{RS}{\sin \theta_2}$$

$$\text{But } PQ = v_1 \Delta t \text{ and } RS = v_2 \Delta t, \therefore \frac{v_1 \Delta t}{\sin \theta_1} = \frac{v_2 \Delta t}{\sin \theta_2}$$

$$\text{Rearranging:} \quad \frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} \quad [1]$$

Alternatively: the ratio $\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$, which is a constant. This is a form of Snell's law,

which is dealt with in Section 2.6.2

2.6.2 Refractive index

Equation 1 in Section 2.6.1 is a general relationship for all wave motions. In this section we deal only with light. The discussion also applies, in principle, to other forms of electromagnetic radiation.

For historical reasons, optical physicists discuss the ability of a material to refract light waves (rays) in terms of its **refractive index**, n , which is defined by $n = \frac{c}{v}$ (see Terms & definitions). Because light waves travel more slowly through materials than through a vacuum, the values of n have a minimum value of 1 exactly for a vacuum (by definition), i.e. $n \geq 1$. Table 2.6.1 gives the refractive indices of a range of common materials.

Exam tip

If the speed of waves **decreases**, the frequency is unchanged so the wavelength **decreases**.

This is related to $v = f\lambda$: the wavelength is proportional to the speed because f is constant.

Self-test

2.6.1

A set of water waves of frequency 4.0 Hz is travelling in water with a speed of 10 cm s⁻¹. It hits a boundary with shallow water, where its speed is 6.0 cm s⁻¹, at an angle of 40°. Calculate:

- The wavelengths of the waves in the deep and in the shallow water.
- The angle the refracted waves make to the boundary.

Study point

If the speed of the waves varies gradually with position, the direction of travel of the waves gradually changes and we can use the equation between θ and v in the form $\frac{\sin \theta}{v} = \text{constant}$.

Example: the direction of seismic waves through the Earth's mantle.

Terms & definitions

The **refractive index**, n , of a material is defined by $n = \frac{c}{v}$, where v and c are the speeds of light in the material and a vacuum respectively.

Study point

The refractive index of a material depends upon the frequency of the radiation. This is the cause of rainbows.

Table 2.6.1 Refractive indices

Material	n
vacuum	1 (exactly)
air (at 0°C)	1.000292
water	1.333*
sea water	1.343*
ice	1.31
glass	1.50 – 1.75
diamond	2.417
glycerine	1.473*
olive oil	1.48*

* at 293 K

2.6.2

Self-test

Calculate the speed of light in:
(a) water and (b) diamond.

Take $c = 3.00 \times 10^8 \text{ m s}^{-1}$.

Study point

Note the following two points from the fish-tank example, which arise only because the glass surfaces are parallel:

1. The light ray in the glass makes the same angle, α , to the normal at both sides.
2. The angle α in the glass can be ignored in the calculation.

2.6.3

Self-test

Calculate the angle in the glass, α , in the fish-tank example.

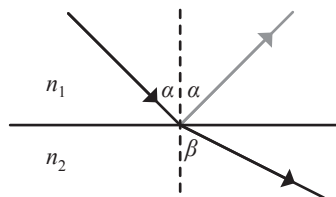


Fig. 2.6.5 Partial reflection

Exam tip

In Fig. 2.6.5,

$n_1 \sin \alpha = n_2 \sin \beta$ as usual.

(a) Snell's law

We shall now write equation 1 in terms of refractive index. The refractive indices of the

two materials in Fig. 2.6.3 are $n_1 = \frac{c}{v_1}$ and $n_2 = \frac{c}{v_2}$.

So equation 1 becomes: $\frac{n_1 \sin \theta_1}{c} = \frac{n_2 \sin \theta_2}{c}$, $\therefore n_1 \sin \theta_1 = n_2 \sin \theta_2$ [2]

This is **Snell's law**. Equation 2 can also be written $n \sin \theta = \text{constant}$, which also can be applied to refraction through a material with a continuously varying refractive index, e.g. the atmosphere, a solution with a varying composition.

(b) Optical density

Many people use the qualitative term *optical density* to describe the refractive property of a material. Diamond is described as having a 'high optical density' and air has a 'low optical density'. This term should not be confused with physical density. There is a rough correlation of physical density with refractive index, e.g. crown glass ($n \sim 1.5$, $\rho = 2.6 \text{ g cm}^{-3}$) and flint glass ($n \sim 1.7$, $\rho = 4.2 \text{ g cm}^{-3}$). However,

- the density of diamond (3.5 g cm^{-3}) is lower than that of flint glass but the refractive index is much higher
- the refractive index of perspex (1.495) is virtually equal to that of crown glass but the density is much lower (1.19 g cm^{-3}).

Example

The diagram shows a light ray entering a fish tank. Use the refractive index values to calculate angle θ_w .

From Snell's law:

$$n_a \sin 50^\circ = n_g \sin \alpha = n_w \sin \theta_w$$

(see Study point)

$$\therefore 1.00 \sin 50^\circ = 1.33 \sin \theta_w$$

$$\therefore \theta_w = \sin^{-1}\left(\frac{\sin 50^\circ}{1.33}\right) = 35^\circ \text{ (2 s.f.)}$$

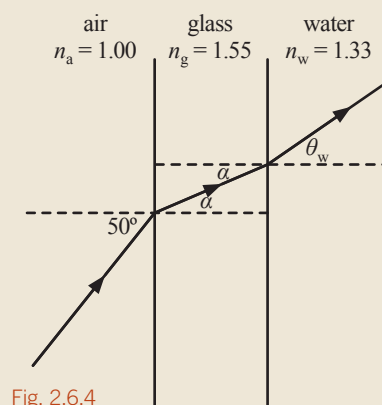


Fig. 2.6.4

2.6.3 Reflection

If a light ray strikes a smooth (polished) boundary between two media, as in Fig. 2.6.5, it is usually partially reflected and partially refracted as shown. This diagram is drawn for $n_1 > n_2$ but the same holds for $n_1 < n_2$, except that, in that case the angle of refraction, β , would be less than the angle of incidence, α . Note that the angle of reflection (i.e. the angle between the reflected ray and the normal) is equal to the angle of incidence.

The fraction of the incident power which is reflected depends upon both the angle of incidence and the refractive indices of the two materials:

- The greater the angle of incidence, the greater the power reflected.
- The greater the difference in refractive indices the greater the power reflected.

(a) Total internal reflection

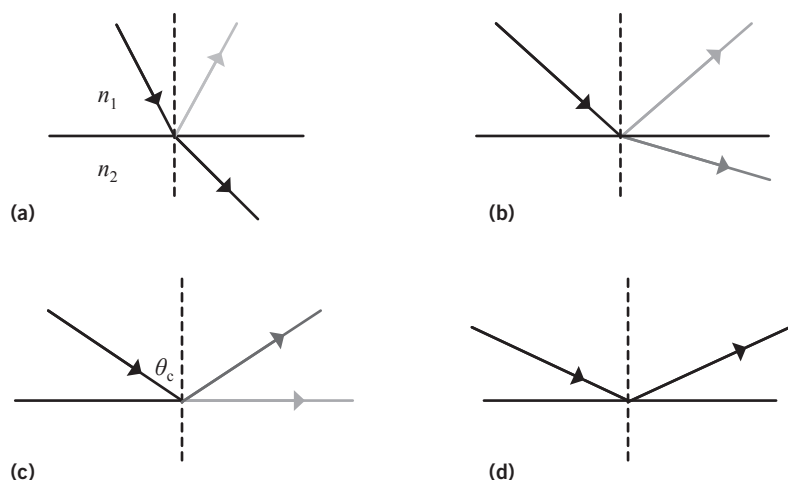


Fig. 2.6.6 Total internal reflection

The sequence of diagrams in Fig. 2.6.6(a) – (d) illustrates qualitatively how the fraction of the reflected power varies with angle of incidence for a light ray incident upon a material **with a lower refractive index**.

- For a small angle of incidence – in (a) – the fraction reflected is small.
- As the angle of incidence increases – the sequence (a), (b), (c) – the fraction reflected increases (and the fraction transmitted decreases).
- At some angle of incidence, θ_c , called the **critical angle**, the angle of refraction is 90° and the refracted power is very low.
- For angles of incidence greater than the critical angle – in (d) – all the incident power is reflected. This phenomenon is known as **total internal reflection (TIR)**.

The relationship between the critical angle, θ_c , and the refractive indices can be found by considering the limiting case in diagram (c).

Applying $n_1 \sin \theta_1 = n_2 \sin \theta_2$ with $\theta_1 = \theta_c$ and $\theta_2 = 90^\circ$

$$\rightarrow n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\text{But } \sin 90^\circ = 1, \quad \therefore n_1 \sin \theta_c = n_2 \quad \text{or} \quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

(b) Examples of TIR

Section 2.6.4 deals with optical fibres, which depend for their operation on total internal reflection. Here are two more everyday examples.

(i) Totally reflecting prisms

Many optical instruments, e.g. binoculars, microscopes and periscopes, use prisms to reflect light and to fold the light path. The advantage of prisms over mirrors is illustrated in Fig. 2.6.7. The multiple reflections in a mirror would cause multiple images in the instrument. In the case of the prisms, the weak partial reflections as the light enters or leaves the prism at right angles would send light back out of the instrument the way it came and wouldn't affect the final image. The glass used for prisms has refractive index in the range 1.5–1.7. You should be able to show that the critical angle is less than 45° .

Exam tip

The conditions for TIR are:

1. The light ray is incident upon a boundary with a material of lower refractive index.
2. The angle of incidence is greater than the critical angle.

Study point

The critical angle for a boundary between a material of refractive index n and air (refractive index = 1.000) is given by

$$\sin \theta_c = \frac{1}{n}.$$

Self-test

2.6.4

A light ray, travelling in perspex of refractive index 1.495, is incident upon a boundary with air. Describe what happens if the angle of incidence is (a) 25° , (b) 35° and (c) 45° .

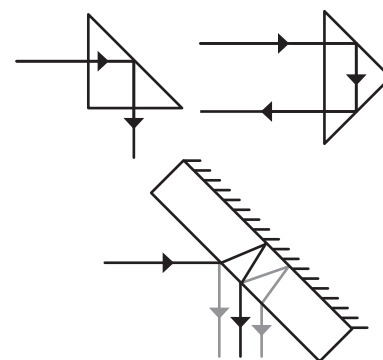


Fig. 2.6.7 Totally reflecting prisms and a glass mirror

Fig. 2.6.8 Desert and highway mirages

Stretch & Challenge

If light hits a boundary between two materials, of refractive indices n_1 and n_2 , at right angles (i.e. $\theta = 0$), the fraction, R , of the power reflected is given by:

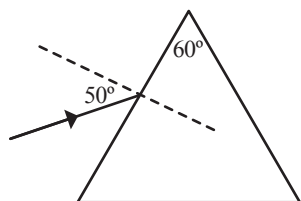
$$R = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

Calculate the fraction of light which is transmitted through a window pane ($n = 1.55$) in air. Remember that the pane has two surfaces.

2.6.5

Self-test

A light ray hits an equilateral glass prism ($n = 1.50$) with angle of incidence 50° . Calculate the angle with which it emerges from the prism.



Study point

Multimode optical fibres typically have core diameters of $\sim 50 \mu\text{m}$. Monomode (single-mode) fibres are typically $8\text{--}10 \mu\text{m}$.

(ii) Mirages

The reflection of light, apparently from a shimmering water surface on a hot summer road or from a non-existent lake in the baking desert, is known as a mirage (strictly an *inferior mirage*). Two examples of this are shown in Fig. 2.6.8.

This happens because the road (or desert) surface absorbs radiation from the Sun and heats up. This warms the air in contact with the surface so that there is a temperature inversion – the air temperature falls with height. The refractive index decreases as a light ray (e.g. from a car, camel or telegraph pole) approaches the surface and, if the ray travels at a glancing angle, there is enough of a difference to cause TIR.

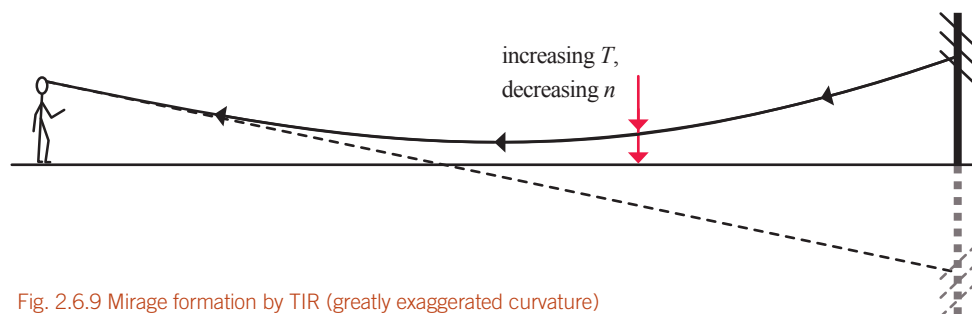


Fig. 2.6.9 Mirage formation by TIR (greatly exaggerated curvature)

2.6.4 Optical fibres

Optical fibres have become ubiquitous since the 1980s. They are used for data transmission in local area networks (LANs), regional networks and long-distance networks (intercontinental). They are also used in remote imaging systems, such as internal medical examinations (endoscopy) and investigating inaccessible locations such as drains and collapsed buildings (in the search for survivors).

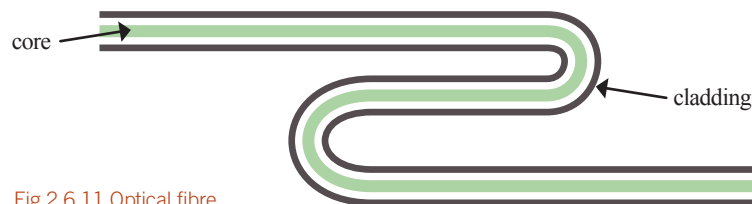


Fig 2.6.11 Optical fibre

Fig 2.6.10 Oesophageal endoscopic image showing reflux damage (Barrett's oesophagus)

This section deals with optical properties of stepped-index optical fibres. A typical optical fibre consists of a single glass thread, the central part of which (the core) carries the light signal and the outer part (the cladding) keeps the signal in the core. Around this is a protective layer of plastic (not shown in Fig. 2.6.11) called the coating. Typically the external diameter of the coating is $\sim 250 \mu\text{m}$, i.e. 0.25 mm . An optical-fibre cable can consist of hundreds of such fibres.

(a) Multimode fibres and TIR

The optical fibre relies for its operation on total internal reflection of light rays at the boundary between the core and the lower refractive index cladding.

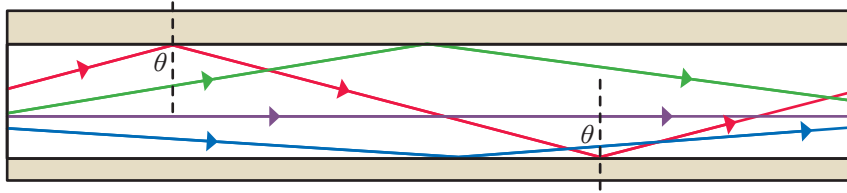


Fig. 2.6.12 Totally reflected light ray in a multimode optical fibre

All light rays that are incident upon the core-cladding boundary at angles greater than the critical angle are totally internally reflected back into the core. They then repeatedly hit opposite sides of the core at the same angle being reflected each time and emerge from the other end of the fibre (unless they are absorbed or scattered by impurities in the glass). The optical fibre is not necessarily perfectly straight but, given that the diameter of the core is $\sim 50 \mu\text{m}$, any reasonable curvature of the cable will hardly affect the angles of the multiple reflections.

This kind of optical fibre is fine for short-distance communication or for imaging applications (endoscopy), but it runs into a problem for digital communication over long distances with rapid switching rates. To see why, consider the time taken for a signal to travel 10 km.

Example

A multimode optical fibre has a core refractive index of 1.6. Calculate the difference in the time taken for a signal to travel through 10 km of optical fibre for light rays travelling parallel to and at 20° to the axis of the fibre.

$$\text{Speed of light in the core} = \frac{c}{n} = \frac{3.00 \times 10^8}{1.6} = 1.875 \times 10^8 \text{ m s}^{-1}$$

$$\therefore \text{Time taken for parallel ray} = \frac{10 \times 10^3}{1.875 \times 10^8} = 53.3 \mu\text{s}$$

$$\text{Distance travelled by signal at } 20^\circ = \frac{10 \times 10^3}{\cos 20^\circ} = 10642 \text{ m}$$

$$\therefore \text{Time taken} = \frac{10642}{1.875 \times 10^8} = 56.8 \mu\text{s}$$

$$\therefore \text{Time difference} = 3.5 \mu\text{s}.$$

This time difference is proportional to the transmission distance, so a 1 km fibre would give a time difference of $\sim 0.4 \mu\text{s}$; for a 100 m fibre Δt would be 40 ns etc. Digital data is transmitted as a series of rapidly oscillating pulses. If the transmission rate is more than 10^5 bits per second [100 kbps], the time between pulses will be less than $\sim 10 \mu\text{s}$, so the time difference between the arrival of the straight-through and 20° will cause the 1s and 0s of the pulses to overlap and cause the signal to be unreadable. As modern data systems operate at Gbps rates [10^9 bits per second] this kind of optical fibre is restricted to a few metres, e.g. in a LAN. This restriction is overcome in monomode fibres.

Study point

The refractive indices of the core and cladding depend on the design, but values of 1.62 and 1.52 respectively are typical.

Self-test

2.6.6

Show that the critical angle for the core and cladding is approximately 70° for the n values given in the Study point.

Exam tip

Note that the speed of light in an optical fibre is not c but $\frac{c}{n}$. This is approximately $2 \times 10^8 \text{ m s}^{-1}$.

Stretch & Challenge

The calculations in the example use more significant figures than justified by the data. This does not matter as this alternative approach shows:

$$t_1 = \frac{d}{v}; t_2 = \frac{d}{v \cos \theta}$$

$$\Delta t = \frac{d}{v} \left(\frac{1}{\cos \theta} - 1 \right)$$

Use the approximation that $v = 2 \times 10^8 \text{ m s}^{-1}$ and this equation to estimate Δt .

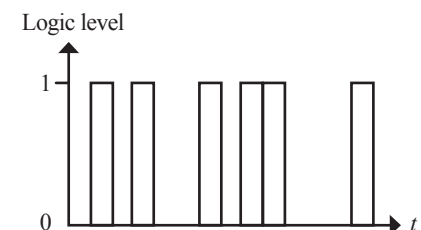


Fig. 2.6.13 Digital signal

Terms & definitions

Data degradation because of the different paths the light rays can take is called **multimode dispersion**.

2.6.7

Self-test

A telephone conversation takes place over a **3000 km** monomode fibre system. Estimate the time delay that the participants experience. State any assumption you make.

Stretch & Challenge

Signal power loss in optical fibres is expressed in decibel per km (dB/km).

The following table relates the percentage loss in signal to the dB scale:

dB/km	fractional loss per km
3	0.5
1	0.2
0.3	0.07
0.2	0.045
0.1	0.02
0.03	0.01

The following formula can be used to relate the output power, P_{out} to the input power P_{in} :

$$P_{\text{out}} = P_{\text{in}} \times 10^{-0.1 \times \text{dB loss}}$$

A **10 mW** signal is injected into a **100 km** fibre with a **0.3 dB/km** loss. Calculate the output power.

(b) Monomode (single-mode) fibres

In Section 2.5 we saw that light spreads out significantly by diffraction when it passes through apertures which are similar in size to the wavelength. In these circumstances we can no longer use the light-ray model and we need to use a full wave analysis (which is beyond the scope of A level physics). For reasons which are dealt with briefly below, optical fibre communication systems use infrared radiation of wavelength (in air) of $\sim 1.5 \mu\text{m}$. The result of the full wave analysis is that, for core diameters of less than about **10 μm** , the light waves cannot take multiple paths and they are effectively restricted to travelling parallel to the axis of the fibre. These fibres are called **monomode fibres**.

Progressive signal loss due to attenuation becomes the main problem to be overcome in monomode systems in boosting transmission rates and distances. For near IR wavelengths this is mainly due to scattering by impurities in the glass of the fibre. This process is called Rayleigh scattering and becomes more serious the shorter the wavelength. This is the same effect that produces the blue of the sky – the short (blue) wavelengths are scattered more by the atmosphere than the long (red) wavelengths. Some molecules and ions in the glass (mainly OH) also selectively absorb particular wavelengths.

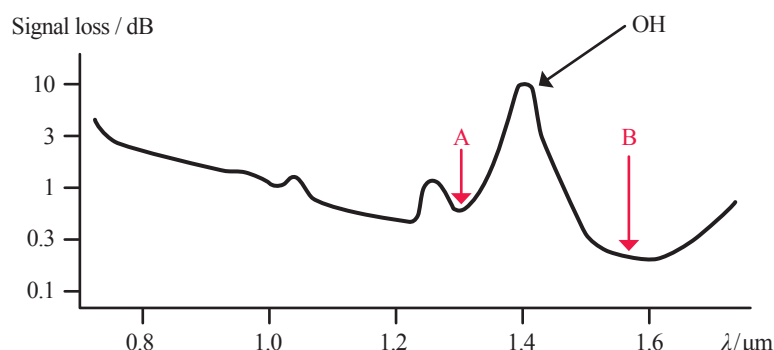


Fig. 2.6.14 Optical fibre signal loss curve

The loss curve – Fig. 2.6.14 – gives a typical graph for a traditional monomode optical fibre. The arrows A and B indicate (vacuum) wavelengths which are used for regional networks and long-distance networks respectively. ZBLAN, a recently developed set of materials, is showing losses as low as **0.01 dB** per kilometre.

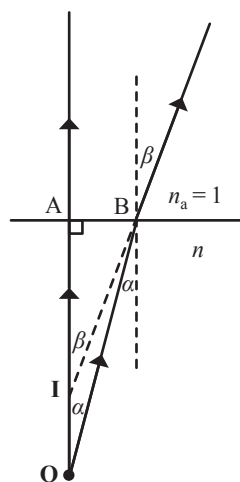


Fig 2.6.15 Real and apparent depth

2.6.5 Determination of the refractive index of a material

All the school laboratory methods of determining the refractive index are carried out in air. They produce results which, at best have an estimated uncertainty of ± 0.01 , so the distinction between the refractive indices of air ($n_a = 1.0003$) and a vacuum ($n = 1$ exactly by definition) can be ignored.

(a) Method 1 – using real and apparent depth

This method is the more accurate of the methods used in A level physics laboratories. It relies on the fact that the ratio of the real depth to the apparent depth of an object under the surface of a refracting material (e.g. water) is equal to the refractive index of the material.

This can be derived using Fig. 2.6.15 which shows two light rays from, **O**, a submerged object emerging into the air at A and B. It is assumed that the angles α and β are small enough for the **small angle approximation** to be valid. Viewed from above the object appears to be at **I**, which is at the intersection of the lines of the emergent light rays.

Snell's law: $n \sin \alpha = \sin \beta$ because $n_a = 1$

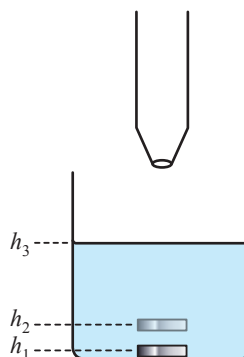
But α and β are small, so $\sin \alpha \approx \tan \alpha = \frac{AB}{AO}$ and $\sin \beta \approx \tan \beta = \frac{AB}{AI}$

In this case, α and β will be typically 0.01 rad or less, so the values of $\tan$ and $\sin$ differ by at most 0.005%.

$\therefore$ Substituting for $\sin \alpha$ and $\sin \beta$, $n \frac{AB}{AO} = \frac{AB}{AI}$. $\therefore n = \frac{AO}{AI} = \frac{\text{real depth}}{\text{apparent depth}}$

To measure the refractive index of a liquid (e.g. water):

1. Stick a coin to the bottom of a beaker and place the beaker on the stage of a travelling microscope.
2. Focus the travelling microscope on the coin and read the vertical scale, h_1 .
3. Without displacing the beaker and coin, add the liquid to a depth of ~ 5 cm. Refocus the microscope and read the scale, h_2 .
4. Sprinkle lycopodium power lightly onto the surface of the liquid, focus on it and read the scale, h_3 .
5. Calculate n using $n = \frac{h_3 - h_1}{h_3 - h_2}$



An improved method is to carry on adding the liquid and obtain a series of values of real depth and apparent depth. A graph of real depth against apparent depth is plotted. The gradient is measured, the value of which is the refractive index.

(b) Method 2 – ray tracing

This method requires the material to be in the shape of a regular block, e.g. a semicircular or rectangular glass or perspex block. The advantage of a semicircular block is that a ray along a radius is at right angles to the curved surface, so it is not refracted at the boundary (see Fig. 2.6.17).

1. The semicircular block is set up on a piece of plane paper on a drawing board on which is drawn a series of lines (drawn faint) at regular angles to the straight face of the block, e.g. $5^\circ - 40^\circ$ in 5° steps.
2. Two pins, P_1 and P_2 , are placed on one of the lines as in the diagram. Sighting these pins through the block, two more pins are placed so that all four pins are in a straight line as **viewed through the block**. The emergent ray (pecked line is drawn in later).
3. Step 2 is repeated for each of the lines from step 1.
4. The block is removed, the ray lines drawn in using the marks left by the pins, and the angles of incidence, θ_1 , and refraction, θ_2 , at the plane surface measured using a protractor.
5. A graph of $\sin \theta_2$ against $\sin \theta_1$ is drawn and the gradient measured. This is the refractive index.

MATHS CHECK

See Section 4.2.4 for the **small angle approximation**:

$$\sin \theta \approx \theta \approx \tan \theta$$

Self-test

2.6.8

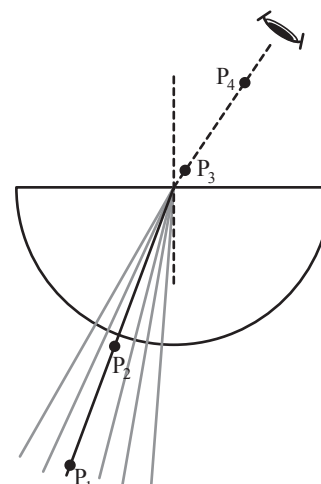
$\theta = 0.01$ rad. Find $\tan \theta$ and $\sin \theta$ and show that the difference is approximately 0.005%.

Fig. 2.6.16 Travelling microscope

Self-test

2.6.9

Typical uncertainties in the values of h_{1-3} are ± 0.03 cm. Assuming a true depth of 6.00 cm, estimate the uncertainty in an obtained value of n for water ($n = 1.33$).



2.6.17 Refractive index by ray tracing

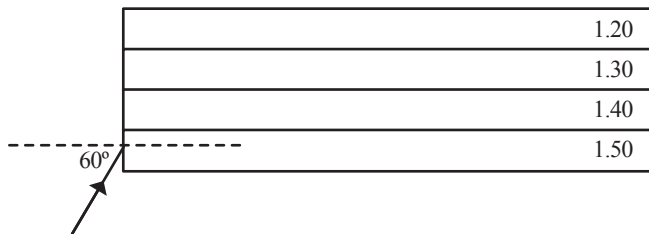
Exercise 2.6

1. A set of waves travelling at 6 m s^{-1} in water of depth 40 m has a wavelength of 10 m . It approaches a beach in a direction at 50° to the normal.
- (a) Calculate the frequency of the waves.
- (b) Assuming the speed of the waves is directly proportional to the square root of the depth, calculate the speed and wavelength of the waves when the water is 10 m deep.
- (c) Calculate the direction of travel of the waves at the 10 m deep point.
- (d) What direction do the waves travel in when they are water of depth 2.5 m ?

2. A light ray is incident at 45° at the middle of a glass block of dimensions $8.0 \text{ cm} \times 15 \text{ cm}$, and refractive index 1.50 . Determine the position and direction at which the light ray emerges from the block.



3. A light ray enters a stack of blocks with refractive indices as shown. By calculating the relevant angles in each layer, decide where total internal reflection occurs. You may assume that the blocks are long enough for this to happen.



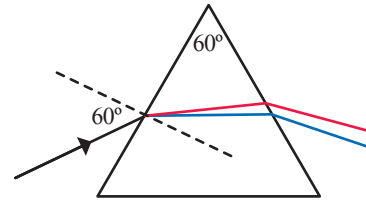
4. A student measures the refractive index of a glass block using the method of real and apparent depth. He draws a cross on a piece of paper, puts the paper onto the stage of a travelling microscope, focuses the microscope and finds that the reading on the scale is $12.52 \pm 0.01 \text{ cm}$. He puts a glass block on top of the paper, refocuses and measures the position as $13.64 \pm 0.01 \text{ cm}$. Finally uses a non-permanent marker to mark the top of the glass block and measures its position as $15.54 \pm 0.01 \text{ cm}$. Calculate the refractive index, giving the estimated uncertainty and expressing your values to an appropriate number of significant figures.
5. The resolution of the scale of the travelling microscope scale in Q4 is $\pm 0.005 \text{ cm}$. Suggest why the uncertainty in position is $\pm 0.01 \text{ cm}$.
6. A student uses method 2 to measure the refractive index of a semicircular perspex block.

Her results are given in the table. θ_g is the angle in the glass and θ_a the angle in the air. All angles are measured to $\pm 0.5^\circ$.

$\theta_g / ^\circ$	5.0	10.0	15.0	20.0	25.0	30.0	35.0
$\theta_a / ^\circ$	7.0	15.0	23.0	31.0	39.0	48.0	59.0

Use the results to draw a graph of $\sin \theta_g$ against $\sin \theta_a$. Use the $\pm 0.5^\circ$ uncertainties to work out the max/min values of $\sin \theta_g$ and θ_a , plot error bars and determine the refractive index of the block together with its uncertainty.

7. A ray of white light, consisting of all wavelengths between 400 nm and 700 nm , hits an equilateral triangular prism with an angle of incidence of 60° . The refractive index of the prism varies with the wavelength of light. For red light of wavelength (in a vacuum) of 700 nm , the refractive index is 1.51 ; for violet light with $\lambda_{\text{vac}} = 400 \text{ nm}$ the refractive index is 1.53 .



- (a) Calculate the wavelengths of red and violet light in the glass.
 (b) By determining the paths of red and violet light through the prism, calculate the angle between the emerging rays of red and violet light.
8. The refractive indices of the core and cladding of a multimode optical fibre are 1.62 and 1.55 respectively.
- (a) Calculate the critical angle between the core and cladding.
 (b) A section of the optical fibre in a LAN is 10 m long. Estimate the travel time difference between the light which passes along the axis of the fibre and the light which travels at the critical angle.
 (c) Estimate the maximum bit-rate by which the fibre can be used for transmitting data, justifying your figure.

Stretch & Challenge

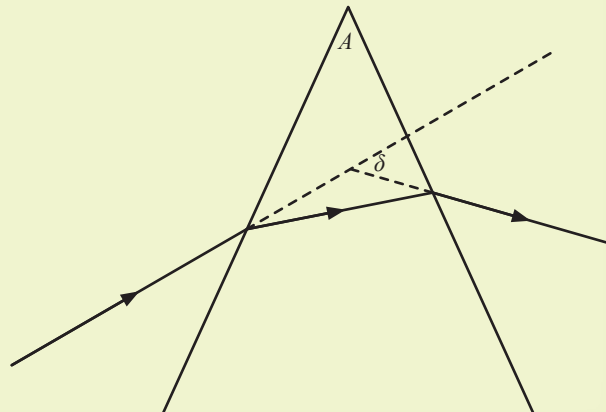
If a light ray passes through a prism without reflection it is deflected by an angle, called the angle of deflection, δ , the magnitude of which depends upon the refractive index, n , of the material of the prism, the prism angle, A , and the orientation of the prism.

Experimentally, if the prism is rotated there is a single minimum in the graph of δ against the orientation of the prism. We deduce that this minimum deviation angle, D , must occur when the light ray passes symmetrically through the prism.

- (a) Explain the deduction in the last paragraph.

(b) Derive the equation:
$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

- (c) Calculate the minimum deviation for a light ray incident upon a prism with $A = 45^\circ$ and $n = 1.7$.



2.7 Photons

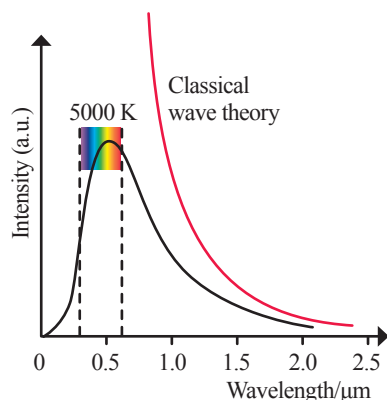


Fig. 2.7.1 The ultraviolet catastrophe

Terms & definitions

The **Planck constant**, h , has magnitude 6.63×10^{-34} J s.

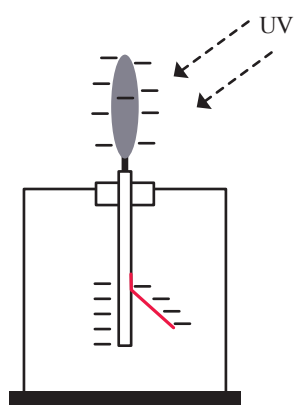


Fig. 2.7.2 Photoelectric effect demonstration

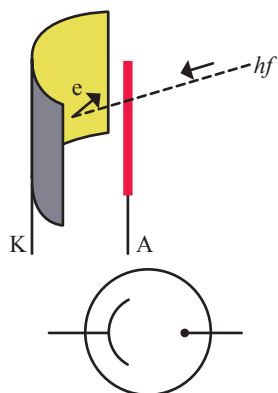


Fig. 2.7.3 The vacuum photocell

By the late nineteenth century, the wave model of light was firmly established. It explained a whole series of phenomena (see Sections 2.5 and 2.6) and the wavelength could be measured using Young's slits. However, this theory could not account for the way that light interacted with matter. According to the classical wave theory, the power radiated by a black body should get progressively higher at shorter wavelengths (actually $\propto \lambda^{-4}$), leading to the conclusion that the total power radiated away across all wavelengths should be infinite! This 'prediction' was known as the *ultraviolet catastrophe*. Planck successfully accounted for the shape of the black-body spectrum by assuming that radiation was absorbed and emitted in discrete packets of energy, rather than continuously as a wave. These packets were of energy $E = hf$, where h is a constant called the **Planck constant**. Einstein took the idea of energy packets literally and postulated that light propagated as a stream of particles (which we now call **photons**) of energy hf and successfully accounted for another phenomenon – the photoelectric effect.

2.7.1 The photoelectric effect

If ultraviolet radiation is shone onto a negatively charged zinc plate, the plate loses its charge. This is easily demonstrated using an electroscope as in Fig. 2.7.2. The gold leaf quickly drops. A positively charged plate shows no effect, suggesting that the UV causes electrons to be given off from the zinc plate. The lack of response from a positive plate can be explained because any electrons given off are attracted back by the positive zinc. The photoelectric effect is not peculiar to zinc.

The vacuum photocell (or phototube) provides a simple way of investigating the photoelectric effect. This was originally used in camera light meters and for reading the optical soundtrack in cinema projectors. It consists of a cylindrical cathode (K) made from a photo-emissive metal and an anode (A) placed in front of it. The whole is surrounded by an evacuated glass envelope. Incoming e-m radiation hits the surface of the cathode, liberating electrons. In the original uses, the anode is connected in a circuit with the cathode positive so that liberated electrons are drawn across and out into the circuit. The current is proportional to the light intensity, enabling it to be used as a light meter. Note that, in the discussion of the photoelectric effect, the word *light* should be taken to include near infrared and ultraviolet electromagnetic radiation.

(a) Photoelectric effect experiments

If we measure the I - V characteristics of a photocell, as in Fig. 2.7.4, we notice that:

- For positive pds above a minimum level the current is constant – all the emitted electrons are collected by the anode.
- This 'plateau current' is proportional to the light intensity.
- There is a positive current for small negative values of V down to a 'stopping voltage' V_s , (the same value for all light intensities).

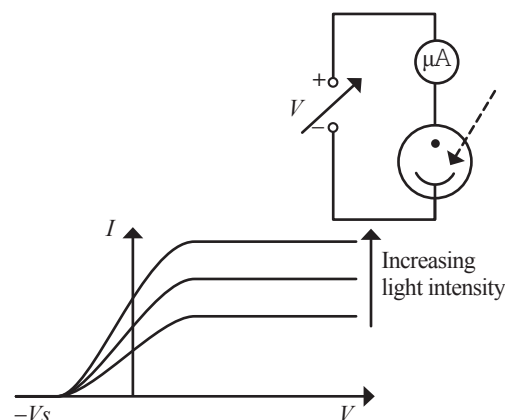


Fig. 2.7.4 Photocell characteristics

If we assume that the **photoelectrons** are emitted with a range of kinetic energies, the value of V_S enables us to measure the maximum value of this, $E_{k \max}$. If an electron with $E_{k \max}$ is just stopped by V_S , then from the definition of potential difference:

$$E_{k \max} = eV_S.$$

The experiments which showed that the radiation did not behave as waves concerned the variation of $E_{k \max}$ with the frequency, f , of the radiation. A suitable circuit is given in Fig. 2.7.5. Notice the polarity of the voltage supply. The photocell is illuminated with monochromatic radiation (i.e. consisting of a single frequency). The pd applied across the photocell is adjusted using the potentiometer until the current just becomes zero and the value, V_S , measured. This is repeated for a range of frequencies, f , and for different metal surfaces (as the cathode in the photocell).

The results of such experiments and the one in Fig. 2.7.4 are as follows:

- If electrons are emitted, there is no measurable time delay.
- For any metal there is a characteristic **threshold frequency**, f_{th} , below which no electrons are emitted, whatever the intensity of the radiation.
- There is a linear relationship between $E_{k \max}$ and the frequency, with the same gradient for all metals (see Fig. 2.7.6).
- If electrons are emitted, $E_{k \max}$ is independent of the radiation intensity.
- If electrons are emitted, the number of electrons emitted per second is proportional to the intensity of the radiation.

(b) Einstein's explanation of the experimental results

The results are incompatible with the idea that the energy of radiation is absorbed continuously, as it would be if light behaved as a wave. There should be no threshold frequency. Low frequency, low intensity radiation could transfer energy gradually to electrons which would eventually build up enough energy to escape from the metal surface, after a time delay. High intensity radiation would be expected to transfer energy more quickly than low intensity and some electrons should gain more energy making $E_{k \max}$ higher.

Einstein proposed the following model to explain the experimental results:

- Electromagnetic radiation consists of discrete packets of energy, photons, with the photon energy given by $E = hf$, where h is a constant called Planck's constant.
- When a photon interacts with an electron in the metal surface its entire energy is transferred.
- An electron only interacts with an individual photon – the probability of two photons interacting with an individual electron is vanishingly small.
- There is a characteristic minimum energy, called the **work function**, ϕ , which is needed to remove an electron from a metal surface.

Taken together, these postulates explain all the results a. to e. above. The maximum energy of the photoelectrons is given by

$$E_{k \max} = hf - \phi,$$

which is known as **Einstein's photoelectric equation**, which is a consequence of postulates 1, 2 and 4; 3 is also involved because, if two or more photons gave their energy to one electron, it could gradually build up enough energy to escape and there should be no threshold frequency.

Table 2.7.1 Miscellaneous values of the work function

Metal	ϕ/eV	Metal	ϕ/eV
Al	4.1	K	2.3
Cd	4.1	Na	2.3
Cs	2.1	Ni	5.0
Ca	2.9	Au	4.5
Mg	3.7	Au	4.6
Cu	4.7	Hg	4.5

Terms & definitions

The electrons emitted by the photoelectric effect are often referred to as **photoelectrons**.

Self-test

2.7.1

If $V_0 = 0.6 \text{ V}$, find the value of $E_{k \max}$ (a) in J and (b) in eV.

[$e = 1.60 \times 10^{-19} \text{ C}$.]

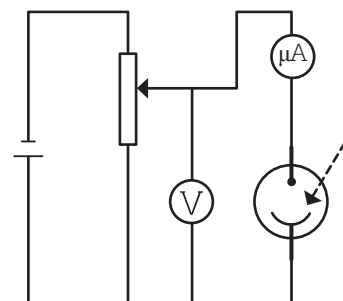


Fig. 2.7.5 Circuit for the photoelectric effect

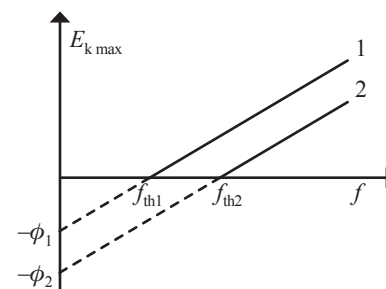


Fig. 2.7.6 Photoelectric graphs for two metals

Self-test

2.7.2

How does Einstein's photon model explain result d?

Terms & definitions

The **work function** is the least energy required to remove an electron from a metal surface. It is usually expressed in **electron volts (eV)**. $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.

Stretch & Challenge

What patterns can you identify in the values of ϕ in Table 2.7.1 and what explanations can you suggest? [Hint: Periodic table and reactivity series.]

Example

Calculate the threshold frequency for photoelectric emission for selenium which has a work function of 5.11 eV.

At the threshold frequency, $E_{k \max} = 0$, $\therefore hf_{\text{th}} = \phi$.

$$\phi = 5.11 \text{ eV} = 5.11 \times 1.60 \times 10^{-19} \text{ J} = 8.18 \times 10^{-19} \text{ J}$$

$$\therefore f_{\text{th}} = \frac{8.18 \times 10^{-19} \text{ J}}{6.63 \times 10^{-34} \text{ J s}} = 1.23 \times 10^{15} \text{ Hz}$$

2.7.3

Self-test

A selenium surface is illuminated by e-m radiation. Calculate the maximum KE of any photoelectrons if the radiation:

- (a) is of frequency $2.5 \times 10^{15} \text{ Hz}$
- (b) is of frequency $1.0 \times 10^{15} \text{ Hz}$
- (c) is a mixture of these two frequencies.

2.7.4

Self-test

For the Sun $L = 6 \times 10^{24} \text{ W}$. Taking 500 nm as representative of all the radiation from the sun, estimate

- (a) the number of photons emitted per second and (b) the number of photons crossing unit area per second at the distance of the Earth (150 million km).

Study point

For very long wavelengths (radio waves) wave properties dominate even when we consider emission and absorption. The higher the frequency, the more the particle properties are seen, though X-ray diffraction is used to investigate crystals.

Exam tip

Learn the typical wavelengths and photon energies of e-m radiation, and the wavelength range of the visible spectrum.

(c) Radiation intensity

We can relate the intensity of a radiation beam to the number of photons crossing an area per second.

Consider a monochromatic beam of radiation, of frequency f , crossing surface. Let N be the number of photons crossing the surface per second.

Then the power in the radiation, $P = NE_{\text{ph}}$, where E_{ph} is the energy of an individual photon.

We can also write this as $P = Nhf = N \frac{hc}{\lambda}$

We can now relate this to the inverse square law by considering a point (or spherical) source of monochromatic radiation of power P . The diagram in Section 1.6.2 shows that, at distance r , this radiation crosses an area of $4\pi r^2$. So the intensity, I , of the radiation [i.e. the power per unit area] is given by:

$$I = \frac{Nhf}{4\pi r^2}$$

Most sources of radiation are far from monochromatic but if the intensity, at all individual frequencies, drops off as r^{-2} , the inverse square law must apply to all radiation whatever the spectral distribution. See Section 1.6.2(a).

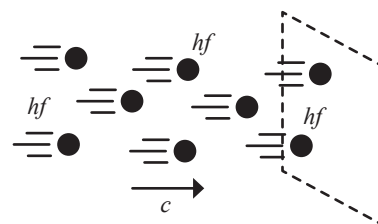


Fig 2.7.7 Photons and intensity

(d) Is light a wave or a particle?

That is a very good question. Light exhibits the wave properties of diffraction and interference. When its speed is measured in materials, it corresponds to the wave model explanation of refraction, which was discussed in Section 2.6. Light is a part of the electromagnetic spectrum. Waves at the low frequency end of the electromagnetic spectrum are produced by oscillating electric and magnetic fields, as predicted by Maxwell's very successful theory of electromagnetism. On the other hand the emission and absorption of light demands a particle model. But the particle properties of energy and momentum (see Section 2.7.4) are calculated using the wave-property values of frequency and wavelength.

So our picture is that light has both wave and particle properties. This is referred to as **wave-particle duality**. We shall see in Section 2.7.4 that objects that we normally think of as particles also exhibit wave properties. This whole topic is the subject of quantum mechanics.

2.7.2 The electromagnetic spectrum

A schematic diagram of the e-m spectrum is shown in Fig. 2.7.8.

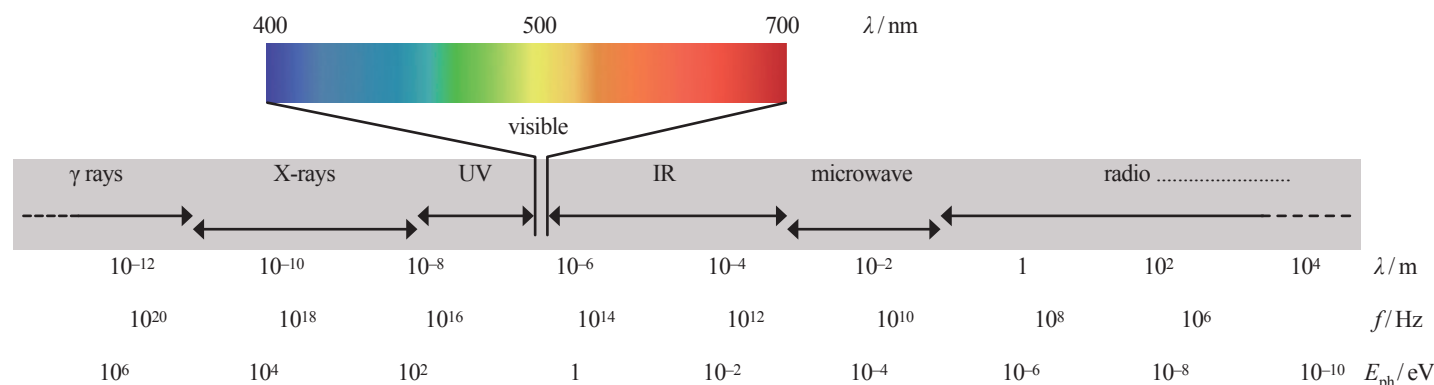


Fig. 2.7.8 The electromagnetic spectrum

Example

Show that $(\lambda/\text{m})(E_{ph}/\text{eV}) \sim 10^{-6}$.

$$E_{ph} = \frac{hc}{\lambda} \therefore \lambda E_{ph} = hc \therefore (\lambda / \text{m})(E_{ph} / \text{J}) = 6.63 \times 10^{-34} \times 3.00 \times 10^8$$

$$\therefore (\lambda/\text{m})(E_{ph}/\text{eV}) = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{1.60 \times 10^{-18}} = 1.24 \times 10^{-6} \sim 10^{-6}.$$

Many aspects of the e-m spectrum are covered in Sections 1.6, Using radiation to investigate stars and 2.5, Wave properties. The regions named in Fig. 2.7.8 are not exhaustive. For example, the infrared region is split into the *near infrared*, *mid infrared*, and *far infrared*; the boundary between the far infrared and the microwave regions is called *terahertz* radiation (radio astronomers refer to these as *sub-millimetre* waves).

2.7.3 Atomic spectra

We saw in Section 1.6.4 that isolated atoms have spectra which consist of a series of wavelengths. We refer to such spectra as line (or discrete) spectra. Atomic gases both emit and absorb radiation at characteristic wavelengths. Part (d) of this section deals with the production of emission and absorption spectra using a diffraction grating and examples are given in Fig. 1.6.10.

(a) Atomic energy levels

The reason why atomic gases have line spectra is to do with the way in which atoms can possess energy. We are used to thinking of systems as being able to possess any level of energy. The kinetic energy of a car could be 13500 J, 13510 J, 13511 J, 13511.1 J, etc. The microscopic world does not behave in this way. Just as light comes in lumps (called photons), atomic systems can only possess particular levels of energy. The simplest atom is the ^1H hydrogen atom because it consists of just two particles (ignoring the quark structure of the proton). Its energy levels (in eV) are as in Fig. 2.7.9. Virtually all the energy shown consists of the sum of the kinetic and potential energies of the electron in motion around the nucleus. An atom is **excited** if it is in an energy state above the ground state. In Fig. 2.7.9, the state with $n = 2$ is called the **first excited state**.

Table 2.7.2 Typical wavelengths and photon energies

Radiation	Typical λ / m	Typical E_{ph} / eV
γ	10^{-12}	10^6
X	10^{-10}	10^4
UV	10^{-7}	10^1
visible	5×10^{-7}	2.5
IR	10^{-5}	10^{-1}
μ -wave	10^{-2}	10^{-4}
radio	10^2	10^{-8}

Self-test

2.7.5

Explain the link between the names terahertz radiation and sub-millimetre waves.

$$\begin{array}{rcl} \infty & \text{-----} & 0 \\ n = 4 & \text{=====} & -0.9 \\ n = 3 & \text{=====} & -1.5 \\ & & \\ n = 2 & \text{-----} & -3.4 \end{array}$$

$$n = 1 \text{ ----- } -13.6$$

Fig. 2.7.9 Energy levels of atomic hydrogen

2.7.6

Self-test

Express the ionisation energy of atomic hydrogen in aJ.

Study point

In atoms with more than one electron, the electrons normally fill up the energy levels from the bottom. The shell with $n = 1$ can hold up to 2 electrons, $n = 2$ can hold 8, $n = 3$ can hold 18, etc. The arrangement of electrons between these energy levels gives rise to the chemical properties of the atoms.

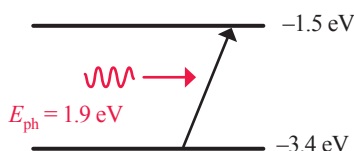


Fig. 2.7.10 Absorption transition

2.7.7

Self-test

Use the formula to show that the H δ line in the spectrum of atomic hydrogen (Fig. 1.6.13) corresponds to a transition from $n = 2$ to $n = 6$.

Conventionally a stationary free electron outside the atom is shown as having zero (0) energy: an electron trapped inside the atom must be given energy to escape so its total energy must be negative. The lowest energy level, called the ground state, has -13.6 eV of energy, so an electron must be given 13.6 eV in order to escape from a hydrogen atom. This is called the **ionisation energy** of hydrogen.

The symbol n is used for the *principal quantum number* of the energy state and corresponds to the *electron shells*. At low temperatures, the electron in a hydrogen atom is most likely to be found in the lowest shell (i.e. $n = 1$) but it could, in principle, be in any of the shells.

(b) Atomic absorption spectra

In a cloud of atomic hydrogen in space, most of the atoms will be in the ground state but some will be in the first excited state (i.e. $n = 2$). The cloud will be flooded with photons of a large range of energies, which come from nearby stars. The difference in energy, ΔE between the 2nd and 1st excited states is given by:

$$\Delta E = -1.5 \text{ eV} - (-3.4 \text{ eV}) = 1.9 \text{ eV}$$

If a 1.9 eV photon hits an atom, which is in the $n = 2$ state, it may be absorbed and giving its energy to the electron and putting it in the higher-energy state, as shown in Fig. 2.7.10. Photons with slightly higher or lower energies (1.8 eV or 2.0 eV) will not be absorbed; so radiation which passes through the cloud will be depleted in photons with this energy, giving rise to one of the dark lines in the spectrum in Fig. 1.6.12. Using ideas from in Section 2.7.1, you should be able to find the wavelength of 1.9 eV photons and identify the absorption line in the visible spectrum of atomic hydrogen to which this corresponds (in Fig. 1.6.13).

For atoms or ions which only have one electron, e.g. H, He $^+$, Li $^{2+}$, Be $^{3+}$ etc (which occur frequently in the atmosphere of stars), the energy levels (in eV) can be calculated from the simple formula:

$$(E_n / \text{eV}) = -13.6 \frac{Z^2}{n^2},$$

where Z is the proton number (atomic number). This formula also approximately gives the energy level of the innermost electron in atoms with more than one electron.

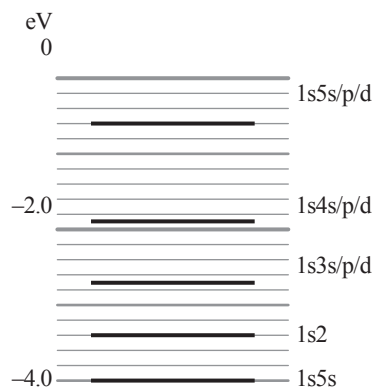


Fig 2.7.11 Helium 2nd electron excited energy levels.

Example

Use the energy diagram for atomic hydrogen (Fig. 2.7.9) to shown that only transitions between the first excited state ($n = 2$) and higher states correspond to e-m radiation in the visible part of the spectrum.

ΔE between $n = 1$ higher energy levels is between 10.3 eV and 13.6 eV . This lies in the UV part of the spectrum. ΔE between $n = 3$ and higher energy levels is between 0.6 eV and 1.5 eV . This is in the near IR part of the spectrum. We have seen (main text and Self-test 2.7.7) that some transitions between $n = 2$ and higher lie in the visible part of the spectrum.

(c) Atomic emission spectra

In a hot cloud of atomic hydrogen, e.g. the one in Section 2.7.3(b), which is heated by absorbing radiation from a nearby star, some of the atoms will be excited, i.e. they will be in energy levels above the ground state. This may also happen as a result of inter-atomic collisions, when some of the kinetic energy of the colliding atoms is lost. If the electron in such an atom falls into a lower energy state it gives out a photon of e-m radiation, of energy equal to the difference in energy of the two states, e.g. the electron which is promoted in Fig. 2.7.10 will subsequently return to the lower energy level, emitting a 1.9 eV photon in the process.

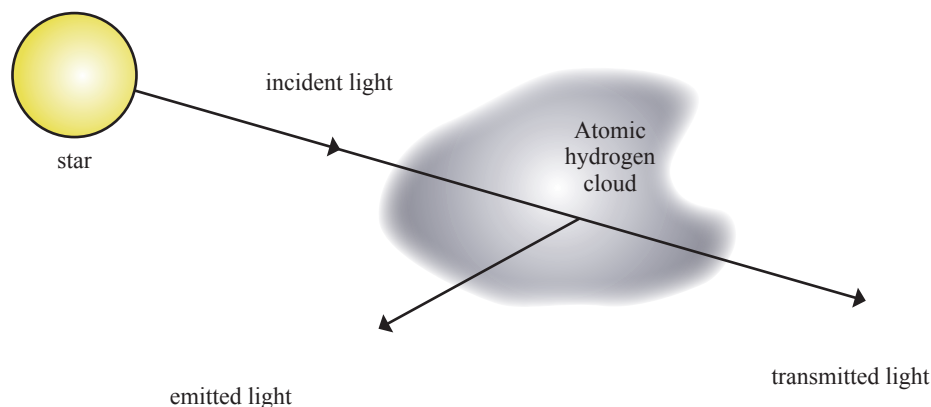


Fig 2.7.12 Emission and absorption by hydrogen atoms

Different observers could see both an emission spectrum and an absorption spectrum from the same object, as shown in Fig. 2.7.12. In principle the two spectra are the same but in practice not all lines will appear, as discussed in Section 1.6.4.

(d) Investigating and displaying atomic spectra

The simplest way to see the emission spectrum of a gas is to use a gas discharge tube. This consists of a sealed glass tube containing a low pressure gas and two high electrical terminals. When a high voltage is applied, the gas partially ionises allowing electrons to pass through it. These collide with the gas atoms and raise them to a range of excited energy states: they then drop down to the lower energy states and emit photons as they do. The discharge tube in Fig. 2.7.13 contains low pressure argon gas.

The lamp can be viewed through a diffraction grating with the results shown: the central image is the zero order spectrum (with all the wavelengths at the same place); the ones on each side are the first order spectra. Alternatively, an image of the discharge tube can be projected onto a screen using a lens and a diffraction grating interposed (Fig. 2.7.16).

Stretch & Challenge

When a helium atom is excited, one of the electrons invariably stays in its ground state: -54.4 eV . The second electron has a ground state of -24.6 eV and its excited energy levels are shown in Fig. 2.7.11.

(a) Show that this information is consistent with the equation

$$(E_n / \text{eV}) = -13.6 \frac{Z^2}{n^2},$$

(b) Investigate whether the above information is consistent with these wavelengths in the spectrum of atomic helium: 388.8 nm 587.4 nm; 667.5 nm; 706.2 nm.

Self-test

2.7.8

Express the typical photon energies in Table 2.7.2 on page 157 in joules, to 1 s.f.

Fig. 2.7.14
Spectra from
an argon
discharge tube

Fig 2.7.13 Argon discharge tube

Study point

When projecting spectra, as in Fig. 2.7.16, the wavelength of a spectral line can be calculated from the separation of the grating lines, the distance from the grating to the screen and the distance from the line to the zero order spectrum. See Section 2.5.4 and Self-test 2.7.9.

The coloured emissions of the aurora borealis are caused by the same mechanism. Charged particles in the solar wind spiral along the Earth's magnetic field lines and enter the atmosphere at high latitudes. They collide with the atoms of the upper atmosphere (chiefly atomic oxygen and nitrogen) ionising them. The radiation is emitted as the electrons recombine with the ions and descend through the atomic energy levels.

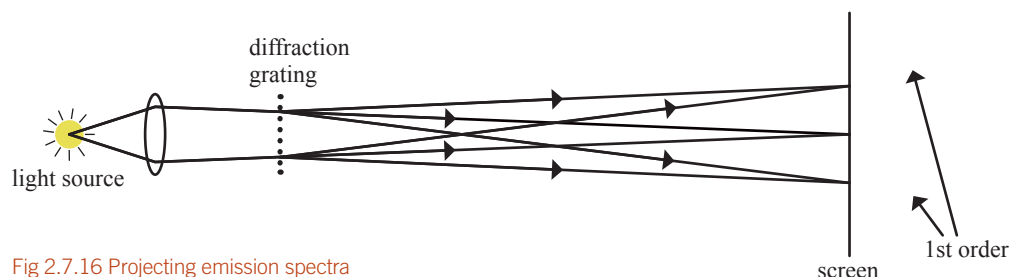


Fig 2.7.16 Projecting emission spectra

To display the **absorption spectrum** of a gas, a suitably shielded bright white light source with a continuous emission spectrum (e.g. a filament lamp) is used and a tube of the gas placed between the light source and the screen (usually between the source and lens). With care this method can also be used to display the absorption spectra of metals by allowing the white light to pass through a Bunsen flame in which a sample of the metal salt has been vaporised (in a flame test).

Fig. 2.7.15 Aurora borealis: atomic oxygen emission

2.7.4 Wave-particle duality

(a) Electrons are waves too

Just as electromagnetic radiation can behave as both waves and particles, objects that we normally think of as particles, such as electrons, protons and even whole atoms have wave-like properties. In other words, particles exhibit diffraction and interference. The single-slit and double-slit experiments are rather difficult to arrange for electrons, although the effect was predicted in the 1920s it was not achieved until 1961. The set-up is shown schematically in Fig. 2.7.17. A stream of electrons (the red balls) is fired through a narrow slit and hit a fluorescent screen: each impact causes a bright spot.

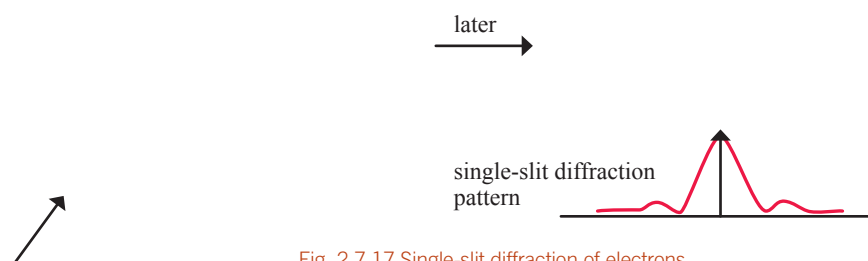


Fig. 2.7.17 Single-slit diffraction of electrons

Initially the scatter of the dots appears random but gradually a pattern emerges which clearly shows similarities to the diffraction pattern of a single slit (see Section 2.5).

To achieve the double-slit pattern the individual slits must be made narrower (to spread out the diffracted electrons), as in Fig. 2.7.18. In a 2008 version of the experiment, the individual slits were only **62 nm** wide and **272 nm** apart! These values were chosen to suit the wavelength of the electrons – see Section 2.7.4(c). The sequence of images in Fig. 2.7.19 indicates how the apparently random spread of electrons eventually resolves itself.

Fig 2.7.18
Electron
double-slit
interference

Fig. 2.7.19 The build-up of an electron interference pattern

(b) The electron diffraction tube

In the school laboratory, the wave-nature of electrons is usually demonstrated using the electron diffraction tube, Fig. 2.7.20, which consists of a vacuum tube with an electron gun at one end, a graphite target (T) and a phosphor screen (P).

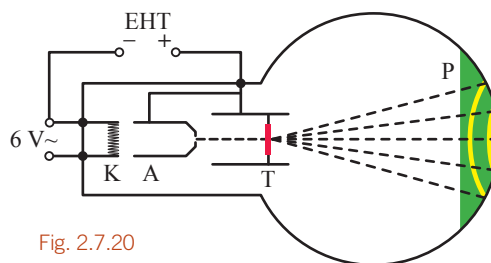


Fig. 2.7.20
Diffraction tube

- A metal-coil cathode (K) is connected to a low voltage a.c. supply which heats it up. This causes it to emit electrons by **thermionic emission**.
- The electrons are accelerated to the anode (A) by an EHT ['extremely high tension', i.e. high voltage] supply [typically 1 – 6 kV].
- A beam of electrons emerges from the hole in the anode and hits the graphite target (shown as a red line)
- The beam is diffracted by the graphite, similarly to light at a diffraction grating and emerges at a series of angles to the forward direction. The electron beams hit the phosphor, producing a display of circular rings.
- If the EHT voltage is increased, the rings are observed to decrease in radius.

The diffraction pattern occurs because graphite consists of regular planes of carbon atoms hexagonally arranged, with an inter-atomic spacing of **0.142 nm**. The planes are separated by **0.335 nm**. Because the crystals in the graphite are randomly orientated, the diffracted beams at any particular angle produce circles rather than an individual dot.

(c) The wavelength of particles

The diffraction tube can be used to measure the wavelength of the electrons. All that is needed is to measure angles that the diffracted beams make with the forward direction. In Fig. 2.7.21 a physics student is doing this by measuring their radii; the distance from the target to the screen also needs to be measured.

In 1924 the physicist Louis de Broglie proposed that the wavelength (λ) of a particle, such as an electron is related to its momentum (p) by the equation:

$$\lambda = \frac{h}{p},$$

where h is the Planck constant. This relationship has since been confirmed – it is consistent with both quantum and relativity theory. The following example shows how we can use the accelerating voltage to calculate the wavelength.

Example

Calculate the wavelength of 5 keV electrons. ($m_e = 9.1 \times 10^{-31}$ kg)

$$5 \text{ keV} = 5.0 \times 10^3 \times 1.6 \times 10^{-19} \text{ J} = 8.0 \times 10^{-16} \text{ J}.$$

$$E_k = \frac{p^2}{2m} \therefore p^2 = 2 \times 9.1 \times 10^{-31} \times 8.0 \times 10^{-16} = 1.46 \times 10^{-45} \rightarrow p = 3.82 \times 10^{-23} \text{ N s}.$$

$$\therefore \text{The wavelength, } \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{3.82 \times 10^{-23}} = 1.7 \times 10^{-11} \text{ m [17 pm]}$$

Notice that the wavelength calculated in the example is of the same order as the inter-atomic spacing in solids and liquids (approximately 10% of the graphite inter-atomic spacing). This means that electron diffraction can be used to investigate the structure of matter. In fact X-rays, which have a similar wavelength, are more often used because difficulties with handling charged particles and the low penetration power of electrons.

Self-test

2.7.9

A diffraction grating with 850 lines per cm is used to display the spectrum of a gas. A line is identified and the separation between the two first order spectra for this line is measured to be **20.0 cm**. If the screen is **2.00 m** from the grating, calculate the line's wavelength.

Fig. 2.7.21 Electron diffraction pattern

Exam tip

The relationship $E_k = \frac{p^2}{2m}$ is useful when calculating the momentum of accelerated particles.

Self-test

2.7.10

Use the de Broglie equation to calculate the wavelength of protons which have been accelerated through 5 kV.

$$[m_p = 1.67 \times 10^{-27} \text{ kg}]$$

Study point

The electron diffraction and interference patterns are not the result of electrons interfering **with one another**. The same pattern is obtained even when the intensity of the electron beam is so low that there is only one electron at a time in the apparatus. Each electron behaves as its own wave and interferes **with itself**!

2.7.11

Self-test

Calculate the momentum of an optical photon of frequency 600 THz.

Stretch & Challenge

Einstein showed that the total energy and momentum of a relativistic particle are related by:

$$E^2 = p^2 c^2 + m^2 c^4$$

Show that this is consistent with

$$p = \frac{E}{c} \text{ for a photon.}$$

2.7.12

Self-test

(a) Show that the pressure exerted by the beam in Fig. 2.7.21 is $2 \frac{I}{c}$ if the surface is perfectly reflecting.

(b) Calculate the pressure if the perfectly reflecting surface is inclined at 45° .

(d) The momentum of photons

De Broglie's equation also applies to photons – another aspect of the duality of wave and particle descriptions. This means that if photons are absorbed or reflected by an object, their momentum changes and so, by conservation of momentum, the object suffers an equal and opposite momentum change. It is often more convenient to express the momentum of a photon in terms of the frequency (f) rather than the wavelength of the radiation:

$$\therefore p = \frac{h}{\lambda} = \frac{hf}{c}$$

Now hf is the photon energy, so the photon momentum is $\frac{E_{ph}}{c}$. One of the consequences of photon momentum is that a beam of radiation exerts a pressure on any surface which it hits. The beam of photons incident on the surface of area A , in Fig. 2.7.22, has intensity I . Thus the total energy delivered in a time Δt is given by $IA\Delta t$.

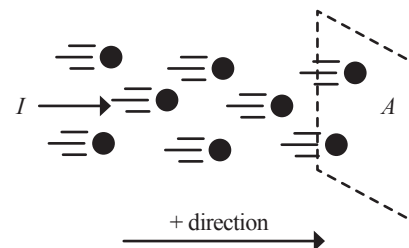


Fig 2.7.22 Radiation pressure

$\therefore$ The momentum of the incident photons in $\Delta t = \frac{IA\Delta t}{c}$

If the surface is totally black, so that all the momentum is absorbed, the rate of change of momentum of the photons is $-\frac{IA\Delta t}{c\Delta t} = -\frac{IA}{c}$. So by the principle of conservation of momentum, the rate of change of momentum of the surface, caused by absorbing the photons is $+\frac{IA}{c}$. In other words, by Newton's 2nd law (N2), the radiation exerts a force $\frac{IA}{c}$ on the surface, i.e. the pressure $= \frac{I}{c}$.

An artist's impression of the solar sail in NASA's proposed *sunjammer* mission is given in Fig. 2.7.23. The name of the mission is a reference to the 1964 story of that title by the science fiction writer Arthur C Clarke. The story concerned an interplanetary race using the pressure of sunlight as a means of propulsion.

Fig. 2.7.23 NASA's sunjammer

2.7.5 Light-emitting diodes

Light-emitting diodes (LEDs) are electronic devices constructed from a small crystal of a semiconductor material, such as gallium arsenide (GaAs). It conducts electricity in one direction (given by the arrow-head direction of the main part of the symbol) and, as it does so it gives out e-m radiation. Many LEDs are designed to give out close to monochromatic radiation, but some (e.g. 'white' LEDs or 'bright' LEDs) emit a broad spectrum.

Pure GaAs has the very high resistivity of about $10^6 \Omega \text{ m}$ at room temperature, which is $\sim 10^{13}$ times as high as most metals. However, the addition of very small quantities (typically 1 atom in 10^6) of specific impurity atoms, in a process called doping, can reduce the resistivity by many orders of magnitude. In particular:

- Doping with **Si** or **Ge** (group IV metals, with four electrons in the outer shell) results in mobile electrons in the GaAs lattice; the resulting crystal is called an n-type (n for negative) semiconductor.
- Doping with **Be** or **Mg** (group II metals, with only two outer electrons) results in mobile regions (called 'holes') with a net positive charge – a p-type semiconductor.

A single crystal is grown with p-type doping at one end and n-type at the other. A small fraction of the mobile electrons and holes diffuse to the centre and combine producing the depletion zone, a region without mobile charge carriers. But if a sufficiently high pd is established across the LED with the p-type positive and the n-negative, the charge carriers are pushed together and the whole crystal becomes conducting; mobile electrons 'fall into' the holes giving out photons of light. This will not be examined.

(a) Determination of h using LEDs

The photon energy of the emitted light is found from the pd, V_0 , at which the LED starts to conduct (see Fig. 2.7.25) and give out light. The photon energy is also given by hf

$$\therefore hf = eV_0 \text{ or, in terms of the wavelength: } \frac{hc}{\lambda} = eV_0$$

So, if there is a range of monochromatic LEDs available with different colours, a graph of V_0 against $\frac{1}{\lambda}$ is a straight line through the origin of gradient $\frac{hc}{e}$.

Experimental details

1. Obtain a range of low power LEDs, with as wide a range of wavelengths as possible; determine the wavelengths using a diffraction grating as in Fig. 2.7.16. If this is not possible, note the manufacturer's values of wavelength.
2. Set up a circuit (see Study point) to determine pairs of (V, I) and plot the I - V graph for each LED.
3. Use the graphs to determine the value of V_0 , as in Fig. 2.7.25, for each wavelength by drawing a best-fit straight line for the steep part of the graph.
4. Plot a graph of V_0 against $1/\lambda$ and determine the gradient, m .
5. Calculate a value for h from $m = \frac{hc}{e}$.

Extension to IR and UV LEDs

LEDs are available with emissions in the near IR (wavelengths 700–900 nm) and UV (wavelength $\sim 390 \text{ nm}$). These can be used to extend the wavelength range. If using these, the wavelength cannot easily be measured in the school laboratory so the manufacture's stated wavelength has to be relied upon.

SYMBOL

LED symbol 

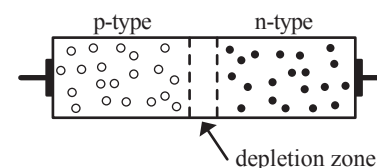


Fig. 2.7.24 LED structure

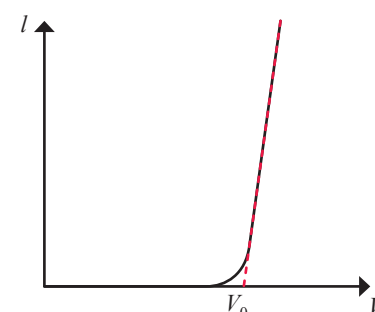


Fig. 2.7.25 LED I - V graph

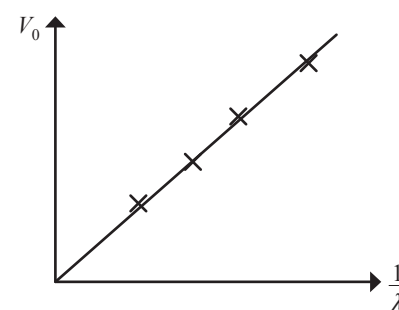


Fig. 2.7.26 Graph of V_0 against $1/\lambda$

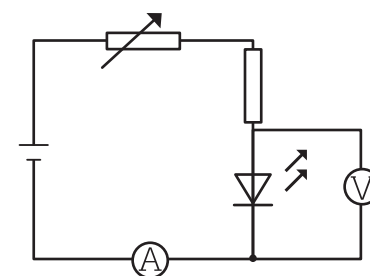


Fig. 2.7.27 I - V circuit

Study point

Section 2.2.8 gives possible circuits for plotting the I - V graph. Fig. 2.7.27 is an alternative. Adjusting the variable resistor changes the led I and V . The fixed resistor is to protect the LED from too large a current.

Exercise 2.7

1. An LED torch has a pd of 3.0 V and takes a current of 50 mA .
If the efficiency is 25%, estimate the number visible photons it emits per second. (Take the mean wavelength to be 550 nm .)
2. The star α Centauri has a luminosity of $L_{\odot}$ where $L_{\odot}$ is the luminosity of the Sun. Its distance is 4.37 light years. Assuming that 70% of the star's emissions are in the visible region, estimate the number of visible photons per second which enter the eyes of an observer on the Earth.
Take the diameter of a dark-adapted pupil to be 7 mm . $L_{\odot} = 6.0 \times 10^{24} \text{ W}$.
3. A metal has a work function of $2.4 \times 10^{-19} \text{ J}$.
(a) Express the work function in eV .
(b) The metal is illuminated by monochromatic e-m radiation of photon energy 2.5 eV . State the maximum energy of the emitted electrons.
(c) The metal is **additionally** illuminated with e-m radiation of frequency $3.0 \times 10^{14} \text{ Hz}$. Explain why the maximum energy of the emitted electrons is the same as in (b).
(d) The metal is built into a photocell and its I/V characteristic plotted as in Fig. 2.7.4. What value of stopping voltage would you expect with the 2.5 eV photons? Explain your answer.
4. Calculate the range of photon energies in the visible spectrum. Express your answer in both J and eV .
5. A glowing gas cloud containing helium is heated to a high temperature by a nearby star, so that some of the atoms are in the excited state $1s2s$ (see Fig. 2.7.11). Visible white light from other stars passes through the cloud. Explain, giving wavelength details, what effect the cloud has on the visible spectrum of the light.
6. Electrons are given off from a heated wire, are accelerated through a pd of 500 V and directed in a narrow beam at a graphite crystal oriented edge on. The crystal planes (separation 0.335 nm) act as the slits of a diffraction grating. An interference pattern is observed on a fluorescent screen 30 cm from the slits.
(a) Explain why an interference pattern is seen on the screen.
(b) Calculate the wavelength of the electrons.
(c) Use the diffraction grating formula to calculate the angular position of the first and second order fringes and hence the position of these fringes on the screen.
Electronic mass, $m_e = 9.1 \times 10^{-31} \text{ kg}$
7. A low voltage accelerator produces beams of electrons and protons by accelerating them through the same pd. Calculate the ratio of the wavelengths of the electrons and protons.
 $[m_e = 9.1 \times 10^{-31} \text{ kg}; m_p = 1.67 \times 10^{-27} \text{ kg}]$.
8. An electron in a stationary hydrogen atom is in the second shell (see Fig. 2.7.9, $n = 2$). The electron drops down to the first shell and, in doing so, gives out a photon. By calculating the momentum of this photon, calculate the recoil speed of the hydrogen atom.
[Mass of a hydrogen atom = $1.67 \times 10^{-27} \text{ kg}$]

9. The solar constant, which is the intensity of the Sun's radiation, is 1.4 kW m^{-2} at the Earth's orbit. Calculate the area of a solar sail which is needed to produce a thrust of 1 N in the vicinity of Mars, given that the radius of the orbit of Mars is 1.5 times that of the Earth. Assume that the sail absorbs all the photons incident upon it.
10. A class of A level students undertakes the LED experiment to determine the value of the Planck constant.

They use the manufacturer's values for the wavelengths of the LED emissions. Their results are as in the table. The estimated uncertainty in the V_0 results is $\pm 0.05 \text{ V}$.

λ / nm	420	460	540	640	660
V_0 / V	2.95	2.74	2.25	1.98	1.85

By plotting a suitable graph, determine a value for h with its estimated uncertainty.

Stretch & Challenge

In an enclosure filled with black body radiation of temperature T , the energy density is given by $\frac{4\sigma}{c} T^4$, where σ is Stefan's constant.

- Starting from Stefan's law of radiation, show that this is dimensionally correct.
- Calculate the energy density in the cosmic microwave background radiation (CMBR), which has a temperature of 2.73 K .
- Estimate the number of CMBR photons in a 1 km^3 volume of space.
- Estimate the mass-energy of CMBR photons in the observable universe.

2.8 Lasers

Study point

In 1962, scientists from both the USA and USSR used the time delay in laser pulses to measure the distance to the Moon. Apollo 11, 14 and 15 and Lunokhod 1 and 2 all left laser reflectors on the Moon; distance monitoring continues using them today.

Stretch & Challenge

A laser sends out 10^{17} photons to the Moon per second. The beam of a laser spreads to a circle 6.5 km in diameter on the Moon. The reflector is 3 m in diameter. Assuming the reflected beam also spreads out to 6.5 km on the Earth, estimate the rate of arrival of photons in a 3 m diameter telescope.

From the first functioning model made in 1960, lasers are now ubiquitous. Most homes have several: in DVD and CD players and optical disc drives, to say nothing of laser pointers. The first public practical use was the bar-code scanner in 1974 and since then they have been pressed into service in a variety of contexts including:

- Surgery – cutting and cauterising, minimising blood loss; also ‘welding’ detached retinas
- Distance measurement, e.g. by estate agents (see Study point)
- Data transmission in optical fibres and free-space
- Laser printers
- research, e.g. into laser-initiated nuclear fusion.

The usefulness of lasers arises from the fact that they produce coherent light. We refer to two light sources as coherent if they have a constant phase difference, e.g. the Young slits, but what does it mean to say that a single source is coherent (with itself)? In fact light from a laser is coherent in two ways. It has:

1. **Spatial coherence:** different points across the width of the laser beam are in phase with one another;
2. **Temporal coherence:** there are no sudden changes of phase.

These properties allow it to be focused down to very small points (~ 1 mm) and to produce very short duration pulses (~ 1 fs). These coherent light sources are made possible by the phenomenon of stimulated emission.

2.8.1 Stimulated emission

We saw in Section 2.7 that atomic and molecular systems exist in a series of discrete energy states. For the moment, we consider just two states, U and L (upper and lower) of a system with energies, E_U and E_L . An atom or molecule can change its state:

- from U to L by spontaneously emitting a photon of frequency f where:
- $$hf = E_U - E_L$$
- from L to U by absorbing the energy of a photon of the same frequency, f , as above.

These two processes, spontaneous emission and absorption, are illustrated diagrammatically in Fig. 2.8.1. The point about referring to ‘spontaneous emission’ rather than just ‘emission’ is that a second process of emission also takes place. This is known as **stimulated emission** and it was initially predicted by Albert Einstein.

In this process, an atom in the upper energy state (U) is stimulated into moving down into the lower energy state by another photon of the same energy: $hf = E_U - E_L$. In doing so it emits a second photon which is **in phase** with the first one and travelling **in the same direction**. If each of these photons now interacts with an atom in the upper state, there will be four photons: the light will be progressively amplified as it passes through the medium, giving rise to the name laser, for **L**ight **A**mplification by the **S**timulated **E**mission of **R**adiation.

Clearly for the light to be continuously amplified in this way, the photons have to carry on meeting atoms, all of which are in the upper state. Is this likely to occur naturally? The answer is, ‘No!’ Let’s see why not.

For convenience we’ll consider a monatomic gas. Suppose we have a gas at room temperature (300 K) with all of the molecules being in the ground state (G). How could

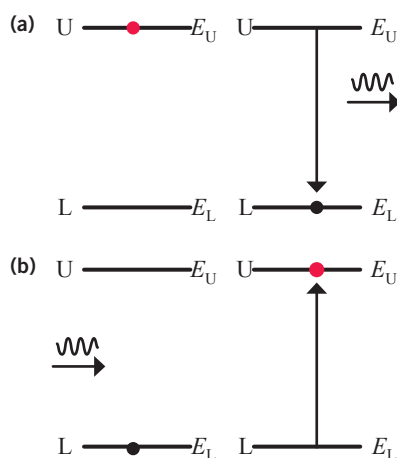


Fig. 2.8.1 (a) Spontaneous emission and (b) absorption

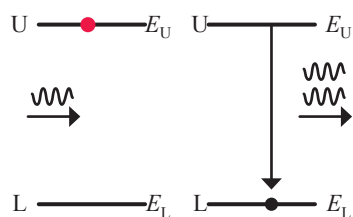


Fig. 2.8.2 Stimulated emission

Study point

Astronomers have discovered naturally occurring **masers** (**m**icrowave **a**mplification ...). See Section 2.8.5.

the molecules be put into an excited state (**E**)? It could happen occasionally by collision: some of the kinetic energy of the molecules is used to put one of the molecules into state **E** (i.e. an inelastic collision). Could this kind of collision happen repeatedly until we have more **E** molecules than **G**? No because the next time the **E** molecule collides, it is likely to be with a **G** and also likely to drop down again into the ground state with the energy reappearing as translational kinetic energy (we could call this a *superelastic* collision). Also the molecule is likely after a short time to lose its extra energy by spontaneous emission. And if we raised the temperature, say to **3000 K**, all we'd achieve is to increase the fraction of molecules in the excited state. The 'best' we can hope as we raise the temperature is to approach 50% of the molecules being excited because at this point the collisions would be equally likely to cause to decrease or to raise the energy level. This is illustrated in the **Stretch & Challenge** below.

Stretch & Challenge

In 1868, the Austrian physicist Ludwig Boltzmann showed how the ratio of the number of particles in two different energy states is related to the (absolute) temperature.

Let N_1 and N_2 be the number of particles in states 1 and 2 respectively and let state 2 have an energy ΔE above state 1. Then:

$$\frac{N_2}{N_1} = e^{-\frac{\Delta E}{kT}},$$

where $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$ (the Boltzmann constant) and T is the kelvin temperature.

The first excited state of sodium atoms is **2.0 eV** above the ground state. In a gaseous cloud containing 10^{15} sodium atoms, estimate the number of atoms in the excited state if the temperature is: (a) **1000 K**, (b) **3000 K** (c) **300 K** (room temperature). Comment on these values.

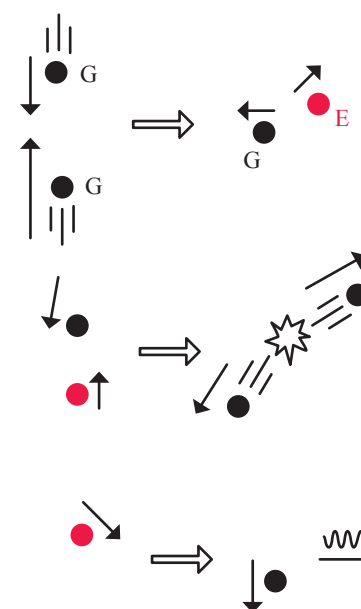


Fig. 2.8.3 Excitation and de-excitation processes

Terms & definitions

Scientists refer to a **population** of atoms (or molecules) when they mean all those with a particular property, e.g. the population of sodium atoms, the population of sodium atoms in the first excited state.

Terms & definitions

Let the populations of atoms in states 1 and 2 be N_1 and N_2 respectively with $E_2 > E_1$. If $N_2 > N_1$ there is said to be a **population inversion**.

Terms & definitions

Pumping is transferring particles to a high-energy state to achieve a population inversion.

2.8.2 Achieving a population inversion

If you have worked through the Stretch & Challenge, you'll have seen that that the **population** of atoms at higher energy levels is normally much smaller than that at lower levels. With energy differences in the **eV** range and temperatures up to **1000 K**, the high-energy population is vastly outnumbered by the low-energy. A laser can only work if the populations are the other way round, i.e. if $N_2 > N_1$. We call this situation a **population inversion**. In order to do this we need to find some non-thermal means of boosting the population of the upper state. We call such processes **pumping**.

It is not normally possible to achieve a population inversion with a two-state system because the upper state will usually empty as fast as it fills. So scientists work with systems of multiple energy states.

(a) Three-state laser systems

Fig. 2.8.4 is of a three-state system. The three energy states are referred to as the ground state (**G**), the pumped state (**P**) and the upper state (**U**).

To understand how a population inversion is achieved in multiple-state systems, you need to know two important features of transitions between energy states:

1. Downward transitions can occur by a variety of routes, not all of which are equally likely. Laser engineers choose systems in which state **P** is much more likely to decay via the intermediate state **U**, rather than directly back to **G**.

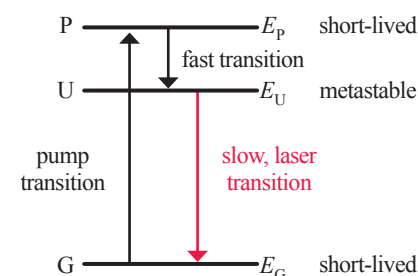


Fig. 2.8.4 Three-state system

Terms & definitions

Optical pumping is raising the state from the ground state to the pumped state using photons of energy ($E_P - E_G$). Other pumping technologies are: electrical, chemical reaction and nuclear fission.

2.8.1**Self-test**

If $E_G = -10.0$ eV, $E_P = -7.5$ eV and $E_U = -8.2$ eV, state the energies of:

- the pumping photons,
- the spontaneously emitted photons and
- the lasing photons.

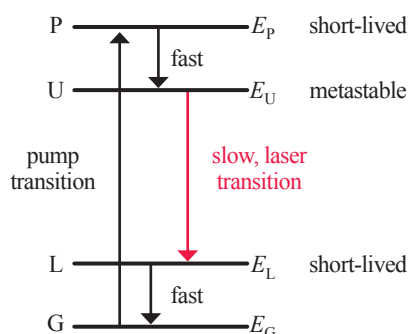


Fig. 2.8.5 Four-state system

2.8.2**Self-test**

Explain why a three-state laser is possible having P as a metastable state and U as a short-lived state.

2.8.3**Self-test**

Calculate the efficiency of energy conversion for a successful pumping event in Self-test 2.8.1.

- Some energy states are very short-lived, i.e. they are populated only for a very brief period of time before decaying. Others, known as **metastable states**, last much longer before decaying. The reasons for these differences are beyond the A level course but engineers choose systems in which P is short-lived (e.g. ~ 1 ns) but U is long-lived (in atomic terms, e.g. ~ 1 ms).

If the laser medium is pumped, e.g. by flooding it with photons of energy ($E_P - E_G$), atoms in the ground state will be raised to the pumped state and rapidly decay into the metastable upper state (partly by spontaneous emission but mainly by collisions). If the pumping is rapid enough, the population of U will exceed that of G, i.e. a population inversion will have been achieved: any spontaneous transition from U to G will produce a photon which will stimulate emissions from other atoms in state U.

Historically, a three-state laser based on a ruby laser medium was the first one to be built (see Fig. 2.8.6). However, because over half the atoms in the ground state must be pumped to achieve laser operation, three-state systems are energy hungry and inefficient. In practice, most lasers use four-state systems.

(b) Four-state laser systems

The characteristics of the pumped and upper states in a four-state laser are the same as in the three-state laser. The additional, lower, state (L), is between the upper and ground states. L is a short-lived state and rapidly decays, mainly by collisions to G.

The advantage of this system over the three-state laser is that L is initially empty, so a population inversion between L and U is present from the first few electrons in U. The short-lived nature of L means ensures that a population inversion is maintained with much lower level pumping, and hence requires less energy input, than the three-state laser. This means that lasing is possible at a much lower power input.

(c) Laser inefficiency

Most of the energy input into the laser is converted into the internal energy (kinetic energy in a gas / vibrational energy in a solid) of the atoms of the amplifying medium rather than raising the energy state of the atoms themselves from G to P. Even for successful pumping events, the energy input is ($E_P - E_G$) but the lasing output is less: ($E_U - E_L$) for the four-state laser; ($E_U - E_G$) for the three-state laser.

2.8.3 Laser construction

The ruby laser illustrated in Fig. 2.8.6 was the first one to be produced. It is introduced here to illustrate generic aspects of lasers. As the diagram shows, it was optically pumped by the quartz 'flash tube' wrapped around the ruby (the amplifying medium). The role of the aluminium cylinder was to reflect stray pumping light back into the ruby to increase the effect.

Fig. 2.8.7 brings out significant features which you should study.

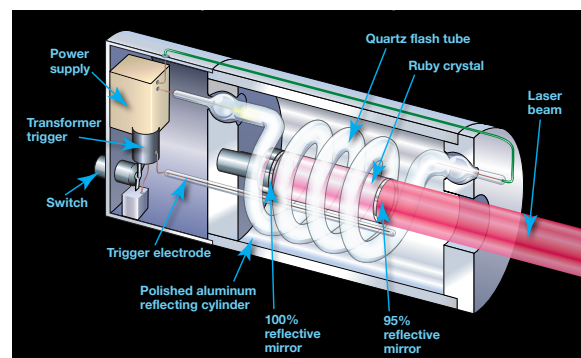


Fig. 2.8.6 The original ruby laser

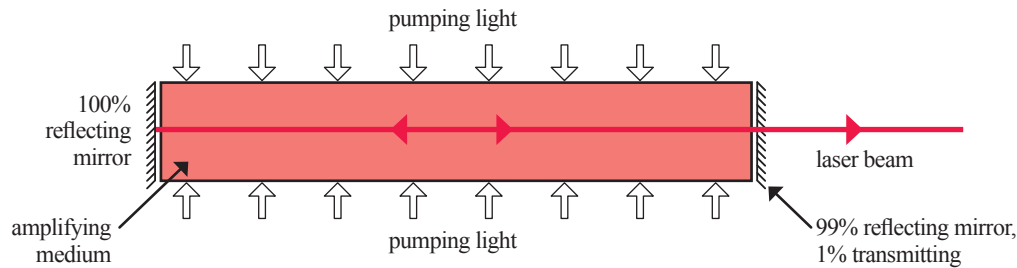


Fig. 2.8.7 Structure of an optically pumped laser

The laser operates as follows:

- The pumping radiation creates a population inversion in the amplifying medium.
- Photons of energy $E_U - E_L$ (as in Fig. 2.8.5) or $E_U - E_G$ (Fig. 2.8.4) are produced by spontaneous emission in the **amplifying medium**.
- These photons pass through the amplifying medium and produce coherent photons travelling in the same direction by stimulated emission. Each photon becomes two, four, eight, etc., in an exponential increase.
- The photons travelling parallel to the axis of the medium (see **Study point**) are reflected backwards and forwards stimulating more photons and eventually escaping through the partly transmitting mirror in the laser beam.
- A dynamic equilibrium is soon established when the rate of escape of photons is equal to the rate of their production by stimulated emission (itself controlled by the rate of pumping).

2.8.4 The semiconductor diode laser

The lasers in everyday household use are almost all semiconductor laser diodes. These are constructed out of small chips of semiconductor material, often gallium arsenide (GaAs). They have many advantages:

- They are electrically pumped and operate at low voltage – some less than 2 V.
- They laser chip is very small (typically $\sim 1\text{ mm}$) and can be incorporated into small standard electrical packages for wiring into circuits, e.g. the pointer laser in Fig. 2.8.8.
- They are very efficient – up to 70% for infrared lasers.
- They can be mass produced cheaply.

Typical uses in the home are DVD and CD reading and writing, blu-ray reading, optical fibre data transfer, computer scanners and printers. The structure of a typical semiconductor diode laser is shown in Fig. 2.8.9.

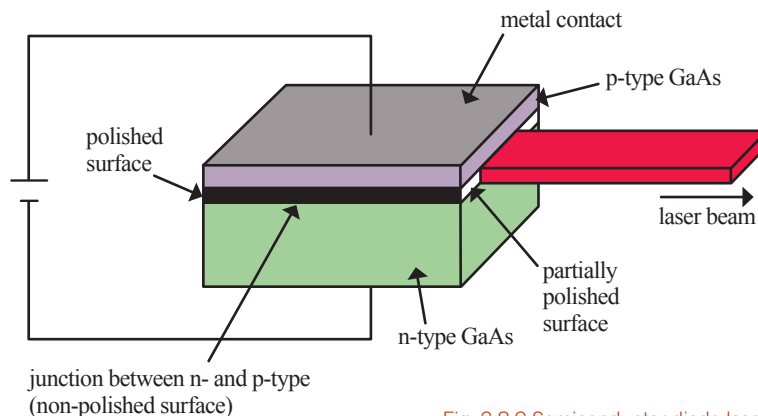


Fig. 2.8.9 Semiconductor diode laser

Terms & definitions

The **amplifying medium** is the material in the laser in which a population inversion exists.

Study point

The mirrors, M_1 and M_2 are accurately parallel. Because of the multiple reflections (for a 1% transmitting mirror, photons will hit mirror M_2 for a mean of 140 times before emission) only the photons travelling accurately parallel to the axis will form the beam. Others will escape from the sides of the medium.

Self-test

2.8.4

Use the information in the **Study point** to estimate the mean distance travelled by a photon through the amplifying medium before it stimulates another one, for a 50 cm long laser. Explain your reasoning.

Fig. 2.8.8 Pointer laser

Study point

Because of the high degree of amplification, the polishing at the end surfaces can be quite poor without stopping the laser action. Diode lasers work effectively with reflection coefficients as low as 40%.

2.8.5

Self-test

Semiconductor lasers exhibit high efficiencies. A **940 nm** laser operates at a 70% efficiency. Such a laser has a pumping current of **20 mA**. Estimate:

- The pumping voltage for this wavelength.
- The number of photons produced per second.

The details of how the diode works are well beyond the scope of this book but a brief description is given at the end of Section 2.7. The power supply inserts new electrons into the n-type and creates new holes in the p-type regions which migrate to the junction, producing a population inversion, where the electrons ‘fall into’ the holes with the emission of photons. As with optically pumped lasers, one photon being released leads to a cascade of photons. The density of the electrons and holes is much higher than population inversions in conventional lasers so very high amplifications can be achieved in a short distance (compare with Self-test 2.8.4). The ‘polished surfaces’ are not required even to approach 100% reflecting, given the levels of amplification achieved.

The advantage of semiconductor laser diodes for optical fibres relates to the spectral purity of the output (practically only one frequency) and the coherence of the output, which allows for very rapid switching. Switching frequencies of 10s of **GHz** are routinely achieved delivering very large data transmission rates.

2.8.5 Nebulium and mysterium

Helium was first identified by its emission lines in the spectrum of the Sun's corona. In 1864 the astronomer Williams Huggins studied the spectrum of the emissions from the Cat's Eye nebula. He found a group of green-blue spectral lines which had never been found in any flame test in a chemistry laboratory. He concluded that this was a previously unknown element, which he dubbed ‘nebulium.’ Investigations in the 20th century showed that, in fact, the lines came from O^{2+} ions (the green lines in Fig. 2.8.11) which are normally not observed because the upper levels of the transitions are metastable and normally the ions lose energy by collisions before they have had a chance to emit photons. However, in the near high vacuum conditions – harder than the best vacuum produced on Earth – the collision frequency is so low that the atoms have time to emit photons with these wavelengths.

Fig. 2.8.11 ‘Nebulium’ emissions

Fig 2.8.10 Cat's eye nebula

2.8.6

Self-test

Calculate the energy difference between the $^1S_0$ and the $^3P_0$ energy levels in nebulium.

Similarly, the molecule ‘mysterium’, was discovered by radio astronomers Howard Weaver and colleagues in 1963. They were looking at emissions from hydroxyl (**OH**) molecules in the Orion nebula, found some lines which were very much stronger than expected (they didn't fit the known pattern of **OH** line strengths). They assumed that they were from an unknown molecule which they called mysterium.

The truth was that they had discovered the first natural maser – like a laser but one which emits microwaves. The emissions were from **OH** which was being pumped into metastable states by infrared radiation from nearby stars. Spontaneous emissions result in stimulated emissions from other **OH** molecules. This could not happen on Earth because, as we've seen from Self-test 2.8.4, a photon has to travel quite a long way before it stimulates an emission; in the high vacuum conditions of space this distance is even longer. But, to quote Douglas Adams, ‘Space is big’. A hydroxide cloud is billions of km across, so there can be enough distance for lasing to occur.

Since then many space molecules have been found to exhibit maser activity, e.g. water, ammonia and hydrogen cyanide, and in 1995 the first atomic hydrogen laser was observed in the swirling disc surrounding the bright blue star MCW 349. The laser emissions are in the infrared at wavelengths corresponding to transitions between highly excited states of atomic hydrogen (see Stretch & Challenge).

Fig. 2.8.12 Orion nebula

The same disc also shows maser emissions as shown in Fig. 2.8.13. The laser emissions are optically pumped from the UV radiation from the star, with the masers being excited by IR radiation which penetrates the disc to greater distances.

These emissions were observed because the radiation is both very intense and highly variable.

The radiation is much more intense than would be expected from a pure thermal emission. The intensity of the masers corresponds to black-body temperatures of up to 10^{15} K, at which temperature the molecules would not exist. The variability is due to the fact that the laser/maser gain is so strongly dependent (exponentially) on the length of the path in the gas cloud with appropriate conditions – so any small variation in this length produces huge changes in output. Both these properties allow astronomers to make detailed measurements of the conditions in the gas clouds.

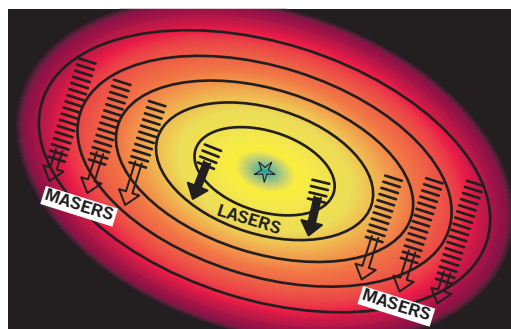


Fig. 2.8.13 Stellar-pumped lasers & masers

Stretch & Challenge

Transitions from level $n + 1$ to the energy level n in hydrogen give rise to waves which are labelled **Hn alpha**. The hydrogen lasers around MCW 349 were at 169 mm and 89 mm. Use the equation

$$(E_n / \text{eV}) = -13.6 \frac{Z^2}{n^2},$$

from Section 2.7.3(b) to show that these are consistent with H15 alpha and H12 alpha.

H10 alpha lasers have also been detected. Calculate the wavelength of the radiation.

Exercise 2.8

The questions in this section are all based around the energy diagram of a helium-neon laser and a short passage describing the laser.

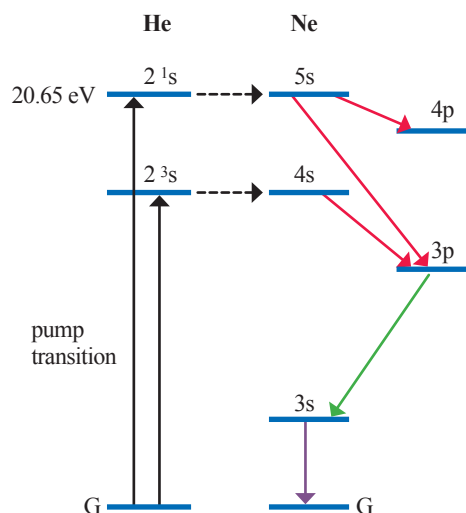
The helium-neon laser

Real lasers have more complicated energy levels than in the textbook notes. The He-Ne laser has a mixture of the two gases helium and neon as the amplifying medium. The two gases at low pressure are sealed into a glass tube and an electric current passed through them. Electrons colliding with the helium atoms excite them to the 2^1s and 2^3s energy states (don't worry about the names of these energy states). These two energy levels happen to be almost the same as the $4s$ and $5s$ energy levels in neon, so energy can easily be transferred from helium to neon atoms by inelastic interatomic collisions.

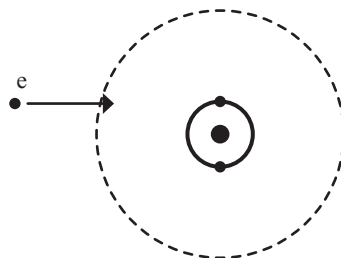
The $4s$ and $5s$ states of neon are *metastable* and this results in laser emissions down to the $4p$ and $3p$ states. The wavelengths of the laser emissions are 633 nm, 1.15 μm and 3.39 μm (shown as red arrows).

The $3p$ and $4p$ states are very short lived. An atom in $3p$ decays very rapidly, by spontaneous emission of wavelength 600 nm (green arrow), to the $3s$ state. The $3s$ atom de-energises by collision (usually with atoms in the container walls) to the ground state. The $4p$ state also decays rapidly by spontaneous emission (not shown) to $3s$.

A data book gives the energy of the helium 2^1s energy state as 20.65 eV above the ground state.



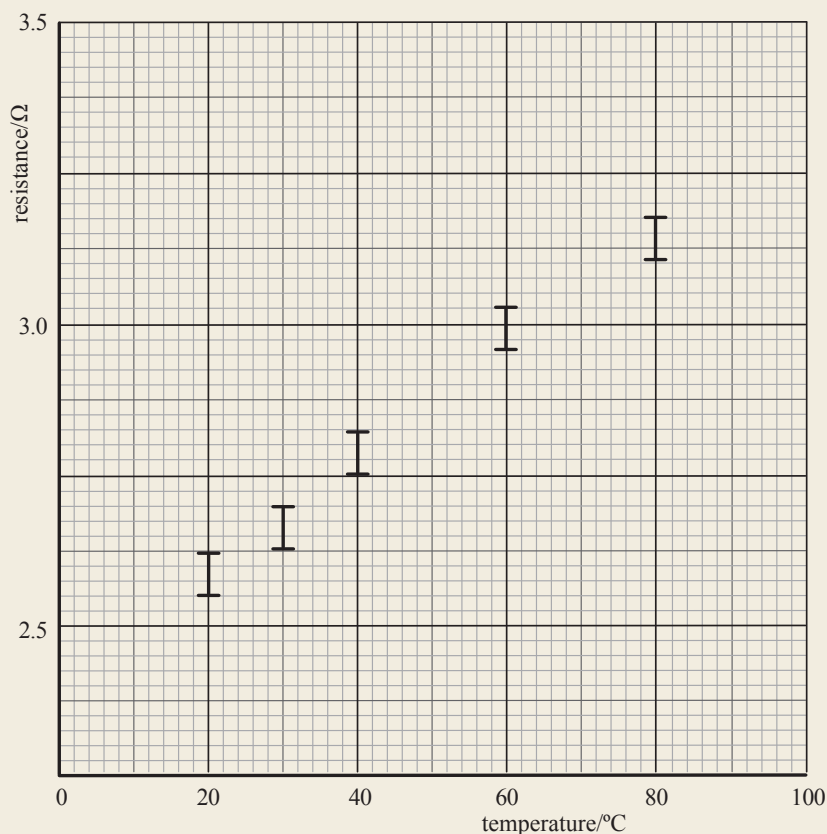
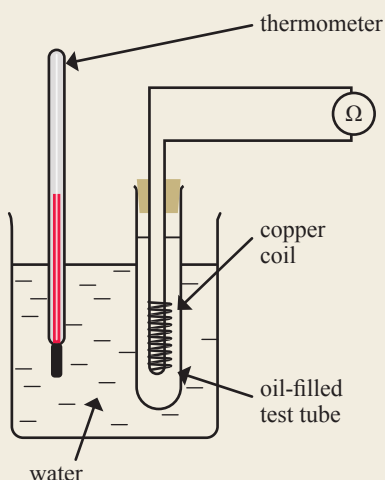
1. Explain what is meant by: (a) an inelastic collision and (b) a metastable state.
2. A neon atom in the 5s state decays to the 3p state by emitting a photon. This photon caused stimulated emission in another neon atom.
 - (a) Draw a diagram to illustrate the stimulated emission event.
 - (b) How do the properties of the second photon compare with the first?
3. Why is important that the 3p state in neon is much shorter lived than the 5s for establishing a population inversion?
4. Identify which of the laser transitions produce the **633 nm**, **1.15 μm** and **3.39 μm** emissions. You may assume that the separations energy states above the 3s state are drawn roughly to scale.
5. Identify the region of the e-m spectrum in which each of these emissions lies.
6. Use your answers to Q4 and the wavelength of the 3p $\rightarrow$ 3s transition to calculate the energies of the excited neon states above the ground state. Give your answers in both **J** and **eV**.
7. Calculate the wavelength of the spontaneous 4p $\rightarrow$ 3s transition. In which region of the e-m spectrum does this lie?
8. Describe in your own words the process of energy transfer between the He 2 ^1s state and the neon 5s state.
9. The diagram shows an electron, with a kinetic energy of $5.3 \times 10^{-18} \text{ J}$, approaching a helium atom (in a He-Ne laser) in the ground state. It collides with one of the electrons in the helium atom and promotes it to the 2 ^1s energy state (shown as a dotted line).



- (a) Calculate the pd through which the incident electron has been accelerated.
 - (b) Express the energy of the incident electron in **eV**.
 - (c) Sketch a diagram showing the atom and electron, with their energies, after the collision.
 - (d) State the minimum accelerating voltage needed to enable an electron to excite a helium atom to 2 ^1s .
10. A He-Ne laser operates at **1.2 kV**. Calculate the number of helium atoms an electron can excite to the 2 ^1s state.

Exam practice questions

1. (a) Use your knowledge of the process of electrical conduction in metals to explain why they possess resistance and why the resistance increases with temperature. [3]
- (b) A pair of students investigates the variation of resistance, R (in ohm), with temperature, θ , of copper wire using the apparatus shown. They heat the water bath using a bunsen burner, and obtained resistance readings for both increasing and decreasing temperature. The results with error bars are shown in the graph.



- (i) In terms of the apparatus used, discuss why it is better practice to obtain resistance readings when heating and cooling rather than heating more than once. [3]
- (ii) Suggest an improvement to the experimental design which would lessen the need to take readings when cooling as well as heating. Give a reason. [1]
- (iii) Label the axes of the graph. [2]
- (iv) The students expect the variation of resistance with temperature to be of the form:

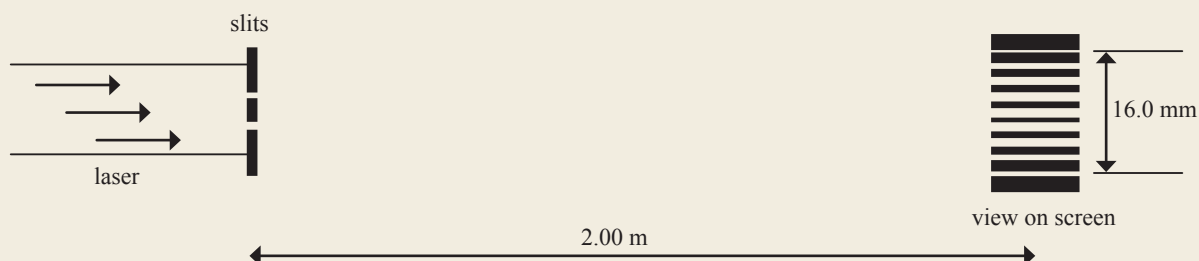
$$R = R_0 (1 + \alpha\theta),$$

where R_0 is the resistance at 0°C and α is a constant.

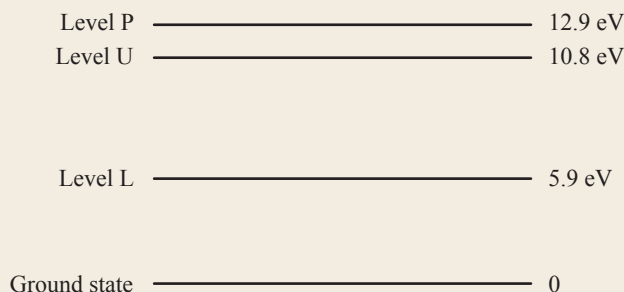
Use the graph to show that the data are consistent with this relationship and determine values for R_0 and α together with their absolute uncertainties. [6]

- (c) Some conductors become superconducting at low temperatures.
- (i) Describe this phenomenon with the aid of a suitable graph. [2]
- (ii) Give an example of the use of superconductors and explain why a superconductor is better than a conventional conductor in this application. [2]

2. (a) Explain what is meant by the principle of superposition. [2]
 (b) A student arranges for a monochromatic red beam of light, of wavelength 650 nm , from a laser to hit a double slit and views the resulting pattern on a screen 2.00 m away.



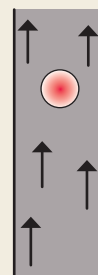
- (i) Calculate the separation of the slits. [2]
 (ii) Explain why, for the pattern to be observed, the slits need to be very narrow and close together and for the screen to be a long way from the slits. [4]
 (iii) Explain how the pattern would be different if green light of wavelength 500 nm were used instead of red. [2]
 (c) The transparent mineral mica of refractive index 1.5 is available in sheets of thickness 0.10 mm .
 (i) A beam of light, with a wavelength in a vacuum of 650 nm , passes at right angles through such a mica sheet. Calculate the number wavelengths of the light in this thickness of mica [2]
 (ii) Explain why the position of the lines would change if a sheet of mica were placed over one of the slits in part (b). [2]
 3. (a) A simplified energy-level diagram is shown for a 4-level laser system:



- (i) On the above diagram, use labelled arrows to show (I) the pumping transition and (II) the laser transition. [1]
 (ii) This laser is *optically pumped*. State the photon energy of the radiation for this purpose and justify your answer. [2]
 (iii) Calculate the wavelength of the laser light produced. [2]
 (iv) In this laser system, electrons only occupy levels P and L for a very short period ($\sim 10^{-9} \text{ s}$) before dropping to the lower level, whilst the natural lifetime of level U is much longer ($\sim 10^{-3} \text{ s}$). Explain in detail how the system is able to amplify light, including the significance of these lifetimes in establishing a population inversion. [6 QER]

- (b) A system is under design in which small reflecting spheres of diameter 0.1 mm and density $1.5 \times 10^{-3} \text{ kg m}^{-3}$, are to be held stationary against the Earth's gravitational field by a laser beam.

- (i) Make a calculated estimate of the intensity of the laser beam required (i.e. the power per unit area). You may treat the spheres as horizontal discs for this calculation.
- (ii) Compare your estimate of the laser intensity with that of the Sun at its surface. [$T_{\text{Sun}} \sim 6000 \text{ K}$].
- (iii) The spheres cannot be made 100% reflecting. Suggest a problem that this is likely to cause.



[3]

[1]

[2]

4. (a) The potential difference between the dome of a van de Graaf generator and the Earth is 120 kV and it stores a charge of $1.2 \text{ } \mu\text{C}$. The dome is discharged safely through a pupil in a time of 0.8 ms . During the discharge the pd falls to zero.

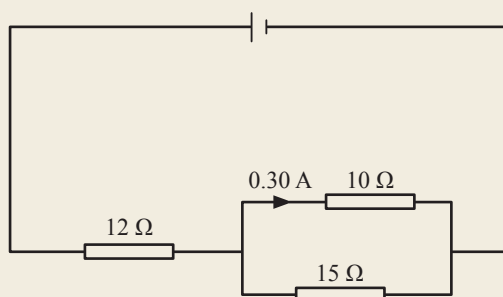
- (i) Define the term potential difference.
- (ii) Calculate the mean current.
- (iii) By considering the mean pd, estimate the total energy transfer during the discharge.

[2]

[1]

[2]

- (b) Current is driven through a resistor network by a battery.



- (i) Show clearly that the pd across the battery is 9.0 V
- (ii) The internal resistance of the battery is $2.0 \text{ } \Omega$. Analyse the energy transfers in the circuit quantitatively.
- (iii) The $15 \text{ } \Omega$ resistor is disconnected. Without any further calculation, explain qualitatively how the pd across each of the $10 \text{ } \Omega$ resistor, the $12 \text{ } \Omega$ resistor and the battery changes.

[3]

[4]

[2]

Overview: Practical skills

Using apparatus, making and recording measurements

p178

- Dealing with zero errors.
- Avoiding parallax errors.
- Using a fiducial mark.
- Resolution.
- Recording and displaying data.
- Number of significant figures in data.

The uncertainty in data

p180

- Estimating uncertainty in a single measurement.
- Best value and uncertainty from several measurements.
- Precision or percentage uncertainty.
- Finding uncertainty in a calculated quantity: multiplying and dividing, adding and subtracting, powers.
- Plotting error bars.
- Best-fit straight lines and error bars.

Using linear graphs to test relationships

p183

- Testing linear relationships using graphs.
- Testing non-linear relationships using linear graphs.

Carrying out and describing experimental work

p184

- Identifying variables; independent variable, dependent variable, control variable.
- Measuring variables.
- Look out for systematic uncertainties.
- Experimental plans and descriptions.

Practical skills



Practical work has several uses for AS/A level physics students:

- It is used by teachers to illustrate theory.
- It enables students to appreciate the experimental foundations of physics.
- It prepares students for the AS/A examination papers.
- It contributes towards the practical endorsement (A level only).

Much detail of individual practical techniques is given alongside the specified practicals in Components 1 and 2.

The aim of this short chapter is to provide a ready reference to the general techniques that you will need to use in your practical work. Whilst dealing with some aspects of measurement taking, it concentrates on the more advanced numerical aspects of physics at AS/A level, such as estimating and combining uncertainties and using graphs to investigate linear and non-linear relationships, including the use of error bars.

Note: Some aspects of graphs are treated in Chapter 4, Mathematical skills.

Content

- 3.1 Introduction
- 3.2 Using apparatus, making and recording measurements
- 3.3 The uncertainty in data
- 3.4 Using linear graphs to test relationships
- 3.5 Carrying out and describing experimental work

3 Practical skills

Exam tip

The specification only covers the titles of experimental investigations. The details of the actual experiments undertaken will depend upon the choices of teachers in your examination centre (i.e. school or college) and the apparatus available.

Exam tip

The ends of metre rules are often damaged. Readings should be taken from a convenient point, e.g. 100 mm, away from the end.

Exam tip

To find the diameter of a wire, it is good practice to measure the diameter at various points along its length and repeat each measurement at right angles in case the wire isn't accurately circular in cross section.

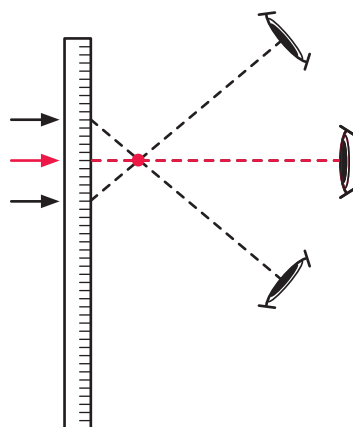


Fig. 3.1 Parallax errors

Exam tip

When measuring the period of oscillations, release the object and allow oscillations to settle before timing from when the object passes the fiducial mark.

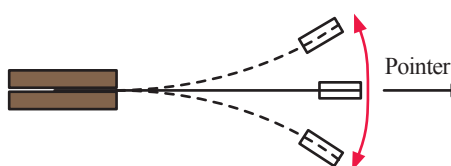


Fig. 3.2 Fiducial mark

3.1 Introduction

As part of the AS course you will be expected to answer questions on a significant number of experimental investigations. These questions will cover experimental design, use of apparatus and techniques, the collection and presentation of data, handling uncertainties in data, drawing conclusions and evaluating outcomes. The titles and details of the investigations are contained in Chapters 1 and 2 in the appropriate unit. This chapter deals with the general skills you need to demonstrate. If you carry on to the full A level, your experimental work will be assessed and reported in the *Practical Completion Certificate*. This will cover the physical demonstration of the same skills. Some higher level mathematical techniques for this are covered in the A level version of this book.

3.2 Using apparatus, making and recording measurements

The list of general apparatus used in AS physics is not long. It includes: distance-measuring equipment such as metre rules, digital callipers, micrometers and (possibly) travelling microscopes; electrical meters, mainly ammeters and voltmeters; digital balances; timers, such as digital stopwatches and light-gate activated timers; thermometers or temperature probes; liquid volume apparatus such as measuring cylinders. At A level, this list is expanded to include signal generators, oscilloscopes, radiation detectors and counters.

3.2.1 Dealing with zero errors

Apparatus should be checked to ensure that it reads zero when it should do. Some equipment can be zeroed, e.g. analogue electric meters. If a piece of equipment cannot be zeroed the zero reading should be subtracted from each reading taken, e.g. a digital calliper reads 0.02 mm with the jaws closed and 0.34 mm when reading the diameter of a wire: the wire diameter should be reported as $0.34 - 0.02 = 0.32$ mm.

3.2.2 Avoiding parallax errors

These affect readings from analogue instruments such as metre rules or electrical meters with pointers and scales. In Fig. 3.1 it is clear that only the middle reading (in red) is correct. The position to be measured should be as close to the scale as possible and the eye should be at right angles to the scale. In 'bouncing ball' experiments the eye should be at the anticipated height of bounce, the approximate value of which can be determined by trial runs.

3.2.3 Using a fiducial mark

In many experiments, it is useful to have a fixed mark from which to take measurements. This is particularly useful in oscillation experiments. The oscillations should be timed when the object crosses the central point. This is where it is moving most rapidly. Also, if the oscillations get smaller because of damping, after a few oscillations the object might not reach a pointer placed at the extreme positions.

3.2.4 Resolution

Always record the resolution of any instrument you use.

For a digital instrument, this is 1 in the least significant figure in the display. The resolution of the ammeter in Fig. 3.3 is 0.1 mA

For an analogue instrument, e.g. the voltmeter in Fig. 3.3, the resolution should be taken as the interval between the smallest graduations. In this case 0.1 V.

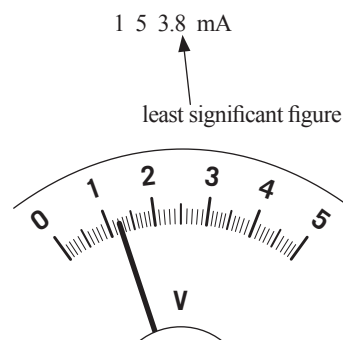


Fig 3.3 Resolution

Terms & definitions

The **resolution** of an instrument is the smallest measurable change that can be recorded using it.

Study point

Resolution is not the same as uncertainty. A stopwatch resolution is often 0.01 s but the best uncertainty that people can achieve in hand timings is ± 0.05 s.

3.2.5 Recording and displaying data

The standard way of displaying systematic data, i.e. results of varying the independent variable, is to use a table. The following table shows the points you should consider when constructing a table:

Headings with units		Over-arching heading			
Systematic sequence	Load / kg	Time for 10 oscillations / s			
		Reading 1	Reading 2	Reading 3	Mean
	0.10	23.65	23.39	23.48	23.5
	0.20	33.39	33.45	33.48	33.4
	0.30				

Readings to the resolution of the instrument

Calculated data to consistent number of s.f. (see later)

Table 3.1 Tabulation

3.2.6 Number of significant figures in data

A stated value of a quantity without an indication of uncertainty is of limited use because we cannot be sure how precise it is. For example, what does a current of stated value 53 mA mean? With no other information, we take 53 mA to mean 'somewhere between 52.5... mA and 53.4... mA.' This means that the range of uncertainty is ± 0.5 mA, which is roughly 1 part in 100 (or 1%).

If we calculate an electrical resistance, from a single pair of values of current and pd, we need to decide how precise the answer is, i.e. how many figures to give in the answer. Suppose the pd is 25.63 V and the current is 53 mA. The value of resistance, R is calculated as follows:

$$R = \frac{V}{I} = \frac{25.52 \text{ V}}{0.053 \text{ A}} = 481.509... \Omega$$

But the current is only known to 1 part in 100. If the actual value of the current were 0.05270 A the calculated resistance would be 484.250... Ω . These two answers are different if we use more than two significant figures, which is the precision of the current, the less precise datum.

If we can estimate the uncertainty in the data we can give a better answer to this problem. This is dealt with in the next section.

Exam tip

Display readings to the resolution of the instrument. In Table 3.1 the time readings are to 0.01 s. You should be consistent even if the reading is (say) 33.00 s.

Study point

Should we concern ourselves with the number of significant figures or the number of decimal places in an answer? Note that 53.2 cm and 0.532 m are the same. They claim the same precision; they have the same number of s.f. but a different number of d.p.

Self-test

3.1

Calculate the value of resistance in Section 3.2.6 if the current is 53.4 mA.

Rule

When multiplying and dividing, express the result to the number of s.f. as the least precise of the data values.

Study point

In professional scientific work, the $\pm$ uncertainty would come with a stated probability of the value lying within the range. The calculation of such an uncertainty requires detailed statistical work, which is omitted in A level physics.

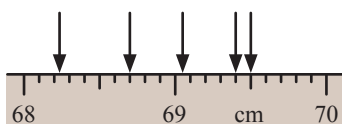


Fig. 3.4 Multiple readings

Terms & definitions

An **outlier** is a result which is very different from the others and is ignored in calculating the best value.

Terms & definitions

The **arithmetic mean** is the sum of the values divided by the number of values. It can be written as $\langle x \rangle$ or $\bar{x}$.

3.2

Self-test

A current, I , is measured 4 times with the following results in mA: 36.7, 37.2, 36.6, 37.0. Give the reported value of I together with its uncertainty.

Terms & definitions

The **precision**, p , is defined as
$$\frac{\text{absolute uncertainty}}{\text{best value}} (\times 100\%).$$
 It is also referred to as the **fractional uncertainty** or **percentage uncertainty**.

3.3

Self-test

Calculate the precision of the wavelength of the microwaves in Section 3.3.1(c).

3.3 The uncertainty in data

No experimental value is known with unlimited precision. A resistor with a marked value of $22 \text{ k}\Omega$ cannot be taken to have a resistance of $22.00000\dots \text{k}\Omega$. Experimental results are quoted together with what is called the **absolute uncertainty**. The resistance could be quoted as $21.6 \pm 0.5 \text{ k}\Omega$, suggesting that the best estimate is that the resistance (in $\text{k}\Omega$) lies between 21.1 and 22.1.

This section introduces the methods employed in AS/A Physics in estimating uncertainties.

3.3.1 Estimating the best value and absolute uncertainty

(a) Uncertainty from a single measurement

The instrument resolution should be used as the uncertainty estimate. For example, when using a metre rule, the uncertainty should normally be given as $\pm 0.001 \text{ m}$ ($\pm 1 \text{ mm}$). It may be possible to estimate readings to better than this, e.g. to half the resolution, but it should be remembered that all length readings are the difference between the readings at each end and hence have two uncertainties associated with them.

(b) Best value and uncertainty from several measurements

In Fig. 3.4, the results of five length measurements, $x_1 \dots x_5$, are given, each to the nearest 0.5 mm . If we have no reason to discard any individual reading, e.g. because we consider it to be an **outlier**, the best value is the **arithmetic mean**, $\langle x \rangle$, of the readings.

$$\langle x \rangle = \frac{x_1 + x_2 + \dots + x_5}{5} = \frac{68.25 + 68.70 + 69.05 + 69.40 + 69.50}{5} = 68.98 \text{ cm}$$

Δx , the absolute uncertainty in x is estimated by dividing the spread of values by 2, in this case:

$$\Delta x = \frac{x_{\max} - x_{\min}}{2} = \frac{69.50 - 68.25}{2} = 0.625 \text{ cm}$$

Because Δx is only an estimate of the uncertainty, we report it to 1 s.f. only. In this case, we report Δx as 0.6 cm and we quote the value of x as $x = \langle x \rangle \pm \Delta x$, which in this case is $69.0 \pm 0.6 \text{ cm}$.

Note that we report the best value of x to the same decimal place as the uncertainty. In this case, the uncertainty is in the 1st decimal place, so we report x to 1 d.p.

(c) Precision

The absolute uncertainty in a quantity does not, by itself, indicate the level of precision with which a value has been determined. For example, an uncertainty in wavelength, $\Delta \lambda$ of 10 nm would be very precise for a microwave ($\lambda \sim 1 \text{ cm}$) but unimpressive for UVB ($\lambda \sim 100 \text{ nm}$) and very imprecise for X-rays ($\lambda \sim 0.1\text{--}10 \text{ nm}$). The **precision**, p , is defined as follows

$$p = \frac{\Delta x}{x} (\times 100\%)$$

For the UVB radiation mentioned above, $p = \frac{\Delta \lambda}{\lambda} = \frac{10}{100} = 0.1 = 10\%$.

It is a matter of taste whether the fraction (0.1) or the percentage (10%) is quoted. The precision is useful when combining uncertainties, which is covered in the next section.

3.3.2 Finding the uncertainty in a calculated quantity

(a) Multiplying and dividing

Many quantities in physics are found by multiplying and dividing others, e.g.

$$\text{speed} = \frac{\text{distance}}{\text{time}}; \quad \text{power, } P = IV; \quad \text{resistance, } R = \frac{\rho l}{A}$$

The uncertainty in the quantity to be calculated (speed, power or resistance) is found by combining the uncertainties in the known quantities. For multiplications and divisions the precisions (fractional or percentage uncertainties) add to give the precision in the answer. So, in the resistance equation:

$$p_R = p_\rho + p_l + p_A$$

To calculate the absolute uncertainty in the resistance, we then use:

$$\Delta R = p_R R.$$

Example

The pd, V , across a component is 5.35 ± 0.02 V; the current, I , is 25.3 ± 0.8 mA. Calculate the resistance of the component together with its absolute uncertainty and report the value correctly

- 1st step: Calculate the best value of R : $R = \frac{V}{I} = \frac{5.35 \text{ V}}{0.0253 \text{ A}} = 211.46 \Omega$
- 2nd step: precision in the voltage, $p_V = \frac{0.02}{5.35} = 0.0037$ [= 0.37%]
precision in the current, $p_I = \frac{0.8}{25.3} = 0.0316$ [= 3.16%]
- 3rd step: Add the precisions: $p_R = p_V + p_I = 0.00373 + 0.0316 = 0.0353$
- 4th step: Calculate ΔR : $\Delta R = R p_R = 211.46 \times 0.0353 = 7.46 \Omega = 7 \Omega$ (1 s.f.)
- 5th step: $\therefore$ Reported $R = 211 \pm 7 \Omega$

Note: In the example, p_V is much less than p_I so that it could really be ignored. In fact ignoring p_V still gives us $\Delta R = 7 \Omega$ (1 s.f.)!

(b) Adding and subtracting

When combining by addition or subtraction, the absolute uncertainties add. For example, if a car accelerates from $12.0 \pm 0.2 \text{ m s}^{-1}$ to $20.5 \pm 0.2 \text{ m s}^{-1}$, the change in velocity is $(20.5 - 12.0) \pm (0.2 + 0.2) = 8.5 \pm 0.4 \text{ m s}^{-1}$. Notice that subtracting quantities tends to produce a result with a much larger percentage uncertainty. In this case the precisions in the velocities are 1.7% and 1.0% but the precision in the change in velocity is 5% (1 s.f.)!

(c) Powers

We often need to square or find the square root of quantities in calculations. If we remember that $A^2 = A \times A$, then we can apply the rule from part (a). So:

$$p(A^2) = 2p_A.$$

We can generalise from this: $p(A^n) = np_A$ and $p(\sqrt[n]{A}) = \frac{1}{n}p_A$. As an example, the area of a circle is given by πr^2 ; π has no uncertainty; so, if we know the radius to a precision of 1%, the precision in the area is $2 \times 1\% = 2\%$.

Study point

If we are calculating ρ from $R = \frac{\rho l}{A}$, we manipulate the equation to give:

$$\rho = \frac{RA}{l}. \text{ Then } p_\rho = p_R + p_A + p_l.$$

Note that this is not a rearrangement of the equation in Section 3.3.2(a).

Uncertainties always add!

Exam tip

It is easier to work with fractional rather than percentage uncertainties when combining them. In the **example** the percentage uncertainties are 0.37 and 3.16 giving a total of 3.53. To convert this to a value of ΔR requires a division by 100, which is not needed if fractional uncertainties are used.

Exam tip

When doing uncertainty calculations, keep 2 or 3 s.f. during your working and only reduce to 1 s.f. in the answer.

Self-test

3.4

Calculate the acceleration in Section 3.3.2(b) if the time taken was 4.0 ± 0.1 s.

Self-test

3.5

The diameter of a sphere is measured as 2.00 ± 0.01 mm. Calculate its volume, $V \pm \Delta V$. Use the formula $V = \frac{4}{3}\pi r^3$ and remember that $\frac{4}{3}\pi$ has zero uncertainty.

3.3.3 Uncertainties and graphs

(a) Error bars

Consider an experiment into the variation of acceleration of a model rocket with height above the ground. At a height of 1.9 m, the acceleration is measured to be $32 \pm 4 \text{ m s}^{-2}$. We plot this information as in the red barred vertical line in Fig. 3.5(a). The line, called an error bar extends to 1.9 m and extends from 28 to 36 m. The best value of 32 m s^{-2} has no special significance apart from being the centre of the error bar. The horizontal bars are not significant and serve only to draw attention to the extent of the error bar. When we come to draw the best-fit graph, we expect it to pass through the bar.

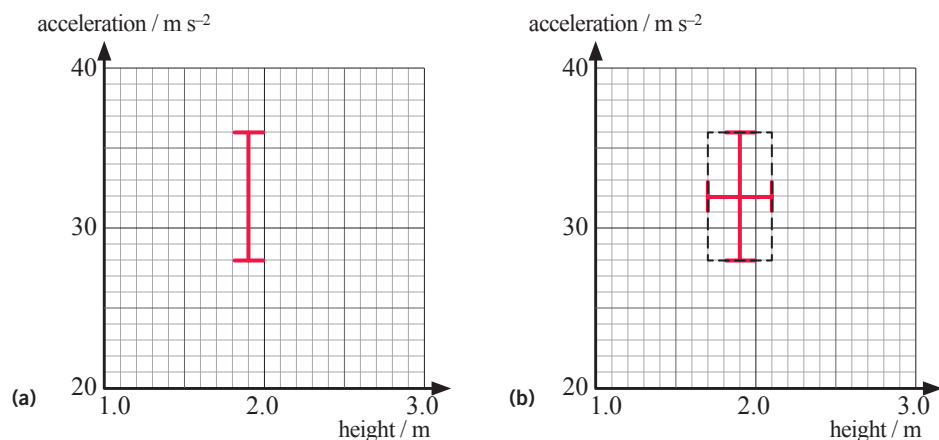


Fig. 3.5 Error bars

If height of the rocket was only known to within 0.2 m we would also plot a horizontal error bar to represent this, as in Fig. 3.5(b). If there were no vertical error bars, the best-fit line would pass through the horizontal error bar. If there are uncertainties in both the x and y directions the best fit graph would pass through the dotted box, which encloses both error bars. In fact, a sensible way of plotting the uncertainties in this case is just to draw the 'error box': this is not conventional but it is perfectly acceptable.

(b) Best-fit straight lines and error bars

We use the plotted error bars to:

- decide whether the results are consistent with a linear relationship
- determine the relationship (with an estimate of the uncertainties) between the variables.

Consider the set of results plotted in Fig. 3.6. It is possible to draw a straight line through all the error bars, so the results are consistent with a linear relationship between y and x . The lines drawn are the extremes – they represent the steepest and least steep lines it is possible to draw through all the error bars.

We can use these extreme lines to find the best values of the gradient and intercept together with their uncertainties. If the extreme gradients and intercepts are m_1 , m_2 , c_1 and c_2 (see Maths check), then:

$$m = \frac{m_1 + m_2}{2} \pm \frac{m_2 - m_1}{2} \text{ and } c = \frac{c_1 + c_2}{2} \pm \frac{c_2 - c_1}{2}$$

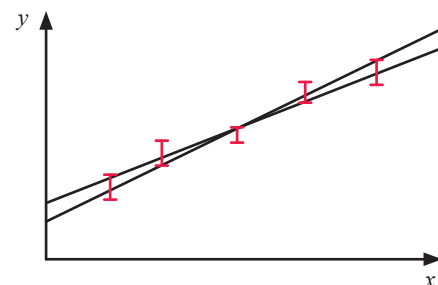


Fig. 3.6 Graphs and error bars

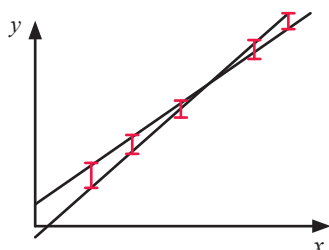


Fig. 3.7 Proportionality?

MATHS CHECK

The equation of a straight line graph between y and x is $y = mx + c$, where m is the gradient and c the intercept on the y axis. See Section 4.5.4.

3.6

Self-test

In a graph of V/V against I/A , the extreme graphs have the following gradients and intercepts:

Gradient: -0.165 , -0.169

Intercept: 9.05 , 9.17

Write the equation between V and I as in Section 3.3.3(b).

i.e. we take the arithmetic means of the extremes of gradient and intercept to be the best values, with the absolute uncertainties being half the ranges. So the equation between y and x is written:

$$y = \left(\frac{m_1 + m_2}{2} \pm \frac{m_2 - m_1}{2} \right) x + \left(\frac{c_2 + c_1}{2} \pm \frac{c_2 - c_1}{2} \right)$$

In Fig. 3.7, the relationship $y \propto x$ is consistent with the error bars because it is possible to draw a straight line through the origin and the error bars. This should not be assumed, however; the extreme graphs should be drawn as above and the best values of m and c together with their uncertainties reported.

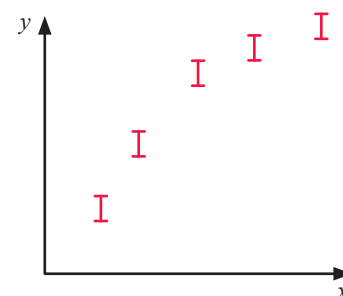


Fig. 3.8 Data inconsistent with a linear relationship

3.4 Using linear graphs to test relationships

This involves comparing the expected relationship between the variables with the straight-line equation, $y = mx + c$, where m and c are constants, the gradient and y -intercept respectively.

3.4.1 For a linear relationship

We shall examine this using a typical AS example, the relationship between terminal pd, V , and current, I , for a power supply: $V = E - Ir$. It is advisable to rearrange the equation so that it is easy to compare with $y = mx + c$.

Rearranging the equation:

$$\begin{array}{ccccccc} V & = & -r & I & + & E \\ \updownarrow & & \updownarrow & \updownarrow & & \updownarrow \\ y & = & m & x & + & c \end{array}$$

The variables in the two equations and the correspondence between them are indicated in red. If the internal resistance, r , and emf, E , of the supply are constant, a graph of V against I , is a straight line with gradient $-r$ and intercept E on the V -axis. Thus the relationship is tested and (assuming the relationship holds) r and E determined.

3.4.2 For non-linear relationships

Many non-linear relationships can be plotted to give a linear graph by a careful choice of variables. Some are rather straightforward; e.g. $v^2 = u^2 + 2ax$, for a constant acceleration. If we plot v^2 against x (rather than v against x), the graph should be a straight line of gradient $2a$ and intercept u^2 on the v^2 axis.

Other relationships are more difficult to linearise. For example $V = \frac{ER}{R+r}$, where E and r are constants as before, is not easy but it can be rearranged to give $\frac{1}{V} = \frac{r}{ER} + \frac{1}{E}$.

This suggests that a graph of $\frac{1}{V}$ against $\frac{1}{r}$ should be a straight line of gradient $\frac{r}{E}$ and intercept $\frac{1}{E}$ on the $\frac{1}{V}$ -axis.

Self-test

3.7

What are the gradient and intercept of a graph of v against t for constant acceleration?

Self-test

3.8

State the values of r and E from Self-test 3.6.

MATHS CHECK

The label of the v^2 axis should be $(\text{m s}^{-1})^2$.

Self-test

3.9

Show that $V = \frac{ER}{R+r}$ can be rearranged to give $\frac{1}{V} = \frac{r}{ER} + \frac{1}{E}$.

3.10

Self-test

The resonance frequency, f , for a pipe which is open at one end, relates to its length, l , by the equation

$$l + \epsilon = \frac{c}{4f}, \text{ where } c \text{ is the speed}$$

of sound waves and ϵ a constant (called the 'end correction'), relating to the diameter of the pipe. What graph, relating f and l , should be plotted to verify the relationship and how should c and ϵ be determined?

Terms & definitions

The distinction between independent and dependent variables is not always useful. In the equation $v^2 = u^2 + 2ax$, we could measure the velocity at a set of displacements (making v the dependent variable) or the displacements at a set of velocities.

Study point

Some variables are almost self-controlling; e.g. when using a wire from a single reel to investigate the effect of length of a wire on resistance, the diameter and composition of the wire are automatically kept the same.

Study point

Consider a wire of approximate diameter 1 mm and length 5 m. If the diameter, D , is measured to ± 0.01 mm that is a precision of 1%. The volume depends upon D^2 , which has a precision of 2%. A 1 cm uncertainty in the length means a precision of 0.2% which is negligible.

3.5 Carrying out and describing experimental work

Experimental work in physics has two main purposes:

1. To test relationships between variables
2. To determine the value of a physical quantity.

In practice these two purposes often overlap. For example, if the purpose is to determine the presumed-constant acceleration of an object, it is sensible to demonstrate that it really is constant, e.g. by plotting a graph of v^2 against x (as in Section 1.3.9) or v against t , showing that the results are consistent with a constant acceleration and using the gradient to determine its value. This procedure has the additional advantage that it allows for a more accurate estimation of the uncertainty in the result.

Physics teachers have additional reasons for carrying out practical work: demonstrating a phenomenon or setting up an event which can be discussed in terms of physical laws. The purpose of a particular piece of experimental work should always be clearly stated.

3.5.1 Identifying variables

All the quantities that need to be measured in experimental work are referred to as variables. This is the case whether or not their values actually change during an investigation – they have the potential to vary. An investigation often involves looking at the effect of varying one or more variables, the **independent variables**, on another variable, the **dependent variable**. Some variables, the **control variables**, need to be kept the same so that the effect of varying the independent variables is isolated. When planning an investigation, all the key variables need to be identified.

Often, when plotting graphs, the dependent variable is plotted on the vertical axis (the ordinate or y -axis) with the independent variable on the horizontal axis (the abscissa or x -axis). This is not necessarily convenient, for example when investigating the resistance of a wire, it doesn't matter whether the current or pd is on the y -axis: if the current is on the x -axis, the gradient is the resistance; current on the y -axis makes the resistance equal to the reciprocal of the gradient.

Unlike living systems, the objects under investigation usually behave in a relatively simple way and we can describe the effect of one variable on another in terms of algebraic or trigonometrical functions

3.5.2 Measuring variables

The measuring technique and the instruments should be chosen which, (a) measure the relevant variables, (b) have appropriate precision and accuracy and (c) are as convenient as possible and (d) give reproducible results. It is not always the case that particular measurements need be very precise – when measuring the volume of a long (~5 m) piece of thin (~1 mm) wire, there is no point in trying to make the length measurement to better than 1 cm uncertainty. Why not? See the Study point. On the other hand, there is every justification for improving the precision of the diameter determination.

3.5.3 Look out for systematic uncertainties

Uncertainties in measurement which have the effect of always making the result too large or always too small are called systematic uncertainties. They are sometimes easy to spot: the relevant distance between the shielded radioactive source and the G-M tube 'hot spot' is unknown but will always differ from the distance x by the same amount, ϵ .

Carefully designed graphs are useful in trapping this kind of uncertainty.

If the inverse square law is being investigated, the count rate, C , is expected to be given

by: $C = \frac{k}{d^2}$, where $d = x + \epsilon$.

The equation can be re-arranged to give $\frac{1}{\sqrt{C}} = \frac{x + \epsilon}{k}$, so a graph of $\frac{1}{\sqrt{C}}$ against x

should be a straight line, if the inverse square law is obeyed. (See Self-test 3.11.) When selecting the graph to plot, the variable with the systematic uncertainty should be linear (i.e. in this case we plot x not x^2). Self-test 3.12 gives another example.

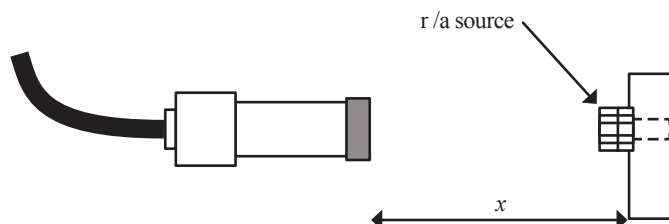


Fig. 3.9
Systematic
uncertainty

3.5.4 Experimental plans and descriptions

The report should be succinct but sufficiently detailed to allow another A level physics student to follow it. Whether you use a numbered list of steps or bullet points is a matter of taste but the report should be clearly sequential. Details of standard items of equipment and their operation need not be given, other than to explain the precautions to reduce random and systematic uncertainties (e.g. measuring the diameter of a wire at several points and in directions at 90°).

The method of analysis should be clear. For example, a statement of the graph to be drawn (such as velocity² against distance), how this relates to any algebraic equation and how features of the graph (linearity, gradient, intercept) will be used.

Example

When determining the Young modulus, Y , of the material of a wire, the diameter, D , and initial length, l_0 , are measured and a graph of force, F against extension, Δl , plotted.

$$F = \frac{\pi D^2 Y}{4l_0} \Delta l, \text{ so the gradient, } m, \text{ of the graph is } \frac{\pi D^2 Y}{4l_0}.$$

The gradient of the graph is measured and Y calculated from $Y = \frac{4l_0 m}{\pi D^2}$.

Self-test

3.11

From the graph of $\frac{1}{\sqrt{C}}$ against x in Section 3.5.1:

- State the values of the gradient and intercept.
- Explain how ϵ can be determined.

Self-test

3.12

The electromagnet in the experiment to determine g in Section 1.2.5 doesn't lose its magnetisation instantly, so hangs on to the ball for an unknown time, τ .



The fall height, h , and true time of fall, t , are related by

$$h = \frac{1}{2} g t^2.$$

Explain how to determine g and τ .

Overview: Mathematical skills

Arithmetic and numeric computation

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- Rules of indices.
- Indices and units.
- Using standard form.
- Using SI multipliers.
- Fractions, ratios and percentages.
- Expressing angles in radians; converting between degrees and radians.

Algebra

p190

- Algebraic symbols, $<$, $>$, $<<$, $>>$ and Δ .
- Direct and inverse proportion; graphs.
- Manipulating algebraic equations.
- Evaluating expressions to solve equations.
- Quadratic equations.

Geometry and trigonometry

p192

- Angles in geometrical figures.
- Areas and volumes of geometrical figures.
- Right-angled triangles.
- Trig ratios for angles outside the range $0 - 90^\circ$.

Graphs

p194

- Graph preparation and point plotting.
- Determination of rate of change from a graph.
- Determination of the 'area' under a graph.
- Equation of a linear graph.
- Finding the equation from a graph.

Mathematical skills



In many ways, mathematics is the language of physics. Physicists develop their models and theories in mathematical terms; they make quantitative predictions about the behaviour of the world; they subject their experiments to mathematical analysis; even the extent to which physicists and engineers are uncertain is expressed mathematically.

As with Chapter 3, this short section of the book is not meant as a substitute for a detailed mathematical course. It provides, rather, a ready reference to the mathematical techniques you will employ in AS physics: use of indices and standard form; algebraic manipulation; angles and trig ratios; graphs.

The topic of Graphs is split between this chapter and Chapter 3. Graphs are sometimes plotted directly from equations; at other times experimental results are plotted, to which best-fit or theoretical lines are added. The final section on graphs in this chapter deals mainly (but not exclusively) with aspects of graphs you will meet in non-experimental work.

Content

- 4.1 Introduction
- 4.2 Arithmetic and numerical computation
- 4.3 Algebra
- 4.4 Geometry and trigonometry
- 4.5 Graphs

4 Mathematical skills

Exam tip

This chapter is not a substitute for a maths text book. For practice in maths techniques you should use a maths textbook, e.g. *Mathematics for A level Physics* (Kelly & Wood)

Exam tip

Remember these:

$$a^0 = 1; a^1 = a; a^{-1} = \frac{1}{a}$$

$$a = \sqrt[n]{a} = a^{\frac{1}{n}}$$

4.1

Self-test

What are

(a) $y^2 \times y$? (b) $y^{-2} \times y$? (c) $y(y^2 + 2a)$

Study point

For acceleration of 1.5 m s^{-2} for 10 s:

$$\begin{aligned} \text{Vel inc.} &= 1.5 \text{ m s}^{-2} \times 10 \text{ s} \\ &= 1.5 \times 10 \text{ (m s}^{-2} \times \text{s)} \\ &= 15 \text{ m s}^{-1}. \end{aligned}$$

Examples

$$\begin{aligned} 5 \times 10^{-3} &= 5 \times 0.001 \\ &= 0.005 \end{aligned}$$

$$\text{and } 5.57 \times 10^{-3} = 0.005 57$$

$$\begin{aligned} 5 \times 10^3 &= 5 \times 1000 \\ &= 5000 \end{aligned}$$

$$\text{and } 5.57 \times 10^3 = 5 570$$

CALCULATOR CHECK

To check that you are entering numbers correctly into your calculator, try:

$$(5 \times 10^{-6}) \times (3 \times 10^7).$$

If your answer is 300 you are doing it right.

4.1 Introduction

Chapters 1 and 2 include mathematical treatment of physics ideas. Basic GCSE level mathematics is assumed. This chapter has two purposes:

- It serves as a quick reference of the necessary skills.
- It introduces some new material, e.g. radians and the use of the 'Δ' symbol, which is more conveniently dealt with away from the main text.

4.2 Arithmetic and numerical computation

4.2.1 Indices

(a) Basic rules

- $a^n = a \times a \times a \dots$ (n times) – where n is a positive integer, i.e. 1, 2, 3 ...
e.g. $5^3 = 5 \times 5 \times 5 = 125$.
- $a^{-n} = \frac{1}{a^n}$, e.g. $10^{-2} = \frac{1}{10^2} = \frac{1}{10 \times 10} = \frac{1}{100} = 0.01$
- $\frac{1}{a^n} = \sqrt[n]{a}$, e.g. $16^{\frac{1}{2}} = \sqrt{16} = 4$ and $81^{\frac{1}{4}} = \sqrt[4]{81} = 3$
- $a^x \times a^y = a^{(x+y)}$, e.g. $10^2 \times 10^3 = 10^5$
- $\frac{a^x}{a^y} = a^x \times a^{-y} = a^{(x-y)}$, e.g. $\frac{4^2}{4^{0.5}} = 4^{1.5}$
- $(a^x)^y = a^{xy}$ e.g. $(5^4)^{0.5} = 5^{4 \times 0.5} = 5^2$

(b) Indices and units

At GCSE we wrote the units of velocity and acceleration as **m/s** and **m/s²** respectively. At A level and higher, the units are written **m s⁻¹** and **m s⁻²**.

When multiplying or dividing two quantities, the units are treated in the same way; e.g. a car accelerates at **1.5 m s⁻²** for a time of 10 s. The increase in velocity is **15 m s⁻¹**. See the Study point for the working.

(c) Standard form

This is used for very large and very small numbers. For example, the speed of light is **300 000 000 m s⁻¹**, which can be written **3.00 × 10⁸ m s⁻¹**; a current of **0.000 015 A** can be written **1.5 × 10⁻⁵ A**. To convert to standard form:

A large number, **xyz00 ... 0** [total number of **digits** = n] = $x.yz \times 10^{n-1}$. Compare this with the speed of light.

A small number **0.0 ... 00xyz** [total number of **zeroes** = n] = $x.yz \times 10^{-(n+1)}$. Compare this with the current above.

The examples in the box show conversions **from** standard form.

Calculator help

To enter **2.5 × 10⁶** into your calculator key: **2 . 5 EXP 6**

To enter **2.5 × 10⁻⁶** key: **2 . 5 EXP +/- 6**

See calculator check.

4.2.2 SI multipliers

Table 4.1 SI multipliers

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^{-18}	atto	a	10^3	kilo	k
10^{-15}	femto	f	10^6	mega	M
10^{-12}	pico	p	10^9	giga	G
10^{-9}	nano	n	10^{12}	tera	T
10^{-6}	micro	μ	10^{15}	peta	P
10^{-3}	milli	m	10^{18}	exa	E
10^{-2}	centi	c	10^{21}	zetta	Z

The multiplier is an abbreviated way of writing $\times 10^n$. For example $53 \mu\text{A}$: From the table, μ means $\times 10^{-6}$, so $53 \mu\text{A}$ is equivalent to $53 \times 10^{-6} \text{ A}$, which is $5.3 \times 10^{-5} \text{ A}$ in standard form – but see Exam tip.

Exam tip

Enter $53 \mu\text{A}$ into your calculator by keying:

5 3 EXP +/- 6

You are less likely to make a mistake than if you convert it to 5.3×10^{-5} first.

Self-test

4.2

A resistor has a voltage of 300 mV across it and passes a current of $50 \mu\text{A}$. Calculate its resistance.

4.2.3 Fractions, ratios and percentages

The fraction $\frac{a}{b}$ means a divided by b . The ratio of a to b is also $\frac{a}{b}$. If $a = 4$ and $b = 10$, the ratio of a to b is $\frac{4}{10} = \frac{2}{5} = 0.4 = 40\%$. These are just different ways of writing the same thing.

We can express a fraction as a **percentage** by multiplying by 100; so

$$\frac{7}{20} = \frac{7}{20} \times 100\% = 35\%. \text{ You should know the following percentage equivalents:}$$

$$\frac{1}{10} = 10\%; \frac{1}{20} = 20\% \dots \frac{1}{90} = 90\%; 1 = 100\%; \frac{1}{4} = 25\%; \frac{1}{3} = 33.3\%; \frac{1}{2} = 50\%;$$

$$\frac{3}{4} = 75\%; 2 = 200\%; 2.5 = 250\% \dots$$

4.2.4 Expressing angles in radians

The SI unit of angle is the **radian** (see Terms & definitions). If r is doubled, l will also double so the ratio $\frac{l}{r}$ does not depend upon the value of r .

To convert between degrees and radians: the circumference of a circle of radius r is $2\pi r$.

$$\text{So } 360^\circ = \frac{2\pi r}{r} \text{ rad} = 2\pi \text{ rad.}$$

Fig. 4.2 illustrates the **small angle approximation** which is only valid if angles are expressed in radians.

The red line is the arc of a circle with radius r .

$$\text{By definition } \sin \theta = \frac{f}{r}; \theta \text{ (in rad)} = \frac{g}{r}; \tan \theta = \frac{h}{r}$$

The three lengths, f , g and h , are very close, with $f < g < h$ and, as $\theta \rightarrow 0$ the ratios $\frac{f}{g}$ and $\frac{g}{h}$ and $\rightarrow 1$. So for small angles we can write $\sin \theta \approx \theta \approx \tan \theta$. For many purposes, angles $< 0.01 \text{ rad}$ [$\sim 6^\circ$] can be considered small.

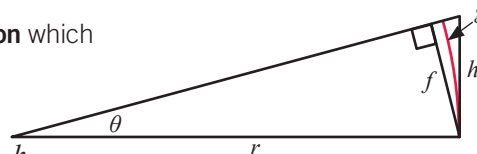


Fig. 4.2 Small angles

Exam tip

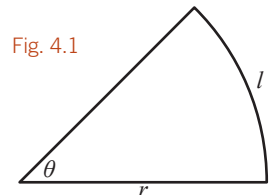
If $a = 3$ and $b = 25$ the ratio of a to b is $\frac{3}{25} = 0.12 = 12\%$. It can be written as 3:25 but this is best avoided.

Terms & definitions

Percent means literally 'in every 100'. So $\frac{1}{2}$ is 50 in every 100, i.e. 50%.

Terms & definitions

Fig. 4.1



$$\theta \text{ (in radians)} = \frac{l}{r}.$$

Exam tip

Remember the following degree / radian equivalents:

$30^\circ = \frac{\pi}{6}$	$135^\circ = \frac{3}{4}\pi$
$45^\circ = \frac{\pi}{4}$	$180^\circ = \pi$
$60^\circ = \frac{\pi}{3}$	$270^\circ = \frac{3}{2}\pi$
$90^\circ = \frac{\pi}{2}$	$360^\circ = 2\pi$
$120^\circ = \frac{2}{3}\pi$	$540^\circ = 3\pi$

4.3 Algebra

4.3.1 Symbols

(a) Less than (<) and greater than (>)

$a < b$ means 'a is less than b'; similarly, $x > y$ means 'x is greater than y'.

Examples: $10 > 5$; $5 \times 10^6 < 2 \times 10^7$. Care should be taken with negative numbers, e.g. $-10 > -20$.

With care, these can be used in the same way as the '=' sign (meaning 'is equal to').

e.g. $p > q$ then: $p + x > q + x$ and $10p > 10q$ but $-5p < -5q$ (!)

(b) Much less than (<<) and much greater than (>>)

These can be used in the same way as < and >. More often they are just used to express that there is a big difference in value without undertaking any algebra.

(c) Direct and inverse proportion

$y \propto x$ ('y is directly proportional to x', or just 'y is proportional to x') means that the ratio $\frac{y}{x}$ is constant, i.e. if x is multiplied by any number (e.g. 2, 3 or π), y is multiplied by the same number.

Sometimes when one variable is doubled, the other one halves. The product, xy , of the variables is constant. In these cases we say that y is inversely proportional to x and write it

$$y \propto \frac{1}{x}.$$

Similarly, y can be proportional to x^2 ($y \propto x^2$), or y can be inversely proportional to x^2 ($y \propto \frac{1}{x^2}$). See the box for examples.

Exam tip

You should know the meaning of the following symbols and how to use them

$=, <, <<, >>, >, \propto, \approx, \Delta$

Study point

If c is the speed of light in a vacuum, v_a the speed of sound in air and v_c the speed of sound in carbon dioxide:

$v_a > v_c$ and $c \gg v_a$

Exam tip

If $y \propto x$, a graph of y against x is a straight line through the origin. If we re-write $y \propto x$ as $y = kx$, then k is the gradient of the graph.

Study point

Proportion relationships can also be written as follows:

$$y \propto x \rightarrow y = kx$$

$$y \propto \frac{1}{x} \rightarrow y = \frac{k}{x}$$

where k is a constant.

Examples of proportion

Hooke's Law $F \propto x$

Waves $\lambda \propto \frac{1}{f}$

Kinetic energy $E_k \propto v^2$

Gravity $F \propto \frac{1}{r^2}$

4.3

Self-test

Identify the constant k in each of the examples above and give its unit.

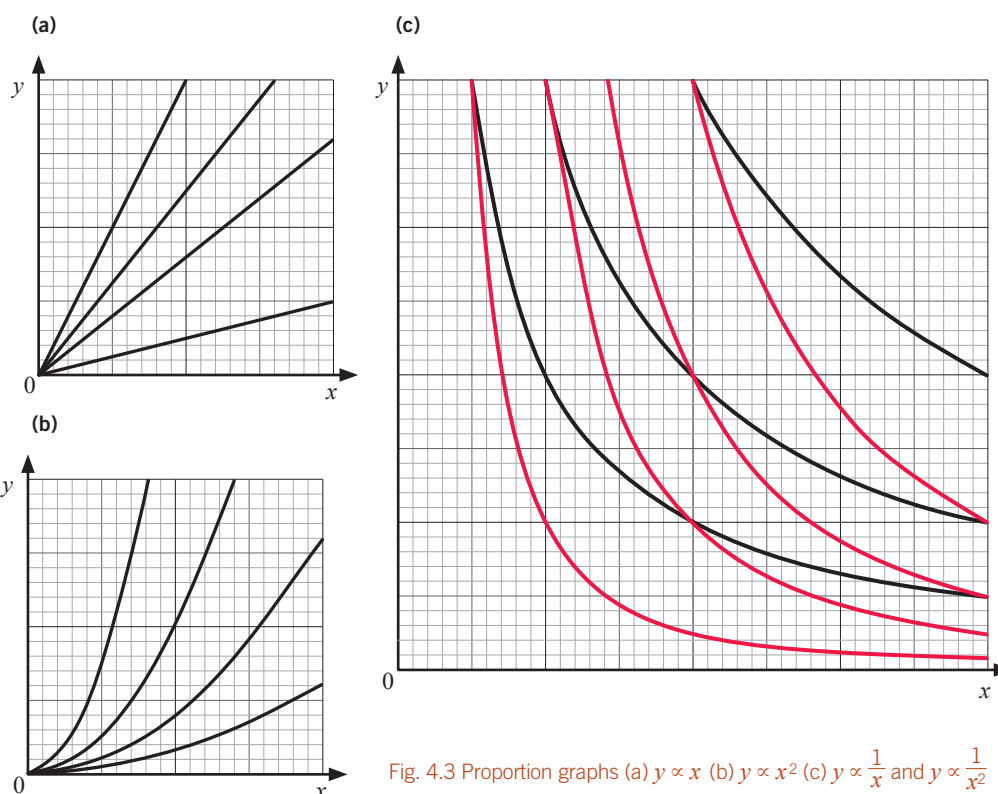


Fig. 4.3 Proportion graphs (a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x}$ and $y \propto \frac{1}{x^2}$

(d) Delta (Δ)

Δ means 'change in', i.e. Δv means 'change in velocity'. If a length, x , changes from $x_1 = 2.5 \text{ cm}$ to $x_2 = 7.6 \text{ cm}$:

$$\Delta x = x_2 - x_1 = 7.6 \text{ cm} - 2.5 \text{ cm} = 5.1 \text{ cm}$$

We always subtract the first value from the second value. So, if $x_2 < x_1$, then Δx will be negative ($\Delta x < 0$).

4.3.2 Manipulating equations

To find the value of a quantity, an equation must be manipulated to make the quantity into the subject, e.g. find a if $10 = 3 + 2a^2$.

In every part of the manipulation, we must perform the same arithmetic operation on the two sides of the equation. Examples:

- Add or subtract the same quantity
- Multiply or divide by the same quantity
- Square both sides or take the square root of both sides.

In our example:

- Subtract 3 from both sides $\rightarrow 7 = 2a^2$
- Divide both sides by 2 $\rightarrow 3.5 = a^2$
- Take the square root of both sides $\rightarrow a = \pm\sqrt{3.5} = \pm 1.87$ (3 s.f.)

4.3.3 Evaluating expressions to solve equations

When evaluating expressions, such as $16 + (6 \times 5^2 - 29)^{0.5}$ the sequence of actions is summarised by the mnemonic BODMAS – bracket, **orders**, division, multiplication, addition, subtraction. With the above expression this works as follows:

- to evaluate the *bracket*, work through the ODMAS sequence within the bracket:
 $5^2 = 25$; $6 \times 25 = 150$; $150 - 29 = 121$
- order: $121^{0.5} = \sqrt{121} = 11$
- addition: $16 + 11 = 27$ (answer)

Another example: $\frac{3}{8} + 4.2$. The $\frac{3}{8}$ term means 3 divided by 8, so start with that:

3 divided by 8 is 0.375. Then add 4.2 to give 4.575.

4.3.4 Quadratic equations

If an unknown quantity, x , satisfies the equation, $ax^2 + bx + c = 0$, where a , b and c are constants, the solutions are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In AS physics, the most common type of question which requires this formula arises in find an unknown time from $x = ut + \frac{1}{2}at^2$. Here, t is the unknown quantity and the constants are x , u and $\frac{1}{2}a$.

Study point

The mean resultant force, F , on an object is defined as the rate of change of momentum, p of the object, i.e. $F = \frac{\Delta p}{\Delta t}$

Exam tip

When solving an equation, e.g. finding x from $v^2 = u^2 + 2ax$, you might find it easier to substitute the numbers into the equation before manipulating the equation.

Self-test**4.4**

Find x if $7x + 16 = 49$.

Terms & definitions

The word 'orders' in BODMAS means powers or indices, e.g. 2^3 , $\sqrt{40} = 40^{0.5}$.

Self-test**4.5**

Evaluate the following:

(a) $8 - (3 + \sqrt{2})^2$, (b) $\frac{24}{8 + 2^2}$, (c) $\frac{1}{3} + \frac{2}{5}$

Self-test**4.6**

If $x = 24 \text{ m}$, $u = 4 \text{ m s}^{-1}$ and $a = 3 \text{ m s}^{-2}$.

(a) From $x = ut + \frac{1}{2}at^2$, show that $1.5t^2 + 4t - 24 = 0$.

(b) Solve the equation using the quadratic formula.

4.4 Geometry and trigonometry

4.7

Self-test

Find the values of all 12 unmarked angles:

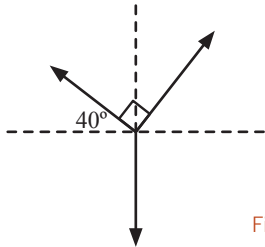
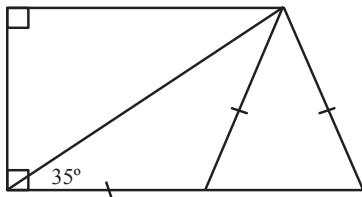


Fig. 4.5



Terms & definitions

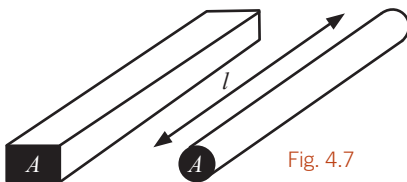


Fig. 4.7

A is called the *cross-sectional area, csa*.

The volume, $V = Al$

For the cylinder, $V = \pi r^2 l$

Exam tip

Converting units of area and volume:

$$1 \text{ m}^2 = (100 \text{ cm})^2 = 10^4 \text{ cm}^2$$

$$1 \text{ m}^2 = (1000 \text{ mm})^2 = 10^6 \text{ mm}^2$$

$$1 \text{ m}^3 = (100 \text{ cm})^3 = 10^6 \text{ cm}^3$$

$$1 \text{ m}^3 = (1000 \text{ mm})^3 = 10^9 \text{ mm}^3$$

4.4.1 Angles in geometrical figures

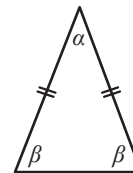
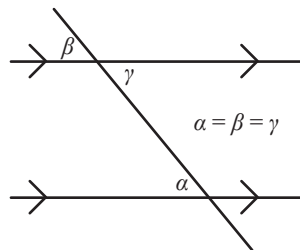
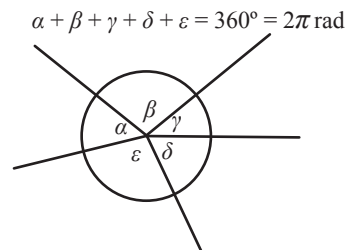
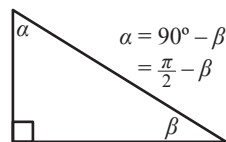
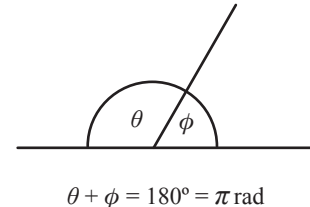
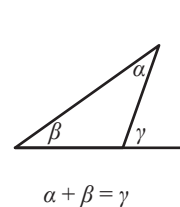
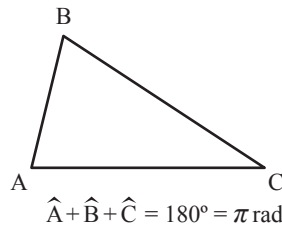
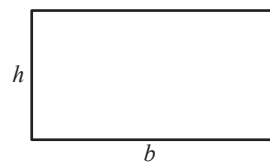
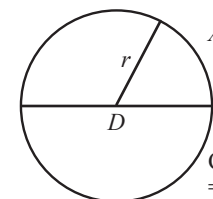
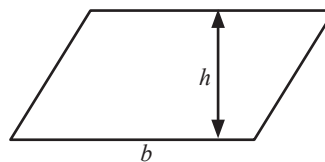


Fig. 4.4 Angles in geometrical figures

4.4.2 Areas and volumes of geometrical figures

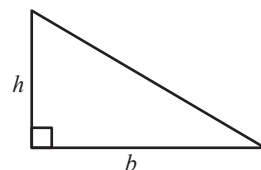


$$A = bh$$

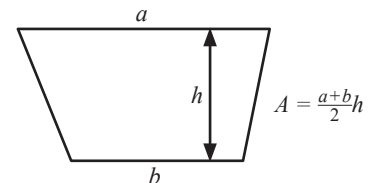
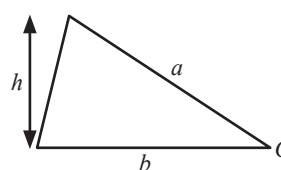


$$A = \pi r^2 = \frac{\pi D^2}{4}$$

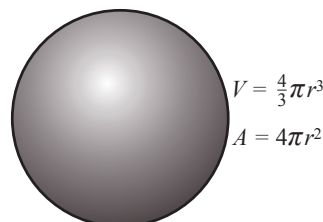
$$\text{Circumference} = \pi D = 2\pi r$$



$$A = \frac{1}{2}bh = \frac{1}{2}ab \sin C$$

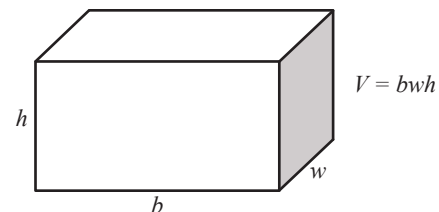


$$A = \frac{a+b}{2}h$$



$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$



$$V = bwh$$

Fig. 4.6 Areas and volumes

4.8

Self-test

What length of wire of $1 \text{ mm} \times 1 \text{ mm}$ cross section could you make from 1 m^3 of metal?

4.4.3 Right-angled triangles

Pythagoras' theorem: $a^2 + b^2 = c^2$,

where c is the long side, the hypotenuse, of the right-angled triangle.

e.g. if $a = 8$ cm and $b = 15$ cm

$$c = \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17 \text{ cm.}$$

A well known *Pythagorean triple* is 3, 4, 5. Others are 5, 12, 13 and 5, 24, 25.

For angles from $0 - 90^\circ$ [$0 - \frac{\pi}{2}$ rad] the trig ratios, sine, cosine and tangent, are defined as follows:

$$\text{Sine: } \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\text{Cosine: } \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\text{Tangent: } \tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

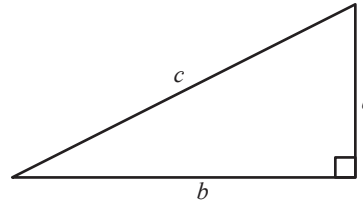


Fig. 4.8 Pythagoras Theorem

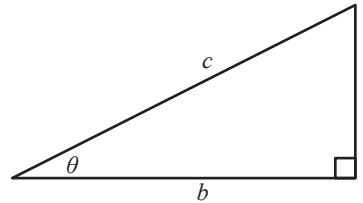


Fig. 4.9 Trig ratios

4.4.4 Trig ratios for angles outside the range $0-90^\circ$

Fig.4.10 shows the sine and cosine functions for angles which cannot be found in a right-angled triangle, i.e. $> 90^\circ$ ($\frac{\pi}{2}$ rad) and negative values.

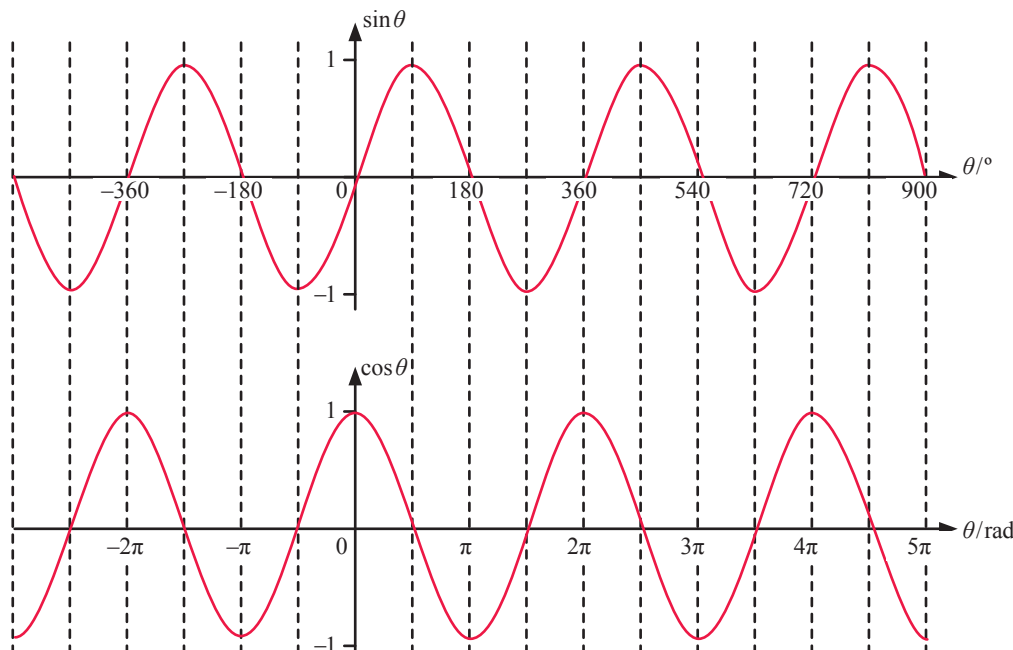


Fig. 4.10 Sine and cosine graphs

Exam tip

Know the following values of trig ratios:

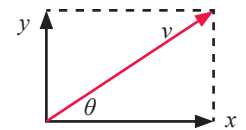
$\theta / ^\circ$	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	0	1	0
30	0.5	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\sqrt{3}}{2}$	0.5	$\sqrt{3}$
90	1	0	∞

Study point

$$\sin \theta = \frac{x}{v}$$

$$\therefore x = v \sin \theta$$

Similarly,
 $y = v \cos \theta$.



Study point

Note the different units in the $\sin \theta$ and $\cos \theta$ graphs. This does not matter because $180^\circ = \pi$ radians, etc. The labels could be swapped or both could use the same.

Exam tip

Learn the graphs and use them when finding angles when you know the value of $\sin \theta$ or $\cos \theta$, e.g. if $\cos \theta = 0.5$, the calculator will give $\theta = 60^\circ$. From the graph, other possible values of θ are: -300° , -60° , 300° , 420° , ...

Self-test

4.9

What are the possible values of θ if $\sin \theta = 0.5$? Give your answer in $^\circ$ and in rad.

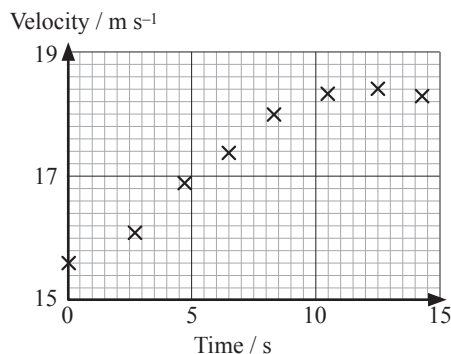


Fig. 4.11 Good practice in graphs

Study point

Fig. 4.6 shows the points plotted as $\times$. Alternatives are $+$, $\odot$.

See Chapter 3 for plotting data with error bars.

4.5 Graphs

4.5.1 Graph preparation and point plotting

Fig. 4.11 illustrates the following rules which you should always follow when graph plotting:

1. The axes are both labelled clearly with the quantity being plotted.
2. The scales increase in equal steps and don't include factors which make the scales difficult to read, e.g. multiples of 3 or 7.
3. The scales are chosen so that the points occupy at least half the grid in both directions.
4. The unit of the quantities plotted are included: the standard method to adopt is $\langle \text{quantity} \rangle / \langle \text{unit} \rangle$. If the quantity is raised to a power, e.g. v^2 then the label should be $(v / m s^{-1})^2$.
5. The points are clearly plotted, with the centre of the cross representing the position of the point.

'Plotting a graph' requires a suitable line to be drawn in addition to setting up the axes, scales and plotting the points. Chapter 3 deals with handling experimental data including decisions on drawing the most appropriate line, e.g. best-fit straight line or best-fit curve.

4.5.2 Determination of rate of change from a graph

The mean rate of change of a quantity y with respect to x is defined by:

$$\text{Mean rate of change} = \frac{\Delta y}{\Delta x}$$

For example, if the gravitational potential energy of an object increases by 500 J when it is raised by 20 m, the mean rate of change of potential energy with respect to height is 25 J m⁻¹.

4.10

Self-test

What do the gradients of the following graphs represent?

- (a) Velocity against time
- (b) Displacement against time

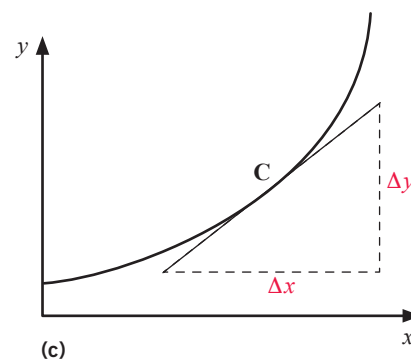
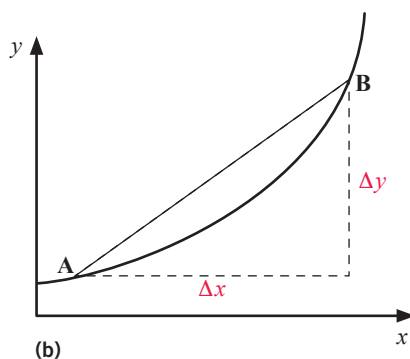
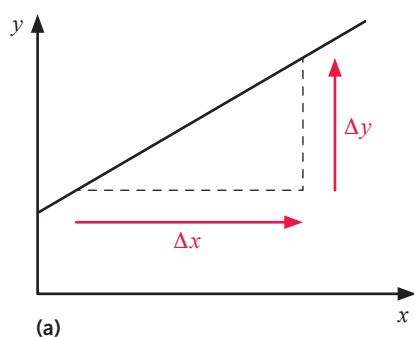


Fig. 4.12 Calculating the rate of change

Terms & definitions

In the expression, 'rate of change' the dependent variable is taken to be *time*, unless otherwise specified. So the 'mean rate of change of velocity' would be:

$$\frac{\Delta v}{\Delta t}, \text{ i.e. the acceleration.}$$

The relationships between the rate of change and the graph are illustrated in Fig. 4.12 (a)–(c). For a straight line variation of y with x , the rate of change of y with respect to x is the gradient of the graph.

For a curved graph, the mean rate of change between **A** and **B** is the gradient of the chord joining **A** and **B**. The instantaneous rate of change of y with respect to x at **C** is the gradient of the *tangent* to the graph at **C**. In principle it doesn't matter how large Δx is in finding the slopes but the larger Δx is the more accurate the value of the rate of change.

4.5.3 Determination of the 'area' under a graph

The area between a velocity–time graph and the time axis is the displacement. This area is significant in many (but not all) graphs. Another examples is the area under a force–distance graph is the work done, i.e energy transfer.

For a graph consisting of linear sections, the area can be divided into triangles and rectangles (or trapeziums). In Fig. 4.13:

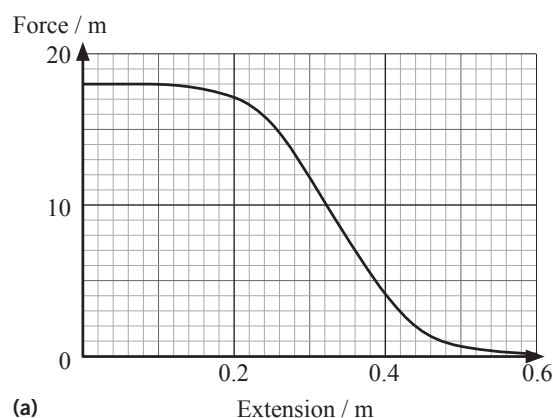
$$\text{Displacement} = A_1 - A_2$$

$$= 204 \text{ m} - 40 \text{ m}$$

$$= 164 \text{ m}$$

$$A_1 = \frac{7+10}{2} \times 24; A_2 = \frac{1}{2} \times 5 \times 16$$

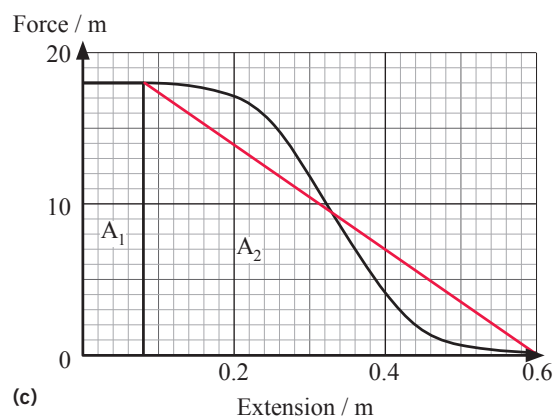
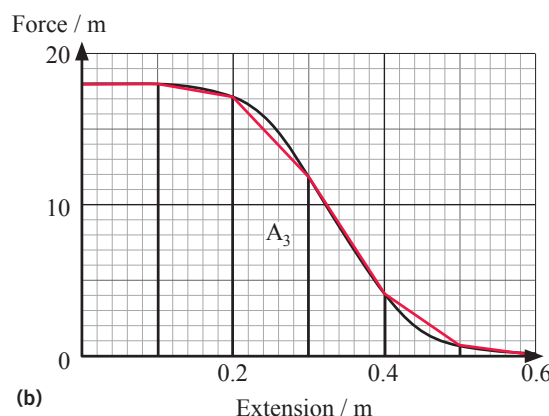
There are several ways of estimating the work done from the area under the force extension curve in Fig. 4.14.



1. Square counting: Each 1 cm square is $5.0 \text{ N} \times 0.1 \text{ m} = 0.5 \text{ J}$. Treating $< \frac{1}{2}$ a square as 0 and $> \frac{1}{2}$ a square as 1, there are 12 such squares $\rightarrow 12 \times 0.5 = 6.0 \text{ J}$.

2. Divide the graph into equal trapeziums, as in Fig. 4.14(b), find the area of each and add to find the total area. For example

$$A_3 = \frac{1}{2} (17 + 11.5) \times 0.1 = 1.425 \text{ J}$$
 See Self-test 4.12.



3. Draw a straight line which cuts the curve into two so that the area above the line is (by eye) equal to the area below the line. In Fig. 4.14(c):

$$A_1 = 18 \times 0.08 = 1.44 \text{ J}$$

$$A_2 = \frac{1}{2} \times 18 \times 0.52 = 4.68 \text{ J}$$

$$\therefore \text{Area} = 6.1 \text{ J (2 s.f.)}$$

Fig. 4.14(a)–(c) Finding the area under a curved graph

Study point

We should really talk about the 'area' between the graph and the horizontal axis – and if the graph is below the axis, the area is negative.

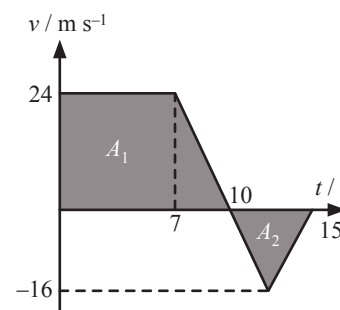


Fig. 4.13 v–t graph

Self-test

4.11

What work do you get if you count 2 mm squares in Fig. 4.14(a)? Each is worth $1.0 \text{ N} \times 0.02 \text{ m} = 0.02 \text{ J}$

Self-test

4.12

Find the area of all the trapeziums in Fig. 4.14(b) and show that the work done is approximately 6.0 J.

Stretch & Challenge

The *trapezoidal rule* states that if you have divided your graph into a series of trapeziums of width Δx and y co-ordinates, $y_0, y_1 \dots y_n$, the area, A , is given by:

$$\frac{1}{2}(y_1 + 2y_2 + 2y_3 + \dots y_n) \Delta x$$

Repeat the calculation of area using this formula.

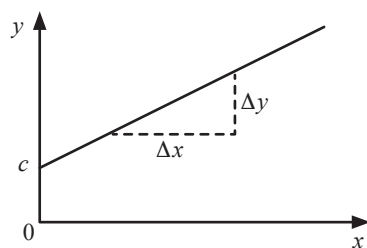


Fig.4.15 Linear graph

Terms & definitions

The value, c , of y when x is 0, is called the **intercept** of the graph on the y axis.

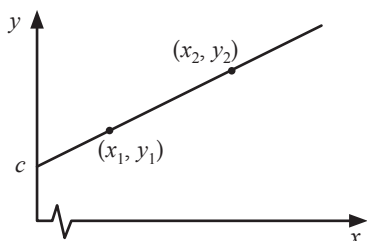


Fig.4.16 Graph remote from the origin

4.13

Self-test

If $v = 55 \text{ m s}^{-1}$ at $t = 25 \text{ s}$ and 34 m s^{-1} at 27.3 s , find the relationship between v and t in the form $v = u + at$. State the values of u and a .

4.5.4 Linear graphs

(a) Equation of a straight-line graph

The graph in Fig. 4.15 represents a linear relationship, i.e. it is a straight line. The equation of the graph is:

$$y = mx + c, \text{ where } m = \frac{\Delta y}{\Delta x}.$$

If the graph passes through the origin $(0,0)$, i.e. $c = 0$, the equation becomes $y = mx$. In this case we say that y is directly proportional to x , $y \propto x$. This is also covered in Section 4.3.1(c).

The relationships $v = u + at$ and $V = E - Ir$ are examples of linear relationships.

- A graph of v against t for constant acceleration is a straight line with gradient a and intercept u on the v axis.
- A graph of V against I for a power supply with a constant internal resistance is a straight line with gradient $-r$ and intercept E on the V axis.

Chapter 3 includes a section on plotting straight line graphs from experimental data including cases where the relationship is non-linear.

(b) Find the equation from the graph

Method 1: If the horizontal axis (the x -axis) of the graph goes back to 0 as in Fig. 4.15, draw a large triangle as shown, measure Δx and Δy , calculate m , read off c from the intercept on the axis.

Method 2: If the data points are all a long way from the origin, as in Fig. 4.16, (e.g. if the values of x are in the range $120 - 150$) then:

1. Locate two well-separated points on the graph (x_1, y_1) and (x_2, y_2) .
2. Calculate the gradient from $m = \frac{y_2 - y_1}{x_2 - x_1}$.
3. The equation is $y - y_1 = m(x - x_1)$.

Insert the values and rearrange into the form $y = mx + c$.

Method 3: As method 2 but after identifying the points, write the equation as:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Insert the values and rearrange into the form $y = mx + c$.

Self-test answers

Component 1

Section 1.1

- 1.1.1** 1. $8a$
 2. $18a^2$
 3. $2\frac{a}{b}$
 4. $36a^2$
- 1.1.2** $V = lbh \quad \therefore [V] = [l][b][h] = \text{m m m} = \text{m}^3$
- 1.1.3** $\text{Pa s} = \text{N m}^{-2} \text{ s} = \text{kg m s}^{-2} \text{ m}^{-1} \text{ s} = \text{kg m}^{-1} \text{ s}^{-1}$ QED
- 1.1.4** $[x] = \text{m}$; $[ut] = \text{m s}^{-1} \text{ s} = \text{m}$; $[\frac{1}{2}at^2] = \text{m s}^{-2} \cdot \text{s}^2 = \text{m}$.
 $\therefore$ The two terms on the RHS have the same units, so can be added together; and the LHS has the same units.
 $\therefore$ The equation is homogeneous.
- 1.1.5** $5 \text{ N at } \tan^{-1} 0.75 = 36.9^\circ \text{ to } 4 \text{ N force}$
- 1.1.6** $F_v = 75 \text{ N}$; $F_h = 130 \text{ N}$
- 1.1.7** $B \sin \theta$
- 1.1.8** $\text{Mass} = 7.9 \text{ g cm}^{-3} \times (10 \text{ cm} \times 5 \text{ cm} \times 4 \text{ cm}) = 1580 \text{ g}$
- 1.1.9** (Wheelbarrow): weight = CWM; lift = ACM
 (Spanner): Force = ACM; friction on nut = CM
- 1.1.10** Total weight = $(2 + 3 + 5.5 + 10) \text{ kg} \times 9.8 \text{ N kg}^{-1} = 201 \text{ N}$
 $\therefore$ For equilibrium, upward force by pivot on plank $\sim 200 \text{ N}$
- 1.1.11** $6 \times 60 = 3 \times 90 + 1 \times 50 + 2(100 - d)$; $\therefore 360 = 520 - 2d$;
 $\therefore 2d = 160$; $\therefore d = 80 \text{ cm}$
- 1.1.12** **A:** $40F = 3 \times 10 + 1 \times 50 + 2d$; $\therefore 20F = 40 + d$ [1]
B: $60F = 3 \times 90 + 1 \times 50 + 2(100 - d)$; $\therefore 30F = 260 - d$ [2]
 Then, e.g., solve [1] and [2] for F by adding to eliminate $d \rightarrow 50F = 300$, etc.
- 1.1.13** By all methods: $F = 11.7 \text{ N}$ at 31.0° to downward vertical
- 1.1.14** $F = 50.3 \text{ N}$ at 9.6° to horizontal

Section 1.2

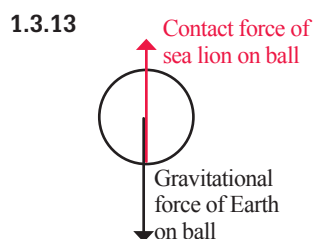
- 1.2.1** 10.8 m s^{-1}
- 1.2.2** $0-3 \text{ s}$; $6-8 \text{ s}$ (0 velocity); $9.6-12 \text{ s}$
- 1.2.3** $0-3 \text{ s}$ constant forward velocity; $3-6 \text{ s}$ decelerating;
 $6-8 \text{ s}$ stationary; $8-9.6 \text{ s}$ accelerating backwards; $9.6-12 \text{ s}$ constant velocity back
- 1.2.4** 3.2 m s^{-1}
- 1.2.5** 29.8 km s^{-1} at 270°
- 1.2.6** (a) 29.8 km s^{-1} at 270°
 (b) 19.0 km s^{-1} at 90°
 (c) 26.9 km s^{-1} at 225°

- 1.2.7** (a) 29.9 km s^{-1} at 330°
 (b) 29.8 km s^{-1} at 30°
- 1.2.8** $20-30 \text{ s}$ (BC): uniform acceleration from 10 to 20 m s^{-1}
 $30-54 \text{ s}$ (CD): uniform velocity of 20 m s^{-1}
 $54-68 \text{ s}$ (DE): uniform deceleration to rest from 20 m s^{-1}
- 1.2.9** 0.4 m s^{-2}
- 1.2.10** Area $0-20 \text{ s}$ (trapezium) = $\frac{1}{2}(15 + 20) \times 10 = 175 \text{ m}$, etc.
- 1.2.11** 72 m s^{-1} at 56.3° to horizontal
- 1.2.12** (a) $10^2 = 26^2 - 2 \times 1.2x$; $\therefore 2.4x = 576$; $\therefore x = 240 \text{ m}$, etc.
 (b) $10 = 26 - 1.2t$; $\therefore 1.2t = 16$; $\therefore t = 13.3 \text{ m s}^{-1}$
 (c) $240 = 26t - 0.6t^2$; $\therefore 0.6t^2 - 26t + 240 = 0$;
 $\therefore t = \frac{26 \pm \sqrt{676 - 576}}{1.2} = 13.3 \text{ s}$ or 30.0 s
- 1.2.13** 63.4 m s^{-1}
- 1.2.14** (2nd part) Because the acceleration is different (and not constant) in the powered stage.
- 1.2.15** (a) 26.0 m s^{-1} ; 15 m s^{-1}
 (b) 26.0 m s^{-1} ; -34.1 m s^{-1}
 (c) 42.9 m s^{-1} at 52.7° below horizontal
 (d) Horizontal: 130 m ; vertical -47.6 m (i.e. 47.6 m below starting point)
- 1.2.16** $t = 2.89 \text{ s}$. $v_v = -8.3 \text{ m s}^{-1} \rightarrow v = 35.6 \text{ m s}^{-1}$ at 13.5° below horizontal
- 1.2.17** (a) $t \sim 0.32 \text{ s}$
 (b) 6% (uncertainty in height is negligible)

Section 1.3

- 1.3.1** (a) $20\,000 \text{ kg m s}^{-1}$ (or 20 kN s)
 (b) $1.8 \times 10^{29} \text{ N s}$
- 1.3.2** $p_1 = 0.15 \times 6 = 0.9 \text{ kg cm s}^{-1}$
 $p_2 = 0.45 \times 2 = 0.9 \text{ kg cm s}^{-1} = p_1$
- 1.3.3** (a) $\text{KE}_1 = 2.7 \times 10^{-4} \text{ J}$;
 $\text{KE}_2 = 0.5 \times 0.3 \times 0.032 = 1.35 \times 10^{-4} \text{ J}$
 $\therefore 50\%$ of KE lost
 (b) 67% of KE lost
- 1.3.4** $1000 \times (-6) + 4000 \times 2 = 5000 v$;
 $\therefore 5000v = 2000$; $\therefore v = 0.8 \text{ m s}^{-1}$
- 1.3.5** KE_1 as in 1.3.3;
 $\text{KE}_2 = 0.5 \times 0.15 \times (-0.02)^2 + 0.5 \times 0.3 \times 0.04^2 = 2.7 \times 10^{-4} \text{ J}$
- 1.3.6** $[F] = \frac{[\Delta p]}{[\Delta t]}$; $\therefore \text{N} = \frac{[\text{p}]}{\text{s}}$; $\therefore [\text{p}] = \text{N s}$ QED

- 1.3.7** (a) 1.4 kN
 (b) ~0.82 kN
 (c) ~4 kN
- 1.3.8** Force = 200 N (so possibly, just)
- 1.3.9** You can show that KE per second = $\frac{1}{2} \frac{\rho V^3}{A^2}$ [as for wind turbine] $\rightarrow \Delta KE = 3 \text{ kW}$
- 1.3.10** (a) F on sand to right because Δp to right
 (b) F on belt to left by N3
- 1.3.11** $[c_d] = \frac{[F_d]}{[\rho][v^2][A]} = \frac{\text{kg m s}^{-2}}{\text{kg m}^{-3} \text{ m}^2 \text{ s}^{-2} \text{ m}^2} = \frac{\text{kg m s}^{-2}}{\text{kg m s}^{-2}}$, i.e. no units
- 1.3.12** 1. Contact force of sea lion on ball
 2. Gravitational force of sea lion on Earth
 3. Contact force of back flipper of sea lion on ground
 4. Contact force of front flipper of sea lion on ground
 5. Frictional force of back flipper of sea lion on ground
 6. Frictional force of front flipper of sea lion on ground



- 1.3.14** 1.0(4) m s^{-2}
- 1.3.15** 72.0 kg [706 N]; 0
- 1.3.16**
-
- 1.3.17** Taking a spread-eagled skydiver, estimating $A = 1 \text{ m}^2$; $m = 90 \text{ kg}$; $\rightarrow v_{\text{term}} \sim 40 \text{ m s}^{-1}$

Section 1.4

- 1.4.1** Initially random vibrational energy of molecules in the rope and winch.
- 1.4.2** 614 J
- 1.4.3** $a = \frac{F}{m} = \frac{1200}{800} = 1.5 \text{ m s}^{-2} \therefore \sqrt{15^2 + 2 \times 1.5 \times 250} = 31.2 \text{ m s}^{-1}$
- 1.4.4** 1 kW h = 3.6 MJ
- 1.4.5** 450 kJ / 0.125 kW h
- 1.4.6** $3.96 \times 10^{19} \text{ J} [= 39.6 \text{ EJ}]$
- 1.4.7** (a) unit of the 0.3 term = $\frac{[F_d]}{[\rho][v^2]} = \frac{\text{kg m s}^{-2}}{\text{kg m}^{-3} \text{ m}^2 \text{ s}^{-2}} = \text{m}^2$. QED
 (b) 6.1 kW

Section 1.5

- 1.5.1** 23.5 N m^{-1}
- 1.5.2** 200 μm
- 1.5.3** 0.1 J
- 1.5.4** $\frac{1}{2} \sigma \epsilon = \frac{1}{2} \sigma \frac{\sigma}{E} = \frac{1}{2} \frac{\sigma^2}{E}$; $\frac{1}{2} \sigma \epsilon = \frac{1}{2} \epsilon E \epsilon = \frac{1}{2} \epsilon^2 E$ QED
- 1.5.5**
-

- 1.5.6** Thickness halves
- 1.5.7** $\epsilon_{\text{rubber}} \sim 10\,000 \times \epsilon_{\text{steel}}$

Section 1.6

- 1.6.1** (a) 970 nm
 (b) IR
 (c) $P = 16 \times 1.7 \sim 27 \text{ kW}$; $\lambda_{\text{max}} = \frac{1}{2} \times 970 \sim 500 \text{ nm}$
- 1.6.2** (a) $\sim 8 \text{ mW m}^{-2}$
 (b) $80 \mu\text{W m}^{-2}$
 (c) 8 nW m^{-2}
- 1.6.3** $L_B = 4L_A$
- 1.6.4** $L = I \times 4\pi a^2 = 42.8 \times 10^{-9} \times 4 \times \pi \times (9.5 \times 10^{16})^2 = 4.9 \times 10^{27} \text{ W}$
- 1.6.5** (a) $7.5 \times 10^{20} \text{ m}$
 (b) $\sim 80 \text{ kly}$
- 1.6.6** 690 000 km
- 1.6.7** Diameter = $9.34 \times 10^{-3} \times 150 \times 10^6 \text{ km} \sim 1.4 \text{ million km}$
- 1.6.8** $T = 3700 \text{ K}$; $r \sim 6.0 \text{ million km}$
- 1.6.9** C – H α , F – H β ; D – NaI; c, E – FeI; K, H – CaII; b – MgI
- 1.6.10** $\lambda_{\text{max}} = 1.1 \text{ mm}$. $\therefore T = \frac{W}{\lambda_{\text{max}}} = \frac{2.898 \times 10^{-3}}{1.06 \times 10^{-3}} \sim 2.7 \text{ K}$

Section 1.7

- 1.7.1** $m_p \sim 0.94 \text{ GeV}/c^2$
- 1.7.2** $f = 1.47 \times 10^{20} \text{ Hz}$; 2.04 pm; $p = 3.3 \times 10^{-22} \text{ N s}$
- 1.7.3** $^{12}_6\text{C}$: 50%; $^{56}_{26}\text{Fe}$: 54%; $^{197}_{79}\text{Au}$: 60% neutrons
- 1.7.4** Neutron decay:
 excess energy = $939.6 - (938.3 + 0.5) = 0.8 \text{ MeV}$
 Proton decay: $m_p < m_n + m_e (+m_\nu)$ $\therefore$ Not enough energy
- 1.7.5** LHS: $L_e = 0$; RHS: $L_e = 0 + 1 + (-1) = 0$. $\therefore L_e$ conserved
- 1.7.6** $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$:
LHS: $L_\mu = -1$. RHS: $L_\mu = 0 + (-1) + 0 = -1$. $\therefore L_\mu$ conserved
LHS: $L_e = 0$; RHS: $L_e = (-1) + 0 + 1 = 0$. $\therefore L_e$ conserved
- 1.7.7** (a) Weak interaction; Neutrino involved which is only affected by the weak interaction.
 (b) Conservation of charge (+e on both sides), [muon] lepton number (0 on both sides), quark number (0 on both sides, but note a change of quark flavour consistent with weak interaction)

Component 2

Section 2.1

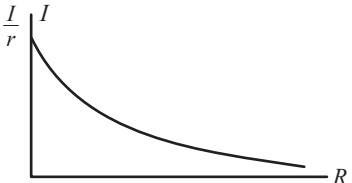
- 2.1.1** 6.25×10^{18}
2.1.2 2×10^{10} electrons are transferred to the polythene rod.
2.1.3 45 A h expresses the charge = 162 kC
2.1.4 v is inversely proportional to A , hence to diameter².
 Hence $4 \times$ the drift velocity = 0.48 mm s^{-1} .

Section 2.2

- 2.2.1** 30 MV
2.2.2 (a) 720 mW (0.72 W)
 (b) 216 C
 (c) 1.3 kJ
2.2.3 $t = \frac{VQ}{P}$
2.2.4 (a) 2Ω
 (b) 11Ω
2.2.5 5 A
2.2.6 2%
2.2.7 0.0063Ω ; 0 Ω
2.2.8 (a) Working resistance $\sim 14 \times R_0$.
 (b) Working temperature $\sim 3200 \text{ K}$
2.2.9 Gradient = $8.29 \Omega \text{ m}^{-1}$; diameter = 0.28 mm.

Section 2.3

- 2.3.1** (a) 36.5Ω
 (b) (i) 0.65 W
 (ii) 2.47 W. Most power wasted in the resistor.
2.3.2 8.3 Ω
2.3.3 4 Ω , 8 Ω , 18 Ω , 36 Ω
2.3.4 $x = 0.5 \text{ A}$, $y = 0.4 \text{ A}$
2.3.5 1%
2.3.6 Roughly 30°C
2.3.7 The brighter the light, the lower the output voltage.
2.3.8 (a) 0.27 A
 (b) 0.41 W (total) [0.37 W exported]
2.3.9 Equations, using $I = \frac{E}{R+r}$: $0.88 = \frac{E}{1.25+r}$ and $0.28 = \frac{E}{5.0+r}$
 $\rightarrow E = 1.54 \text{ V}$; $r = 0.50 \Omega$
2.3.10 (Correct orientation) $I = 1.75 \text{ A}$, $V = 3.49 \text{ V}$; (cell wrong-way round) $I = 0.58 \text{ A}$, $V = 1.16 \text{ V}$

- 2.3.11** 
 $I(R=0) = \frac{E}{r}$; $I(R=r) = \frac{E}{2r}$; $I(\infty) = 0$

- 2.3.12** (a) $V = 5.92 \text{ V}$, $r = 123 \Omega$

(b) 0.018 A, 3.67 V

- 2.3.13** $E = \frac{1}{\text{gradient}}$; $r = \frac{\text{intercept}}{\text{gradient}}$

- 2.3.14** $E = \frac{1}{\text{intercept}}$; $r = \frac{\text{gradient}}{\text{intercept}}$

Section 2.4

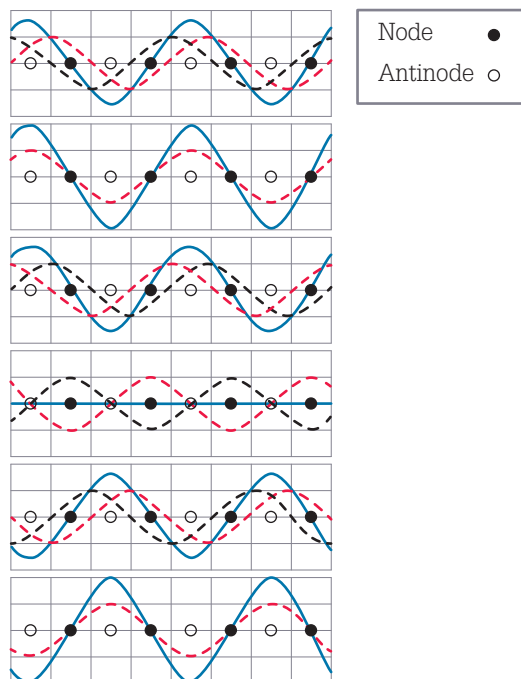
- 2.4.1** (Examples) Radiation from the sun; electricity
2.4.2 $n = 0.5$
2.4.3 ρ is density; the unit is kg m^{-3}
2.4.4 (a) 0.156 s
 (b) 1.6 Hz
2.4.5 The horizontal axis represents time, so the peak-to-peak interval cannot be a length. He should have said that this represents the period.
 The amplitude is the maximum displacement, so 'maximum amplitude' is a tautology. He should have said that A was the amplitude.
2.4.6 0.6 m
2.4.7 $\lambda/4 \sim 8 \text{ cm}$, so this is possible.

Section 2.5

- 2.5.1** Roughly 25° on either side
2.5.2 The low frequencies have long wavelengths which spread out a lot by diffraction through the open window. The higher frequencies, which are important for understanding do not diffract to the same extent. Taking $f > 3 \text{ kHz}$ (for consonants), this gives $\lambda < 0.1 \text{ m}$. Doorways and window gaps $\sim 1 \text{ m}$, so diffraction spreading small.
2.5.3 The string still has kinetic energy – just to the left of the centre line, the string is moving upwards; just to the right it is moving downwards.
2.5.4 They do not affect each other.
2.5.5 $\lambda = 3.7 \text{ mm}$
 $S_1R = 31 \text{ mm}$; $S_2R = 34.5 \text{ mm}$; $S_2R - S_1R = 3.5 \text{ mm}$
 $\therefore$ agreement
2.5.6 $S_1U - S_2U = 2\lambda$; $S_1V - S_2V = 2.5\lambda$
2.5.7 Path difference = $120 \sin 30^\circ = 60 \text{ m} = 2.5\lambda$. $\therefore$ destructive interference.
2.5.8 The intensity of the diffracted beams from the slits decreases with angle from the centre.
2.5.9 Fringe spacing = 1.36 mm; $\lambda = 680 \text{ nm}$.
2.5.10 470 nm
2.5.11 Slit width

- 2.5.12** 

2.5.13



2.5.14 2nd harmonic



3rd harmonic



$$f_n = nf_1$$

 2.5.15 (a) $f_1 = 170 \text{ Hz}$; $f_2 = 510 \text{ Hz}$

 (b) With closed-end pipe: $f_1 = 340 \text{ Hz}$ and $f_2 = 680 \text{ Hz}$

 2.5.16 (a) % uncertainty $\sim 9\%$

 (b) absolute uncertainty $\sim \pm 50 \text{ nm}$

 2.5.17 $d = 5 \text{ }\mu\text{m}$

 2.5.18 (a) $d = 2.76 \text{ }\mu\text{m}$

 (b) grating constant $= 3.63 \times 10^5 \text{ m}^{-1} = 3630 \text{ cm}^{-1}$.

Section 2.6

 2.6.1 (a) deep $= 2.5 \text{ cm}$, shallow $= 1.5 \text{ cm}$

 (b) 23°

 2.6.2 (a) $2.26 \times 10^8 \text{ m s}^{-1}$

 (b) $1.24 \times 10^8 \text{ m s}^{-1}$

 2.6.3 29.6°

 2.6.4 (a) Ray emerges at 39° ; weak reflected ray at 25°

 (b) Weaker refracted ray at 59° , stronger reflected ray at 35°

 (c) Total internal reflection – strong reflected ray at 45°

 2.6.5 47.2°

 2.6.6 $\sin c = \frac{1.52}{1.62}$. $\therefore c = \sin^{-1} \frac{1.52}{1.62} = 69.8^\circ$

 2.6.7 0.03 s , assuming the glass has a refractive index of 1.5 .

 2.6.8 $\sin \theta = 0.0099998$; $\tan \theta = 0.0100003$

$$\therefore \frac{\tan \theta - \sin \theta}{\tan \theta} \approx \frac{5 \times 10^{-7}}{0.01} = 5 \times 10^{-5} = 0.005\%$$

 2.6.9 Uncertainty in $n = \pm 0.03$

Section 2.7

 2.7.1 $E_{k \text{ max}} = 9.6 \times 10^{-20} \text{ J} = 0.6 \text{ eV}$

2.7.2 All photons have the same energy (for a particular frequency). A change in intensity will affect only the number of electrons emitted, not their energy, because an electron is only emitted by one photon.

 2.7.3 (a) $5.2 \text{ eV} = 8.4 \times 10^{-19} \text{ J}$

(b) no electrons emitted

(c) same as (a).

 2.7.4 (a) $1.5 \times 10^{45} \text{ photons s}^{-1}$ (b) $5.3 \times 10^{21} \text{ m}^{-2}$

 2.7.5 If $f = 1 \times 10^{12} \text{ Hz}$, $\lambda = 3 \times 10^{-4} \text{ m} = 0.3 \text{ mm}$. So THz radiation has a wavelength of less than 1 mm .

 2.7.6 2.18 aJ

 2.7.7 $E = 13.6 \left(\frac{1}{2^2} - \frac{1}{6^2} \right) = 3.02 \text{ eV} = 4.84 \times 10^{-19} \text{ J}$

$$\therefore \lambda = \frac{hc}{E} = \frac{6.630 \times 10^{-34} \times 3.0 \times 10^8}{4.84 \times 10^{-19}} \text{ m} = 411 \text{ nm}.$$

 Fig 1.6.12 gives H δ has $\lambda = 410 \text{ nm}$.

 2.7.8 2×10^{-13} , 2×10^{-15} , 2×10^{-18} , 4×10^{-19} , 2×10^{-20} , 2×10^{-23} , 2×10^{-27}

 2.7.9 590 nm

 2.7.10 410 fm

 2.7.11 $1.3 \times 10^{-27} \text{ N s}$.

 2.7.12 (a) The momentum of the incident photons in $\Delta t = \frac{IA\Delta t}{c}$

 The momentum of the reflected photons in $\Delta t = -\frac{IA\Delta t}{c}$
 $\therefore$ The momentum change of the incident photons in $\Delta t = -2\frac{IA\Delta t}{c}$
 $\therefore$ The force exerted by the photons on the plane $= \frac{\Delta p}{\Delta t} = 2\frac{IA}{c}$ by N2 and N3

 $\therefore$ The pressure exerted by the photons on the plane $= 2\frac{I}{c}$

 (b) pressure $= \frac{I}{c}$!

Section 2.8

 2.8.1 (a) 2.5 eV (0.4 aJ)

 (b) 0.7 eV (0.11 aJ)

 (c) 1.8 eV (0.29 aJ)

2.8.2 The pumping will produce a population in the P state. Any electrons which drop to U will quickly drop further to G, so there will be a population inversion between P and U, allowing stimulated emission to produce amplification.

 2.8.3 0.72 ($= 72\%$)

 2.8.4 For a laser operating with a constant beam, the rate of production of photons must equal their rate of loss. Hence a photon must travel ~ 280 lengths of the medium before stimulating an emission. This is 140 m in this case.

 2.8.5 (a) Photon energy $= 1.3 \text{ eV}$. Pumping voltage $\sim 1.3 \text{ V}$

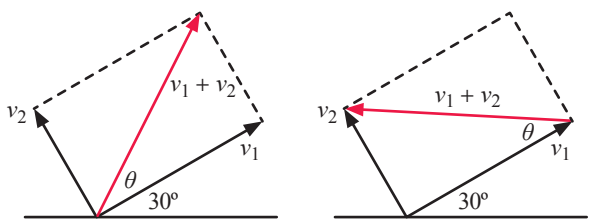
 (b) $8.6 \times 10^{16} \text{ photons s}^{-1}$.

 2.8.6 $8.59 \times 10^{-19} \text{ J}$ (5.37 eV)

Exercise answers

Component 1

Exercise 1.1

- $[G] = [F][d^2][M_1M_2]^{-1} = \text{N m}^2 \text{ kg}^{-2}$
 - $\text{N} = \text{kg m s}^{-2} \therefore [G] = \text{kg m s}^{-2} \text{ m}^2 \text{ kg}^{-2} = \text{kg}^{-1} \text{ m}^3 \text{ s}^{-2}$
- $V = \text{kg m}^2 \text{ s}^{-3} \text{ A}^{-1}$
 - Use $R = \frac{V}{I} \therefore \Omega = V \text{ A}^{-1} = \text{kg m}^2 \text{ s}^{-3} \text{ A}^{-2}$
- 34 kg
- $2.2 \times 10^{11} \text{ m}$ [It could be $2.20 \times 10^{11} \text{ m}$ if the original figure was to 3 s.f.]
- Circumference of Earth $\sim 40\,000 \text{ km} \rightarrow 85\,000 \text{ km}$
- Taking right as positive: $v_{1H} = 30 \cos 30^\circ = 26.0 \text{ m s}^{-1}$;
 $v_{2H} = 20 \cos 120^\circ = -10.0 \text{ m s}^{-1}$
 Taking upwards as positive: $v_{1V} = 30 \sin 30^\circ = 15.0 \text{ m s}^{-1}$;
 $v_{2V} = 20 \cos 30^\circ = 17.3 \text{ m s}^{-1}$
- $(v_1 + v_2)_H = 26.0 + (-10.0) = 16.0 \text{ m s}^{-1}$.
 $(v_1 + v_2)_V = 15.0 + 17.3 = 32.3 \text{ m s}^{-1}$
 $\rightarrow v_1 + v_2 = 36.0 \text{ m s}^{-1}$ at 63.6° to horizontal
 - $(v_2 - v_1)_H = -10.0 - 26.0 = -36.0 \text{ m s}^{-1}$.
 $(v_2 - v_1)_V = 17.3 - 15.0 = 2.3 \text{ m s}^{-1}$
 $\rightarrow v_2 - v_1 = 36.0 \text{ m s}^{-1}$ at 3.7° to left horizontal
 $[176.3^\circ$ to right horizontal]
- 

Magnitudes of $v_1 + v_2$ and $v_2 - v_1$ are both $\sqrt{30^2 + 20^2} = 36.0 \text{ m s}^{-1}$.
 $\theta = \tan^{-1} \frac{20}{30} = 33.7^\circ \rightarrow$ same directions as in Q7
 (with a rounding error in Q7).
- $3.75 \text{ g cm}^{-3} / 3750 \text{ kg m}^{-3}$
- 100 g
 - 2.9 N [taking g as 9.81 m s^{-2}]

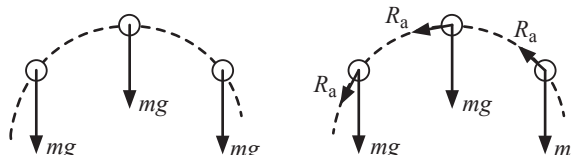
Exercise 1.2

- 5.0 m s^{-1}
 - 4.4 m s^{-1}
 - 4.7 m s^{-1} [$\neq 0.5 (5.0 + 4.4)$]
 - 0.53 m s^{-1} downwards
 - 18.9 m s^{-1} upwards

- Using $x = ut + \frac{1}{2}at^2$, $5 = 0 + \frac{1}{2}a \times 1^2 \therefore a = 10 \text{ m s}^{-2}$
- Maximum speed = 45 m s^{-1} .
 Distance travelled in first 90 s = $22.5 \times 90 = 2025 \text{ m}$
 Time at maximum speed = 100 s;
 Time to decelerate = $1800 / 22.5 = 80 \text{ s}$
 $\therefore$ Total distance = $2025 + 4500 + 1800 \text{ m} = 8325 \text{ m}$;
 total time = $90 + 100 + 80 = 270 \text{ s}$
 $\therefore$ Mean velocity = $8325 \text{ m} / 270 \text{ s} = 30.8 \text{ m s}^{-1}$
- $\Delta v = 20\sqrt{2} \text{ m s}^{-1}$ SE. $\therefore \langle a \rangle = 4\sqrt{2} = 5.7 \text{ m s}^{-2}$ SE [i.e. N135°]
- 1.0 m s^{-2}
 - 250 m
 - [From gradient of tangent] -0.60 m s^{-2}
 - Distance est = 355 m. $\langle v \rangle = 8.9 \text{ m s}^{-1}$
- 123 m,
 - 49.1 m s^{-1} ,
 - 57.5 m s^{-1} at 59° to horizontal
- Estimate time = 0.25 s. Vertical speed at 0.25 s = 2.5 m s^{-1} .
 $\therefore$ Horizontal speed = 2.5 m s^{-1} . $\therefore$ distance between vertical lines 0.10 m
- 31.9 m
 - 2.54 s
 - 43.3 m s^{-1} horizontal
 - 220 m
- Time = 40 s; distance = 1200 m
- h against t^2 : best fit straight line has gradient $4.25 (\text{m s}^{-2})$ and intercept $-0.045 (\text{m}) \rightarrow g = 8.5 \text{ m s}^{-2}$
 $\sqrt{h}$ against t : best fit straight line has gradient $2.21 (\text{m}^{0.5} \text{ s}^{-1})$ and intercept $-0.086 (\text{m}^{0.5}) \rightarrow g = 9.77 \text{ m s}^{-2}$ and $\Delta t \sim 0.04 \text{ s}$

Exercise 1.3

- 800 kg m s^{-1} (N s) to the right
 - 200 N s to the right
 - 580 N s at N31°E
- 7750 J
- Let m be mass of a rider. Initial momentum = $2m \times 6 = 12m$;
 final momentum = $3m \times 4 = 12m$. Equal initial and final momenta, $\therefore$ as predicted
- 33%
- 28 kN s
 - 11.2 m s^{-1} (in the direction of the first car), assuming no resultant horizontal external force acts

6. 5.0 m s^{-1}
7. (a) $2.8 \times 10^5 \text{ m s}^{-1}$
(b) $3.4 \times 10^{-13} \text{ J}$ [$\sim 2.1 \text{ MeV}$]
8. A 17 m s^{-1} ; B 18 m s^{-1} both in reverse directions to their original ones
9. Change of momentum of football = $[(-25) - 30] \times 0.45 = 24.75 \text{ N s}$. $\therefore$ Mean force exerted by the wall on the ball is $\frac{-24.75}{0.04} = -620 \text{ N}$ [2.s.f] by N2, i.e. -620 N in the original direction of the ball
 $\therefore$ By N3 the mean force exerted by the ball on the wall = $-(-620 \text{ N}) = 620 \text{ N}$ in the ball's original direction
10. (a) $\alpha_{\max} = 11.0 \text{ m s}^{-2}$; $\alpha_{\min} = 4.5 \text{ m s}^{-2}$,
(b) 8.4 m s^{-2} at 22.6° to the 12 N force.
11. (a) 0.392 N
(b) (i) $0.392 - T = 0.040a$; (ii) $T = 0.200a$
(c) 1.63 m s^{-2}
12. (a) $4.9 \times 10^4 \text{ m s}^{-2}$
(b) Assuming $\langle F_{\text{res}} \rangle = \frac{1}{2}F_{\max} \rightarrow 186 \text{ m s}^{-1}$
[constant $F_{\text{res}} \rightarrow 260 \text{ m s}^{-1}$]
13. (a) Without air resistance (b) With air resistance
- 
14. (a) Graph of x/t against t is a straight line, $\therefore$ constant acceleration
(b) Gradient = $14.2 \text{ (cm s}^{-2})$ $\therefore$ acceleration = 28.4 cm s^{-2}
(c) 0.17 kg .
15. Gradient = $14.2 \pm 1.6 \text{ (cm s}^{-2})$ $\therefore$ acceleration = $28 \pm 3 \text{ cm s}^{-2}$ (i.e. 11%). $\therefore \Delta M = \pm 0.02 \text{ kg}$.

Exercise 1.4

1. (a) 540 kJ
(b) 5.0 kJ
2. (a) 500 kJ
(b) 5.0 kN
3. (a) 16.7 kJ
(b) 19.8 m s^{-1}
(c) 62 N
4. (a) 0.5 J
(b) 16 m s^{-1} assuming all PE transferred to KE in sphere
5. (a) 1.62 m s^{-1}
(b) 42.9° assuming no loss in energy, i.e. same height as originally (0.134 m)
6. 1.69 m
Horizontal velocity after rolling down a height h_1 given by

$$\frac{1}{2}mv^2 = 0.714mgh_1, \text{ i.e. } v = \sqrt{1.428gh_1}$$

$$\text{Time to drop a step height } h_2 \text{ given by } t = \sqrt{\frac{2h_2}{g}},$$

$$\therefore \text{Horizontal distance} = vt = \sqrt{1.428gh_1} \times \sqrt{\frac{2h_2}{g}} = \sqrt{2.856h_1h_2},$$

which is independent of g

7. (a) 84.9 m s^{-1}
(b) Height = 92 m , range = 640 m

Exercise 1.5

1. (a) 24.5 N m^{-1}
(b) 0.078 J
(c) 0.157 J
2. Extension = 16.0 cm
(a) 12.25 N m^{-1}
(b) 0.157 J
(c) 0.304 J
3. (a) 0.885 m s^{-1}
(b) 16.0 cm
(c) 9.81 m s^{-1}
4. (a) end-to-end $k = 12 \text{ N m}^{-1}$. Total extension double for any given load
(b) side-to-side $k = 48 \text{ N m}^{-1}$. Total load double for any given extension
5. $F_{\max} = 650 \text{ N}$, $\Delta l = 0.14 \text{ mm}$
6. (a) 24.5 N
(b) 156 GPa
7. The following table gives the masses, m , loads needed to produce given deflections, x assuming Hooke's law is obeyed:

x/cm	1.0	1.5	0.2	2.5	3.0	4.0	4.5	5.0
m/g	12	42	100	195	337	799	1140	1560

Supplementary question: by plotting a suitable graph, show that $m \propto x^3$.

The deflection for $\epsilon = 1.0 \times 10^{-3}$ is 4.47 cm , which is produced by a load of 1120 g .

8. (a) $58\,900 \text{ N}$
(b) 295 kJ
(c) 7.4 m
(d) 540 m
9. Any surface scratch will close up and not be under tension until the glass is stretched so that the compression in the surface layer is overcome.
10. $T_A = T_B$; $\Delta l_B = 2\Delta l_A$; $\sigma_B = \sigma_A$; $\epsilon_B = 2\epsilon_A$;
Energy_B = $2 \times$ Energy_A.
11. $\frac{2}{3}E$
12. $\frac{T_2}{T_1} = \frac{\sigma_2}{\sigma_1} = \frac{E_2}{E_1}$; $\Delta l_1 = \Delta l_2$; and $\epsilon_1 = \epsilon_2$;
Energy in 1 = $2 \times$ Energy in 2.
13. $E_{\text{eff}} = \frac{E_1 + E_2}{2}$

Exercise 1.6

- (a) $1.2 \times 10^{25} \text{ W}$
(b) 120 nm
- $A_{\odot} \sim 10^4 A_*$, $T_{\odot} \sim \frac{1}{4} T_*$,
 $\therefore L_{\odot} \sim \frac{10^4}{4^4} L_* = \frac{10^4}{256} L_* \sim 40 L_*$; $\lambda_{\odot} = 4\lambda$
- Using Wien's law $\lambda_{\text{max}} \sim$ (a) 0.3 mm
(b) 3 μm
(c) 30 nm
(d) 300 pm
This suggests (a) microwave
(b) (far) IR
(c) UV
(d) X-ray
- The intensity axis needs to be shrunk by a factor of ~ 100 ($\sim 3^4$) and the wavelength axis needs to be expanded by a factor of ~ 3 .
- $L_{\text{red giant}} = 10^6 \times L_{\text{red dwarf}}$. $\therefore$ At same distance the red giant appears $10^6 \times$ as bright. To reduce the brightness of the red giant by a factor of 10^4 (so it only appears $10^2 \times$ as bright) it needs to be moved $10^2 \times$ the distance. So the red giant is 100 times as far away as the red dwarf.
- Nearly all the hydrogen outside stars has been used up (or blown away by radiation pressure).
21 cm emission is only from neutral hydrogen atoms.
- Visible light, of wavelength 0.4–0.7 μm , is strongly scattered by interstellar dust. IR radiation, especially far IR, has a longer wavelength than the size of the dust particles so is not scattered by them and can therefore better penetrate the dust clouds within which the stars are being formed.
- $\lambda_{\text{max}} \sim 10 \mu\text{m}$ so observations in the far IR shown a stronger emission if the star is surrounded by such a disc than in its absence.
- The black body radiation from the stars contains photons with the correct energy to raise the energy of electrons within the hydrogen atoms to the first and second excited states. These excited atoms drop down into lower energy states, by emitting the characteristic wavelengths as they do so.
- The hot spot emits X-rays with a wavelength of around 300 pm (see Q3). Hence, a burst of X-rays will be observed once in each revolution (i.e. $\sim$ once per second). If the observer is directly in the plane of the accretion disc, this might not be observed because of absorption within the disc.

Exercise 1.7

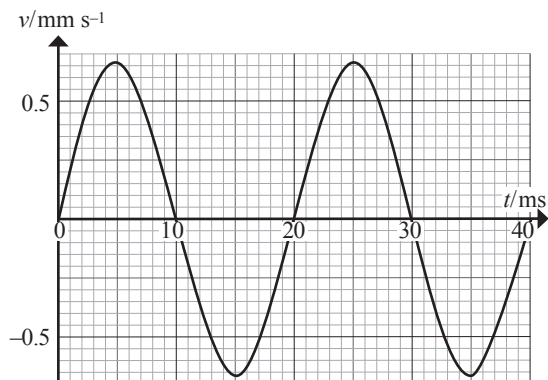
- (a) $E_p = E_e = 500 \text{ eV}$; $E_a = 1000 \text{ eV}$ [1 keV]
(b) $E_p = E_e = 8 \times 10^{-17} \text{ J}$; $E_a = 1.6 \times 10^{-16} \text{ J}$
- (a) $1.674 \times 10^{-27} \text{ kg}$
(b) 940.6 MeV/ c^2
- (a) Electron neutrino ν_e . Must be neutral (conservation of charge) and $L_e = 1$.
(b) Weak decay because it involves the neutrino (strong and e-m interactions don't affect neutrinos).
(c) Conservation of mass/energy would be violated
 $-m_n > m_p$.
- $\Delta^0 \rightarrow p + \pi^-$ / $udd \rightarrow uud + \bar{u}\bar{d}$
 $\Delta^+ \rightarrow n + \pi^+$ / $uud \rightarrow udd + \bar{u}\bar{d}$
 $\Delta^{++} \rightarrow p + \pi^+$ / $uuu \rightarrow uud + \bar{u}\bar{d}$
Very short decay time; quark flavour conserved.
- (a) The long decay time and change of quark flavour both indicate a weak interaction.
(b) $\Lambda^0 \rightarrow p + \pi^-$ / $uds \rightarrow uud + \bar{u}\bar{d}$ or
 $\Lambda^0 \rightarrow n + \pi^0$ / $uds \rightarrow udd + \bar{u}\bar{u}$ (or $\bar{d}\bar{d}$)
(c) Must be the second one because a π^- cannot decay into two photons (conservation of charge).
- (a) Neutron in Cl-37 is changed into a proton. Number of protons increases by 1; number of neutrons decreases by 1.
(b) $n + \nu_e \rightarrow p + e^-$ / $udd + \nu_e \rightarrow uud + e^-$
(c) Weak interaction $\therefore$ unlikely to occur.
- [Slightly unfair question!]. The reactions $n + \nu_{\mu} \rightarrow p + \mu^-$ and $n + \nu_{\tau} \rightarrow p + \tau^-$ are both possible but only if the neutrinos have enough energy. The μ^- and τ^- particles are much heavier than electrons: solar neutrinos do not have enough energy to create these particles.
- (a) X must be a meson because the total quark number cannot change. It starts off as zero [$1 + (-1)$] so the products must have 0. The π^+ has a quark number of zero, so X must also have 0. It therefore consists of a quark-antiquark pair, i.e. it is a meson. By conservation of charge it must be a π^0 .
(b) The decay is $\bar{u}s \rightarrow \bar{u}\bar{d} + u\bar{u}$ (or $\bar{d}\bar{d}$)

Component 2

Exercise 2.1

- $+1.47 \times 10^{-17} \text{ C}$ [14.7 aC]
- $3.93 \times 10^5 \text{ C}$
- 100 pA
- (a) 110 C
(b) 100 mA h = 360 C, so the label is misleading
- (a) 1.3 mA
(b) $\sim 2.0 \text{ mA}$ [from the gradient at $t = 0$]
- $t(20 - 10 \text{ mC}) = 8.3 \text{ s}$; $t(27 - 13.5 \text{ mC}) = 8.5 \text{ s}$;
 $t(12 - 6 \text{ mC}) = 8.0 \text{ s}$. $\therefore$ All very close to equal times.
 $t(24 - 6 \text{ mC}) = 19.0 - 3.0 = 16.0$. $\therefore$ $\frac{1}{2}$ -life = 8.0 s.
- (a) Ionic volume = $1.047 \times 10^{-30} \text{ m}^3$
 $\therefore N \sim \frac{3}{1.047 \times 10^{-30}} = 2.9 \times 10^{30} \text{ m}^{-3}$
(b) $2.7 \times 10^{-5} \text{ m s}^{-1}$
- 0.1 m s^{-1}

9. (a) $I_{\max} = 7.0 \text{ A} \rightarrow v_{\max} = 0.66 \text{ mm s}^{-1}$
 $\therefore$ Graph as shown



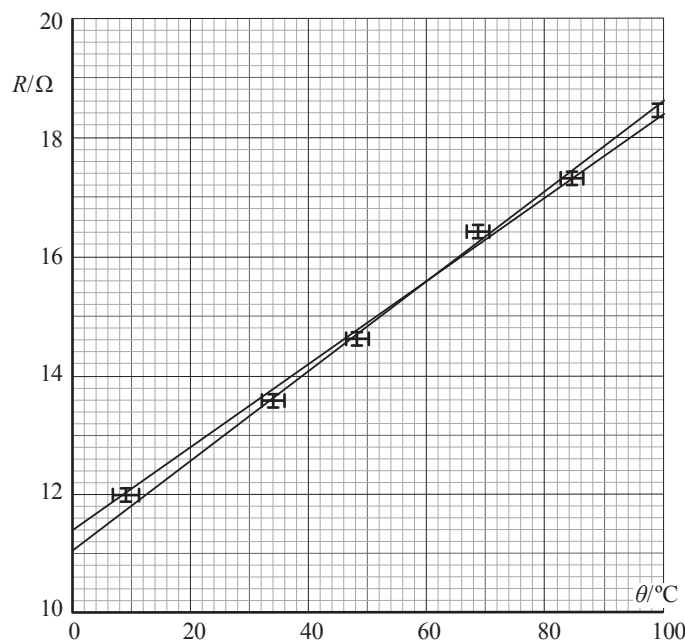
- (b) ~ 80 squares under $\frac{1}{2}$ -cycle
 Each square $= 1 \times 10^{-3} \text{ s} \times 0.05 \text{ mm s}^{-1}$
 $= 5 \times 10^{-5} \text{ mm}$
 $\therefore$ Drift distance $\sim 0.004 \text{ mm}$. ($4 \mu\text{m}$)

Comment: The electrons each travel much further than this by their own random thermal motion. The magnitude of this distance emphasises the point that the electrons in wires all move in concert.

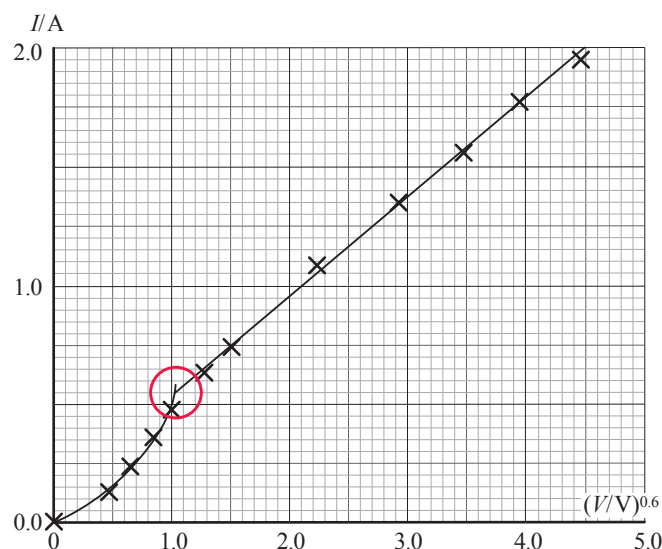
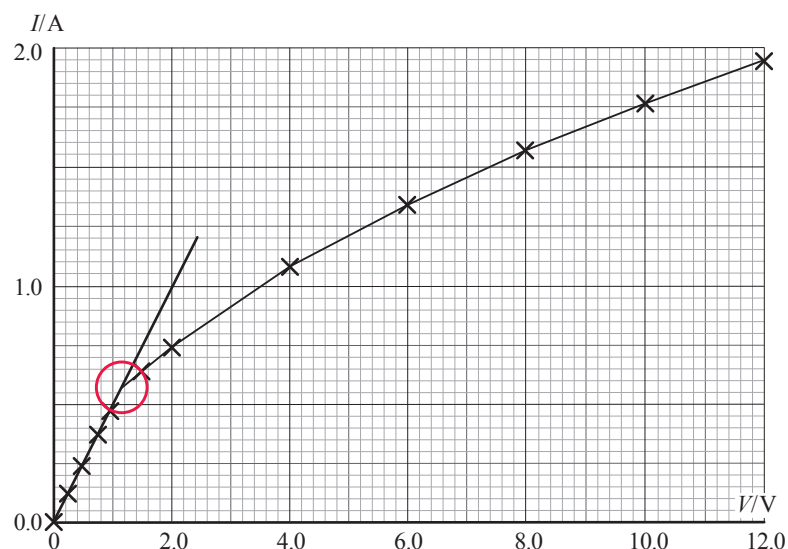
Exercise 2.2

- (a) 10.9 A
 (b) 21Ω
- (a) 2.7 kW in UK
 (b) 2.3 kW in Germany. The kettle will take $\sim 20\%$ longer to boil in Germany.
- 60 W bulb current $= \frac{P}{V} = \frac{60}{240} = 0.25 \text{ A}$. $\therefore$ Both bulbs take 0.25 A , so should operate OK in series.
Warning: Do not try this at home! If these two bulbs are connected in series to a 240 V supply and the current abruptly switched on, the torch bulb will explode. This is because the resistance of the bulbs is much less at room temperature than at the operating temperature, so the initial current is much higher. The mains bulb is designed to allow for this.
- (a) 0.8 W
 (b) 1 keV ($= 1.6 \times 10^{-16} \text{ J}$)
 (c) $1.9 \times 10^7 \text{ m s}^{-1}$
 (d) 0.76 mA [$= 95\%$ of 0.8 mA]
- (a) $1.71 \times 10^{-23} \text{ N s}$
 (b) The electrons collide with the anode, passing on their kinetic energy and raising its temperature. The power given up is 95% of $0.8 \text{ W} = 0.76 \text{ W}$,
 (c) Change of momentum of photons per second is $1.71 \times 10^{-23} \times 95\% \times 5 \times 10^{15} \text{ N} = 81 \text{ nN}$.
 By N2, this is the force exerted by the anode on the electrons. So by N3 the electrons exert an opposite force of 81 nN on the anode.
- $1.4 \times 10^6 \text{ m s}^{-1}$

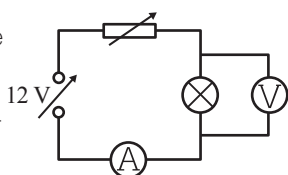
- (a) $P_E = \frac{P_N}{1 - v^2}$, where v is the fraction of the speed of light.
 $\therefore$ Rearranging: If $v = 1 - \frac{P_N^2}{P_E^2}$ If $v < 0.14$
 then $1 - \frac{P_N^2}{P_E^2} < 0.0196$, $\therefore \frac{P_N^2}{P_E^2} > 0.9804$,
 $\therefore \frac{P_N}{P_E} > 0.99$ QED.
 (b) (i) $\sim 5 \text{ kV}$ (ii) $\sim 9 \text{ MV}$
- (a) See graph. NB Ideally it should be plotted on a 10×10 grid.



- (b) $R_{0 \max} = 11.40 [\Omega]$
 $R_{0 \min} = 11.05 [\Omega]$
 $\therefore R_0 = 11.23 \pm 0.15 \Omega$
 $m_{\max} = \frac{18.60 - 11.05}{100} = 0.0755$
 $m_{\min} = \frac{18.38 - 11.40}{100} = 0.0698$
 $\therefore m = 0.073 \pm 0.003 [\Omega \text{ K}^{-1}]$
- (c) (i) Unit: $^{\circ}\text{C}^{-1}$ or K^{-1}
 (ii) $\alpha = \frac{m}{R_0} = \frac{0.073 \pm 0.003}{11.23 \pm 0.16}$
 $= (6.5 \pm 0.4) \times 10^{-3} \text{ K}^{-1}$
 This is consistent with iron.
- (d) If the graph remained linear at low temperatures, the resistance would become zero at $= -154^{\circ}\text{C}$.
 Hence the graph cannot be linear.
- (a) At low voltages the power will be very low and so the temperature will be approximately constant. Thus Ohm's law will be obeyed. At higher voltages the temperature will increase, raising the resistance, so Ohm's law will not be obeyed. The operating temperature is expected to be $\sim 2000 \text{ K}$ or more (to give a whitish emission).



- (b) The series variable resistor is only needed in the low voltage range. Below 2 V, the resistor is adjusted: increasing its resistance, decreases the current in the circuit and therefore the pd across the bulb, allowing pairs of values of V and I for $V < 2$ V to be obtained.



- (c) From the graph of I against V (above), the behaviour changes at ~ 1.2 V.
- (d) $R = \frac{2.0 \text{ V}}{0.99 \text{ A}} = 2.0 \Omega$ at low temperatures
 $R = \frac{12.0 \text{ V}}{1.95 \text{ A}} = 6.2 \Omega$ at operating voltage.
- (e) Transition voltage ~ 1.03 V. Above this the graph (above right) is almost a straight line but with a slight downward curve. Suggesting that the value of n is slightly less than 0.6.
- (f) Table of $\ln$ values of I and V .

$\ln (V/\text{V})$	$\ln (I/\text{A})$
-1.38	-2.14
-0.69	-1.45
-0.28	-1.04
0.00	-0.76
0.41	-0.45
0.69	-0.30
1.38	0.08
1.79	0.29
2.08	0.45
2.30	0.57
2.48	0.69

Using the dotted triangle: gradient $= n = \frac{1.34}{2.5} = 0.54$ (2 s.f.)

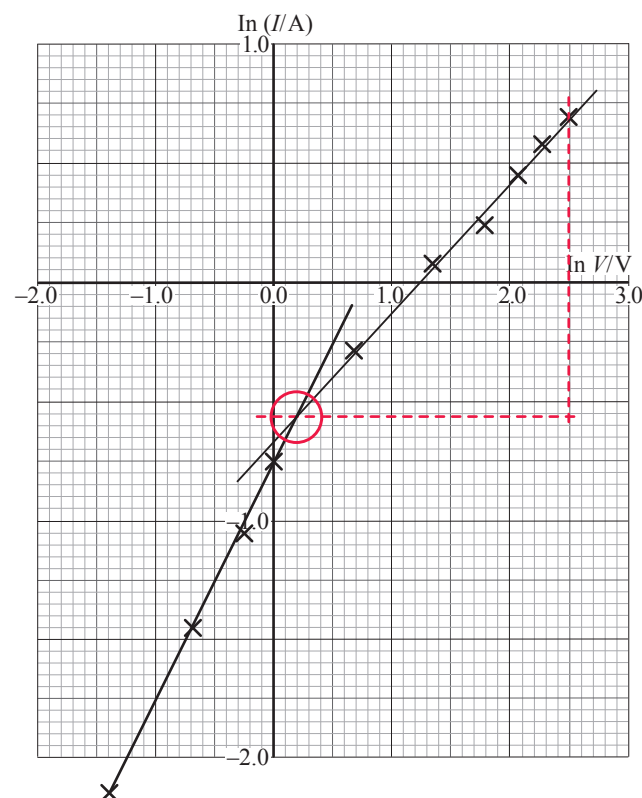
$$\therefore \ln I = \ln k + n \ln V$$

$$\therefore \ln k = \ln I \text{ intercept} = -0.66$$

$$\therefore k = 0.52, \text{ i.e. } (I/\text{A}) = 0.52 (V/\text{V})^{0.54}$$

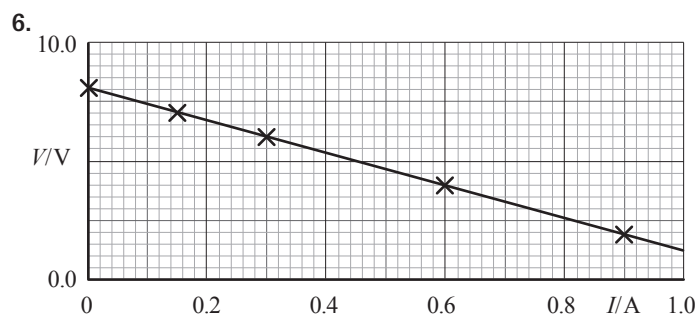
Cross-over when $\ln V = 0.16$

$$\therefore V = 1.17 \text{ V (1.2 V to 2 s.f.)}$$

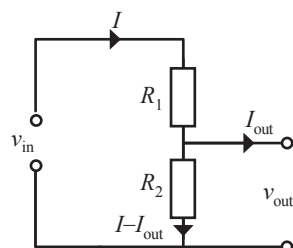


Exercise 2.3

- $I_{10} = I$, $I_{60} = \frac{1}{3}I$, $I_{30} = \frac{2}{3}I$
 - $\frac{V_{60}}{V_{10}} = 2$
 - $\frac{P_{10}}{P_{30}} = 0.75$
 - (i) 4.5 V, (ii) 0.675 W
- 160 Ω
- 10.7 m
- 8 L
 - 8 L
 - 4 L
- 8 V
 - 6 V
 - 7 V



7. pd across $R_1 = IR_1$;
pd across $R_2 = (I - I_{\text{out}})R_2$
- $$V_{\text{in}} = IR_1 + (I - I_{\text{out}})R_2 \quad [1]$$
- and $V_{\text{out}} = (I - I_{\text{out}})R_2 \quad [2]$
- From [1] $I = \frac{V_{\text{in}} + I_{\text{out}}R_2}{R_1 + R_2}$



Substituting into [2] for $I \rightarrow$

$$V_{\text{out}} = \left(\frac{V_{\text{in}} + I_{\text{out}}R_2}{R_1 + R_2} - I_{\text{out}} \right) R_2.$$

which rearranges to $V_{\text{out}} = \frac{V_{\text{in}}R_2}{R_1 + R_2} + I_{\text{out}} \frac{R_1R_2}{R_1 + R_2}.$

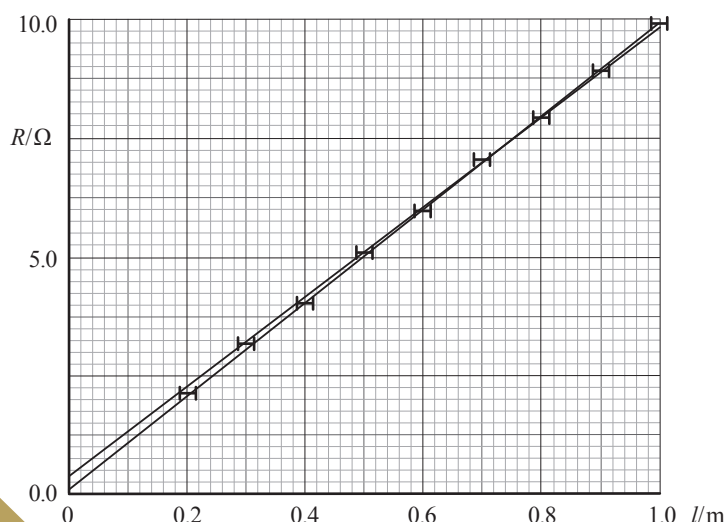
$\therefore$ By comparison with $V = E - Ir$:

Equivalent emf $= \frac{V_{\text{in}}R_2}{R_1 + R_2}$ and equivalent internal

resistance $= \frac{R_1R_2}{R_1 + R_2}.$

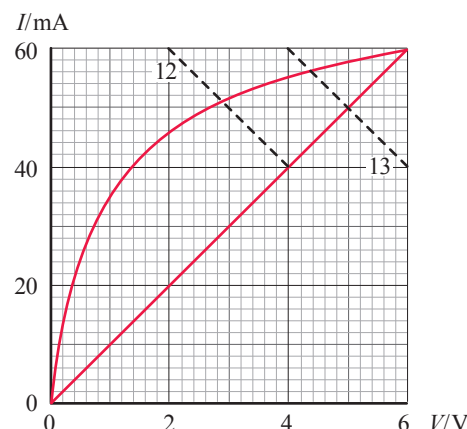
8. (a) 150Ω
(b) Red LED: 150Ω , 180Ω ;
Yellow LED: 120Ω , 150Ω , 180Ω ;
Green LED: 100Ω , 120Ω , 150Ω ;
Blue LED: 100Ω , 120Ω .

9. Gradient $= 6.50 \pm 0.03 \Omega \text{ m}^{-1}$
Intercept $= 0.02 \pm 0.01 \Omega$
 $\text{csa} = (7.77 \pm 2\%) \times 10^{-8} \text{ m}^2$
 $\rho = \frac{AR}{l} = A \times \text{gradient}$
 $\therefore \rho = (5.05 \pm 0.05) \times 10^{-7} \Omega \text{ m}$



10. (a) 100Ω , (b) 44Ω
11. (a) 6.5 V , (b) 96.0 mA

12. Method 1 (following the hint, using the additional red graph): look for the pd, V , for which the currents add to 80 mA . At 3.0 V , $I_R = 30.0 \text{ mA}$ and $I_B \sim 51 \text{ mA}$ (actually a little more) so the required voltage is a little less than 3.0 V (2.90 V ?)



13. Method 1 (as in Q12): look for the current, I , for which the pds add to 10 V . At $I = 56 \text{ mA}$, $V_R = 5.6 \text{ V}$ and $V_B = 4.4 \text{ V}$, so this is the required current.

A more elegant method for Qs 12 and 13:

12. If the current through the bulb is I_B , the current through the resistor is $0.08 - I_B$, so the pd, V_R , across the resistor is given by $V_R = 8 - 100I_B$. For I_B in mA, this gives $I_B = 80 - 10V_R$. This is the equation of the dotted line labelled 12. The required answer is where this line crossed the bulb characteristic, i.e. 2.90 V , $I_B = 51.0 \text{ mA}$, $\therefore I_R = 29.0 \text{ mA}$.
13. If the pd across the bulb is V_B , the pd across the resistor is $10 - V_B$. So the current in the bulb is $(10 - V_B)/100$. So I in mA given by $I = 100 - V_B$. This is the equation of the dotted line labelled 12. The required answer is where this line crosses the light bulb characteristic, i.e. $I = 56.0 \text{ mA}$, $V_B = 4.40 \text{ V}$, $\therefore V_R = 5.60 \text{ V}$.
14. The lower the external resistance, the lower the terminal pd. Two ways of using the data are:
- (a) Plot a graph of V against I , using the relationship $V = E - Ir$;
- (b) Plot a graph of $\frac{1}{V}$ against $\frac{1}{R}$ using the relationship $\frac{1}{V} = \frac{1}{E} + \frac{r}{E} \frac{1}{R}$.

Assume the uncertainties in V are negligible. The percentage uncertainty in the resistance values is 2% , so that is also the percentage uncertainty in I and $1/R$. The table gives the data required to draw both graphs. The graph of V against I is reproduced at the top of the next page.

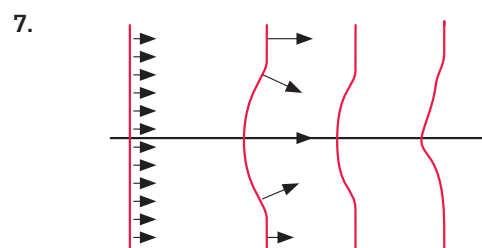
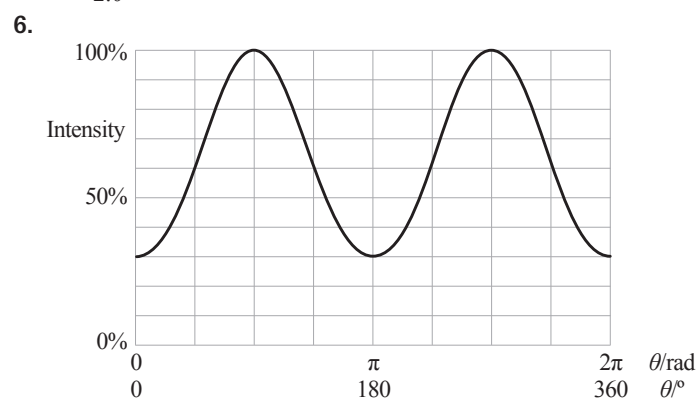
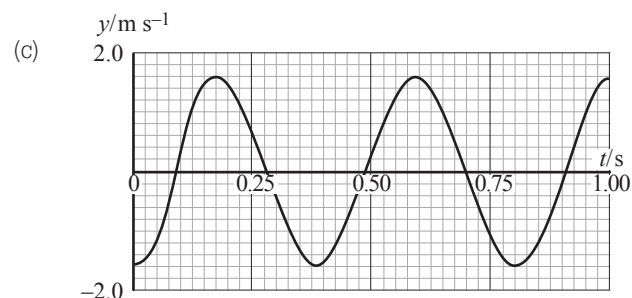
R/Ω	3.33	5.00	6.67	10.0	15.0	20.0	3.00
$I/(R/\Omega)$	0.300	0.200	0.150	0.100	0.067	0.050	0.033
$\Delta(I/R)$	0.066	0.004	0.003	0.002	0.001	0.001	0.001
V/V	6.07	6.88	7.36	7.92	8.34	8.58	8.82
$V/(V/\Omega)$	0.165	0.145	0.139	0.126	0.120	0.117	0.113
I/A	1.82	1.38	1.10	0.792	0.556	0.429	0.294
$\Delta I/\text{A}$	0.04	0.03	0.02	0.016	0.011	0.009	0.006



Exercise 2.4

- wavelength = 0.400 m; amplitude = 6.00 mm
 - The distance travelled is not known; it could be 0.08 m, 0.48 m, etc.
 - 320 m s^{-1} .
 - $f = 800 \text{ Hz}$; $T = 1.25 \text{ ms}$
- (i) 0.100 m, 0.300 m, 0.500 m, 0.700 m;
(ii) 0.200 m, 0.600 m;
(iii) 0.000 m, 0.400 m, 0.800 m.
 - Vertical speed = gradient $\times$ horizontal speed
 $\sim 25 - 30 \text{ m s}^{-1}$ (max gradient where graphs cross axis).
- (i) 0.55 m, 0.95 m, 1.35 m, 1.75 m
(ii) 0.35 m, 0.75 m, 1.15 m, 1.55 m
- (a) $\lambda = 1.00 \text{ cm}$, (b) 500 Hz, (c) 0.002 s
 - On expanded diagram: mean separation of planes = 0.88 cm. Maximum deviation from mean separation = 0.31 mm. This represents 0.029 cm on actual size diagram.
- From the information, $f = 2.5 \text{ Hz}$ and $T = 0.4 \text{ s}$.
 - Gradient = $\frac{0.20 \text{ m}}{0.64 \text{ m}}$
 $\therefore v_{\text{max}} = \frac{0.20}{0.64} \times 5.0 = 1.56 \text{ m s}^{-1}$

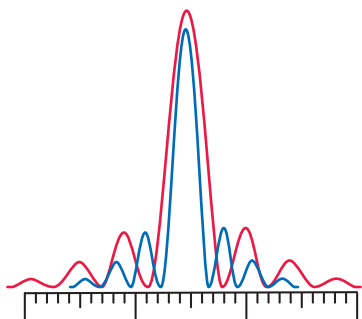
For A level candidates who have done shm:
the equation of motion of a point in the wave is given by $x = A \sin(2\pi ft + \epsilon)$. The velocity is given by $v = 2\pi f A \cos(2\pi ft - \epsilon)$, so $v_{\text{max}} = 2\pi f A = 1.57 \text{ m s}^{-1}$, in good agreement with the graphical method.



8. $A = 10 \text{ cm}$; $\omega = 1000\pi = 3140 \text{ (rad s}^{-1}\text{)}$; $\phi = \pm\pi$.

Exercise 2.5

- Diffraction
 - The point at O receives diffracted waves from both slits. The waves add by superposition. The waves are in phase because the path lengths from the two slits are equal. Because of this the waves interfere constructively, i.e. the resultant amplitude is twice the amplitude in part (a) – so the intensity is $4 \times$ the intensity in part (a).
 - As the probe moves towards P the probe continues to receive waves from both slits but the waves from the slits are increasingly out of phase because the path length from the bottom slit is greater than from the top slit. When this path difference is $\lambda/2$ the waves are exactly out of phase and so (almost) cancel each other out. Further motion in the same direction results in the waves approaching λ path difference at which point they fully reinforce again. This effect is called interference.
 - Young formula $\lambda = 2.94 \text{ cm}$; Pythagoras $\lambda = 2.76 \text{ cm}$. The Young formula which is derived for $d \gg D$ gives quite an accurate answer even when $d = 8 \text{ cm}$ and $D = 50 \text{ cm}$.
- Angles for the known wavelengths are consistent with a grating slit separation of $2.02 \times 10^{-6} \text{ m}$. Using this value, the wavelength for the mystery line is 546 nm , in good agreement.
- (a), (b) Diffraction pattern for the shorter wavelength light is more compressed. The extent of the pattern is proportional to the wavelength. The intensity scale for the 450 nm pattern is not to the same scale as the 650 nm .



- Slit width $\sim 0.33 \text{ mm}$.
- $\frac{\lambda}{4} = 188 \pm 3 \text{ mm}$ (uncertainty 1.4%)
 $\therefore v = 331 \pm 5 \text{ m s}^{-1}$
 - The equations are: $188 \pm 3 = \frac{\lambda}{4} - \epsilon$ and $576 \pm 3 = \frac{3\lambda}{4} - \epsilon$
 Subtracting and multiplying by 2 $\rightarrow \lambda = 776 \pm 12 \text{ mm} = 0.776 \pm 0.012 \text{ m}$. This gives $v = 341 \pm 5 \text{ m s}^{-1}$
 And $\epsilon = 8 \pm 6 \text{ mm}$!
 - $v = \sqrt{\frac{T}{\mu}}$, with $T = mg$ and $\mu = \frac{M}{l}$, so $v = \sqrt{\frac{mgl}{M}}$.
 For the fundamental vibration, the standing wave has
 $l = \frac{\lambda}{2}$, $\therefore \lambda = 2l$.
 $\therefore f = \frac{v}{\lambda} = \frac{1}{2} \sqrt{\frac{mg}{Ml}}$

- The slit separation, $x = 0.64 \text{ mm}$.
 - ~ 10 fringes. The interference pattern is mainly visible in the central diffraction maximum. $10 \times 2 \text{ mm}$ gives 2 cm .
 - The central maximum would be $\sim 1.6 \text{ cm}$ wide and would contain 10 fringes 1.6 mm apart. Both the diffraction and interference pattern spreads are proportional to the wavelength.
 - No interference pattern is seen but the diffraction patterns overlap to give a mixture of red and green (appearing as yellow) in the centre shading to red at the edges where the green diffraction pattern central fringe finishes.
 - The interference pattern is not affected but is half the intensity because of the absorption of the light which is polarised at right angles.
 - No interference pattern is seen. Two light beams at right angles cannot superpose to give a zero resultant. [NB. It is instructive to investigate a photon explanation for this effect.]

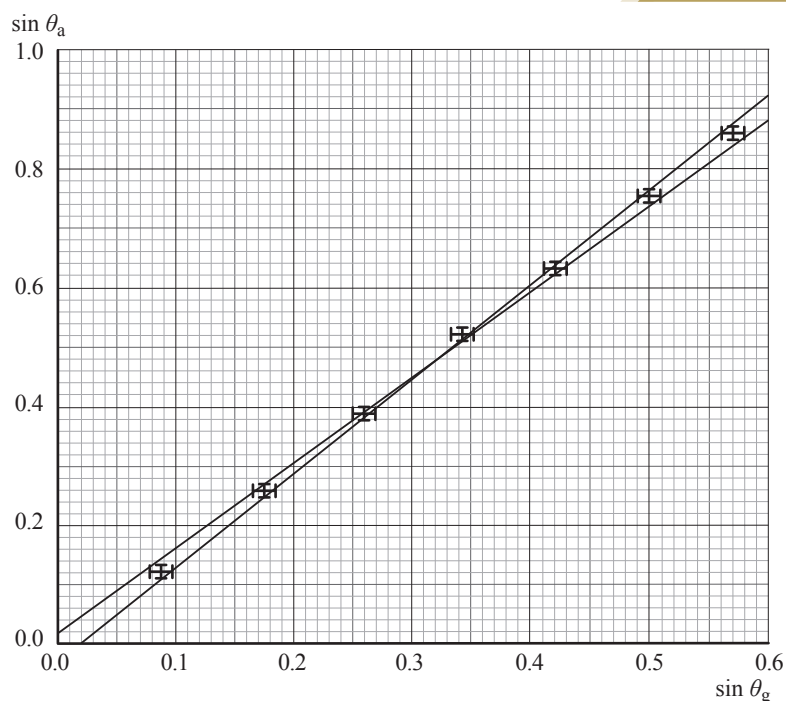
Exercise 2.6

- $f = 0.6 \text{ Hz}$
 - $v = 3 \text{ m s}^{-1}$; $\lambda = 5 \text{ m}$
 - 22.5° to normal
 - 11.0° to normal
- Ray emerges from middle of opposite face (4.02 cm from top) at 45° below the normal.
- Angles of incidence on the horizontal boundaries are respectively: 54.7° , 61.0° , 70.4° . TIR occurs at the boundary between the 1.30 and 1.20 layers, with an angle of incidence of 70.4° .
- $$\text{Refractive index} = \frac{\text{real depth}}{\text{apparent depth}}$$

$$= \frac{(15.54 \pm 0.01) - (12.52 \pm 0.01)}{(15.54 \pm 0.01) - (13.64 \pm 0.01)} = 1.59 \pm 0.03$$
- Because it relies on the accuracy of focusing, which is a judgement.
- (See graph on next page.) With these max/min lines
 $m_{\text{max}} = 1.586$
 $m_{\text{min}} = 1.435$
 $\therefore n = 1.51 \pm 0.08$
- $\lambda_r = 464 \text{ nm}$
 $\lambda_v = 261 \text{ nm}$
 - angle between red and violet rays $= 1.6^\circ$
- $c = 73.1^\circ$
 - 2.4 ns
 - Bits which are less than $\sim 3 \text{ ns}$ apart could start to overlap. $\therefore$ Max safe bit rate $\sim 10^8 \text{ bit s}^{-1}$ (0.1 Gbps)

Exercise 2.7

- $1.0 \times 10^{17} \text{ s}^{-1}$
- 21 000
- 1.5 eV
 - 1.0 eV

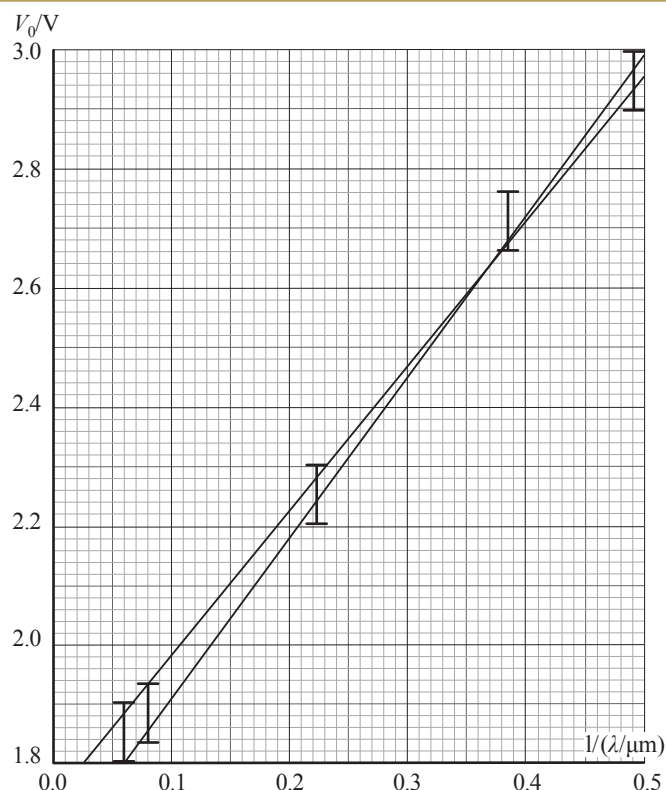


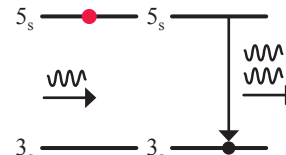
- (c) The photon energy of the 3×10^{14} Hz radiation is 2.0×10^{-19} J. This is less 2.5 eV; indeed it is less than the work function. Hence no electrons are emitted by these photons. The photon energies do not combine: photons cannot co-operate to cause photo-emission.
- (d) 1.0 V. The electrons would lose 1.0 eV with this pd.

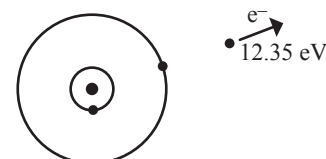
4. Assuming 400 nm – 700 nm wavelength range: 2.8×10^{-19} J (1.8 eV) – 5.0×10^{-19} J (3.1 eV).
5. The visible photons (up to 3.1 eV) are energetic enough to promote the 1s2s electrons to the 1s2p, 1s3s and 1s4s energy levels but not the 1s5s level. In falling down the energy levels only the 1s4s → 1s2s transition is in the range 1.8 eV – 3.1 eV and gives rise to a visible photon (2.1 eV, 590 nm). If some electrons are promoted to the 1s5s level either by a subsequent absorption or by UV photons, then visible photons with of energy 2.6 eV (480 nm) from the 1s5s → 1s3s transition are possible.
6. (a) The electrons behave as waves. These waves are diffracted by the gaps in the planes and the diffracted waves interfere.
- (b) 5.5×10^{-11} m.
- (c) First order: 9.4° , 0.165 rad; 5.0 cm from midpoint
Second order: 19.2° , 0.334 rad 10.4 cm from midpoint
7. $\lambda = \frac{h}{p} = \sqrt{\frac{h}{2mE_k}}$. The values of E_k are the same:
 $\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = 42.8$
8. 3.3 m s^{-1} .
9. $4.8 \times 10^5 \text{ m}^2$. [= area of a square of side ~700 m]
10. gradient = $(1.29 \pm 0.07) \times 10^{-6} \text{ V m}$
→ $h = (6.9 \pm 0.4) \times 10^{-34} \text{ J s}$.

Exercise 2.8

1. (a) An inelastic collision is one in which kinetic energy is lost.
(b) A metastable state is one which lasts a relatively long time before decaying into a lower energy state.



2. (a) 
- (b) The second photon has the same frequency and polarisation as the first and is in phase with it.
3. The 3p state needs to be shorter lived than the 5s because it needs to empty quickly, so that its population is always less than that of the 5s.
4. 633nm: 5s → 3p; 1.15 μm: 4s → 3p; 3.39 μm: 5s → 4p (shorter wavelength = greater energy difference)
5. 633 nm: visible (red); 1.15 μm and 3.39 μm: IR
6. 4p = 20.28 eV
4s = 19.85 eV
3p = 18.77 eV
3s = 16.70 eV
7. 350 nm, near UV
8. A helium atom in the excited state collides with a neon atom in the ground state. The excited electron in the helium atom transfers energy to a ground state electron in the neon, promoting it to the 5s state.
9. (a) 33 V
(b) 33 eV
(c) (see sketch)
(d) 20.65 V
10. 58 (max)

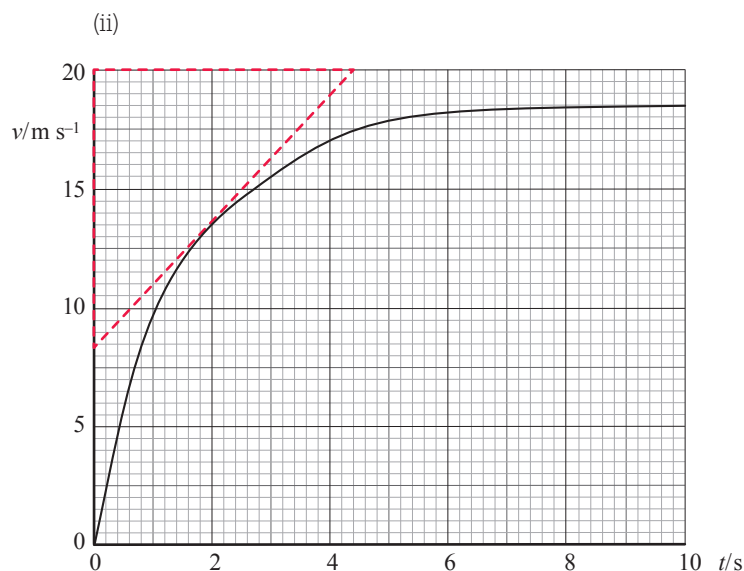


Exam practice answers

Component 1

1. (a) (i) Mean speed = $\frac{[\text{Total}] \text{ distance travelled}}{\text{time [taken]}}$
 (ii) Time taken for first 6 km = $\frac{6 \text{ km}}{90 \text{ km/h}} = 0.0667 \text{ h}$
 Minimum total time for 10 km = $\frac{10 \text{ km}}{80 \text{ km/h}} = 0.125 \text{ h}$
 $\therefore$ Minimum time for the last 4 km =
 $0.125 - 0.0667 = 0.0583 \text{ h}$
 $\therefore$ Maximum speed for last 4 km =
 $\frac{4 \text{ km}}{0.0583 \text{ h}} = 68.6 \text{ km/h}$.

- (b) (i) The resultant force steadily decreases from its initial maximum value, reaching zero at $\sim 7.0 \text{ s}$. After this time the resultant force is zero.



Acceleration at 2.0 s = gradient of tangent at 2.0 s
 $= \frac{11.6 \text{ m s}^{-1}}{4.40 \text{ s}} = 2.63 \text{ m s}^{-2}$.

$\therefore$ Resultant force, $F = ma = 350 \times 2.63 = 921 \text{ N}$

- (c) (i) Power = work done per unit time
 $= \frac{\text{work done}}{\text{time taken}} = (\text{Force} \times \text{distance}) \text{ per unit time}$
 $= \text{Force} \times \text{distance moved per unit time}$
 (for force and motion in same direction)
 $= Fv$

- (ii) $2450 \text{ W} = F \times 18.5 \text{ m s}^{-1}$
 $\therefore F_{\text{res}} = \frac{2450}{18.5} = 132 \text{ N}$

- (iii) Velocity at 2.0 s. Assuming $F_{\text{res}} \propto v$,

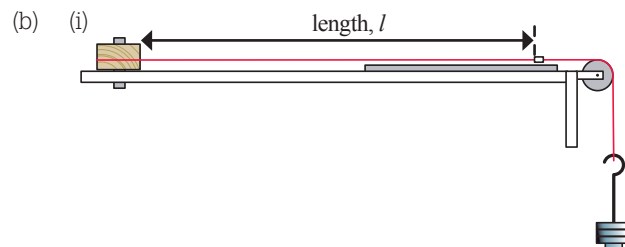
$$F_{\text{res}}(2.0 \text{ s}) = \frac{13.5}{18.5} \times 132 = 96 \text{ N}$$

$\therefore$ Driving force at 2.0 s = $921 + 96 \text{ N} = 1017 \text{ N}$

$\therefore$ Power output = $1017 \times 13.5 \text{ W} = 13.7 \text{ kW}$

- (d) Work done by braking force = decrease in kinetic energy
 $\therefore F \times 25 = \frac{1}{2} \times 350 \times 18.5^2 = 59\,900 \text{ J}$
 $\therefore$ Force = 2400 N .

2. (a) tensile stress, $\sigma = \frac{\text{tensile load}}{\text{cross-sectional area}} = \frac{mg}{\frac{1}{4}\pi d^2} = \frac{4mg}{\pi d^2}$
 tensile strain, $\epsilon = \frac{\text{increase in length}}{\text{original length}} = \frac{\Delta l}{l_0}$
 Young modulus, $E = \frac{\sigma}{\epsilon}$
 $\therefore E = \frac{4mg}{\pi d^2} \times \frac{l_0}{\Delta l} = \frac{4mgl_0}{\pi d^2 \Delta l}$
 $\therefore$ (re-arranging) $\Delta l = \frac{4l_0 mg}{\pi d^2 E}$ OED



- (ii) $mg = A\sigma$, $\therefore m = \frac{\pi \times (0.3 \times 10^{-2})^2 \times 250 \times 10^6}{4 \times 9.81} = 1.8 \text{ kg}$

- (iii) For this length of wire, using $\sigma = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$:

$\epsilon = \frac{\sigma}{E}$, $\therefore$ the maximum extension, $\Delta l_{\text{max}} =$

$$\frac{l_0 \sigma}{E} = \frac{4.0 \times 250 \times 10^6}{200 \times 10^9} = 0.005 \text{ m} = 5 \text{ mm}$$

If the mm scale is read by the naked eye, the best uncertainty is $\pm 0.5 \text{ mm}$, giving a percentage uncertainty of 10%.

If the diameter is measured using a digital micrometer, the uncertainty is $\pm 0.01 \text{ mm}$, giving a percentage uncertainty of 3%. This is squared in the calculation.

Hence the minimum percentage uncertainty in the value of E is $\sim 16\%$, which is rather poor. If Δl is measured using a travelling microscope to $\pm 0.01 \text{ mm}$, the total percentage uncertainty could be as low as 8%.

(iv) I. The extension Δl is proportional to l_0 so the percentage uncertainty in Δl will be lower.

II. The extension is inversely proportional to, A , the cross-sectional area of the wire. However, the maximum load is also inversely to A so these effects cancel out and reducing the diameter of the wire has no effect on the uncertainty in the value of E . If we consider the uncertainty in the value of A itself, the smaller the diameter the greater the percentage uncertainty in d and hence A [$p_A = 2p_d$]. This will increase the uncertainty in E .

$$(c) p_d = \frac{0.005}{0.272} = 0.018; p_l = \frac{2}{1535} = 0.001 \text{ (which is negligible)}$$

$$p_{\text{gradient}} = \frac{0.06}{1.27} = 0.047$$

$$E = \frac{4l_0g}{\pi d^2} \times \frac{1}{\text{gradient}} = \frac{4 \times 1.535 \times 9.81}{\pi \times (0.272 \times 10^{-3})^2} \times \frac{1}{1.27 \times 10^{-3}} = 2.04 \times 10^{11} \text{ Pa}$$

$$p_E = 2 \times 0.018 + 0.001 + 0.047 = 0.084, \\ \therefore \Delta E = \pm 0.084 \times 204 \text{ GPa} = \pm 17 \text{ GPa}$$

$\therefore$ The Young modulus, $E = 200 \pm 20 \text{ GPa}$ [could also be expressed as $204 \pm 17 \text{ GPa}$]

3. (a) (i) $\lambda_{\text{max}} = 420 \text{ nm}$ from the graph.

$$\therefore \text{Using Wien's law: } T = \frac{2.898 \times 10^{-3} \text{ m K}}{420 \text{ nm}} = 6900 \text{ K} \sim 7000 \text{ K}$$

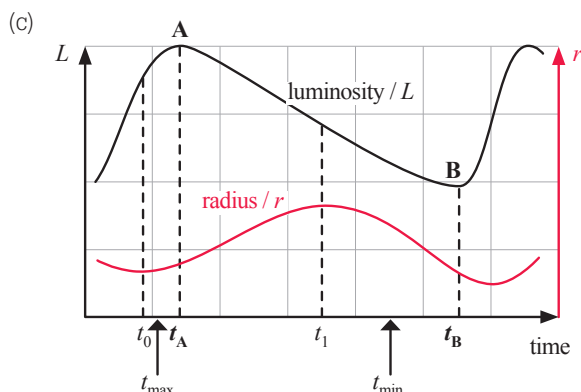
$$(ii) T = \frac{2.898 \times 10^{-3} \text{ m K}}{530 \text{ nm}} = 5470 \text{ K}$$

(iii) In the visible range [$\sim 400 - 700 \text{ nm}$] the spectral intensity is approximately constant at the lower temperature, giving $\delta \text{ Cep}$ a white appearance. At the higher temperature, the intensity graph is skewed towards the low wavelength end giving it a bluish colour.

- (b) Surface area, $A = 4\pi r^2 = \pi d^2$.

$$\text{From Stefan's law: } L = \pi d^2 \sigma T^4$$

$$\therefore d = \frac{1}{T^2} \sqrt{\frac{L}{\pi\sigma}} = \frac{1}{6900^2} \sqrt{\frac{1.46 \times 10^{30}}{5.67 \times 10^{-8} \pi}} = 6.0 \times 10^{10} \text{ m [60 million km]}$$



- (i) Around t_0 the radius is roughly constant (it is the minimum of the radius curve) and the luminosity is rising. This implies that the temperature is rising.

Around t_1 , the radius is again roughly constant but the luminosity is falling, implying that the temperature is falling.

- (ii) At point **A** on the luminosity graph, i.e. at time t_A , the luminosity is roughly constant but the radius is increasing, suggesting that the temperature is falling. Thus the maximum temperature must be in between t_0 and t_A . At point **B** on the luminosity graph, i.e. time t_B , the luminosity is again roughly constant but the radius is decreasing suggesting that the temperature is rising. Thus t_{min} must be between t_1 and t_B .

4. (a) The electron and the electron neutrino are fundamental particles – leptons.

The proton and the π^- meson are hadrons: they are composed of quarks. The composition of the proton is **uud** (two up-quarks and one down-quark); the π^- meson is composed of $\bar{u}d$ (anti-up quark and down quark).

- (b) (i) Only one of the protons in the ${}^7_4\text{Be}$ nucleus is involved. One of its up quarks is turned into a down quark, transforming the proton into a neutron.

$$\text{hence: } u + e^- \rightarrow d + \nu_e$$

$$\text{Or, including the other quarks: } uud + e^- \rightarrow udd + \nu_e$$

- (ii) The weak interaction is involved:

1: There is a change of quark flavour (u to d).

2: A neutrino is involved and neutrinos only feel the weak force.

- (iii) Conservation of charge is illustrated. Taking the $u + e^- \rightarrow d + \nu_e$ form of the reaction, the total charge on the left is $\frac{2}{3}e + (-e) = -\frac{1}{3}e$; on the right, the total charge is $-\frac{1}{3}e + 0 = -\frac{1}{3}e$.

Conservation of quark number: one quark on each side [or three if considering the form of the equation: $uud + e^- \rightarrow udd + \nu_e$]. Alternatively, baryon number is conserved: one baryon on each side of the equation.

- (c) The mass gain in the reaction is:

$$36.966\,77 \text{ u} - 36.965\,90 \text{ u} = 0.00087 \text{ u}$$

$$\text{Converting to SI units, } \Delta m = 0.00087 \times 1.67 \times 10^{-27} \text{ kg} = 1.45 \times 10^{-30} \text{ kg}$$

$$\text{The energy required to create this } \Delta m \text{ is } 1.45 \times 10^{30} \times (3.00 \times 10^8)^2 = 1.31 \times 10^{-13} \text{ J}$$

Hence the neutrinos in part (b) cannot be detected by this means.

5. (a) For an initial velocity (u) of 0 the equations become:

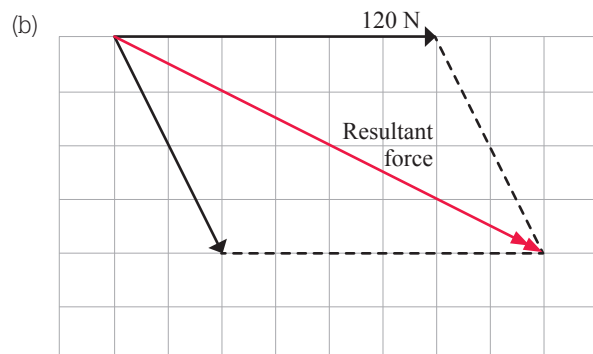
$$v = at \quad [1]$$

$$\text{and } x = \frac{v}{2}t \quad [2]$$

$$\text{Substituting for } v \text{ from [1] into [2]} \rightarrow x = \frac{at}{2}t, \\ \therefore x = \frac{1}{2}at^2 \text{ QED}$$

$$\text{From [1] } t = \frac{v}{a}. \text{ Substitute into [2] for } t \rightarrow x = \frac{v}{2} \frac{v}{a} = \frac{v^2}{2a}$$

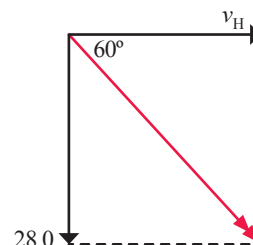
$$\text{Rearranging } a \rightarrow 2ax = v^2, \text{ i.e. } v^2 = 2ax \text{ QED}$$



- (i) The scale is 1 cm = 20 N
The resultant force is 179 N [180 N] at 26.6° [27°] to the 120 N force.
- (ii) Vector quantities.

- (c) The vertical component of the velocity when the stone hits the ground is given by

$$v_v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 40} = 28.0 \text{ m s}^{-1}$$



The horizontal projection velocity v_H is given by:

$$\tan 60^\circ = \frac{28.0}{v_H}$$

$$\therefore v_H = \frac{28.0}{\tan 60^\circ} = 16.2 \text{ m s}^{-1}$$

Component 2

1. (a) Metals conduct electricity because they possess free electrons [a.k.a. conduction electrons or delocalised electrons] which are free to move through the lattice of positive metal ions. They possess resistance because the conduction electrons interact with the lattice ions, colliding with them and transferring energy. The higher the temperature, the stronger the vibrations of the lattice ions and the stronger the interaction between the conduction electrons and the ions. If a p.d. is applied the effect of the increased interaction strength is to reduce the drift velocity of the electrons, reducing the current and therefore increasing the resistance.
- (b) (i) The thermometer is in the water, which is unlikely to be at exactly the same temperature as the copper coil. When the water is being heated, the temperature of the coil is likely to be lower than that of the water; when it is cooling down, the converse is true. Hence taking the mean of heating and cooling resistances is likely to achieve a value which more accurately reflects the true resistance at that temperature.
- (ii) Including the thermometer in the test tube, alongside the coil, would help because the temperature reading should accurately reflect the temperature of the coil.
- (iii) See labelled graph
- (iv) It is possible to draw straight lines through all the error bars, so the data are consistent with a linear relationship between R and θ , which can be expressed in the form: $R = R_0 (1 + \alpha\theta)$

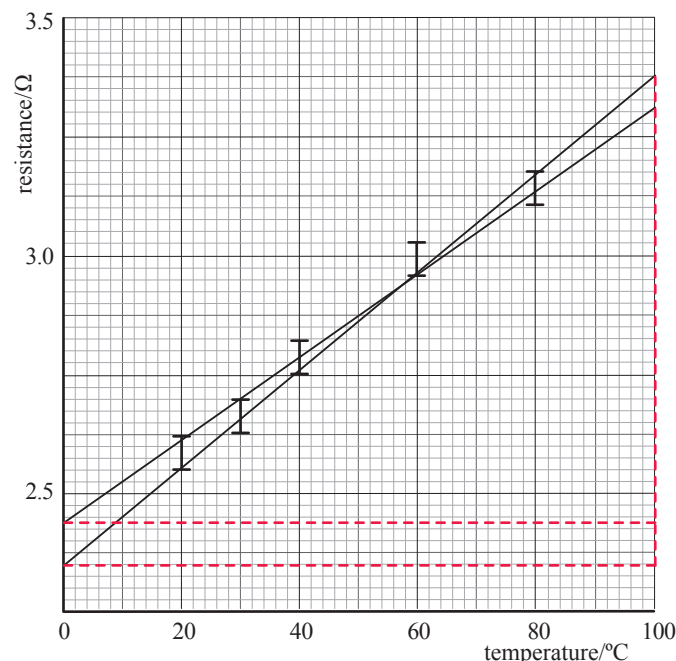
From the graph: $R(0^\circ)$ range = $2.35 \Omega - 2.44 \Omega$

and $R(100^\circ)$ range = $3.31 \Omega - 3.38 \Omega$

$$\therefore R_0 = \frac{2.44 + 2.35}{2} \pm \frac{2.44 - 2.35}{2} = 2.40 \pm 0.05 \Omega$$

$$\text{max gradient} = \frac{3.38 - 2.35}{100} = 0.0103 \Omega \text{ K}^{-1}$$

$$\text{and min gradient} = \frac{3.31 - 2.44}{100} = 0.0087 \Omega \text{ K}^{-1}$$



$$R_0\alpha = \text{gradient} = 0.095 \pm 0.0008 \Omega \text{ K}^{-1}$$

$$\therefore \alpha = \frac{0.095}{2.40} = 3.96 \times 10^{-3} \text{ K}^{-1}$$

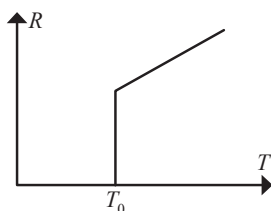
$$P_{R0} = \frac{0.05}{2.40} = 0.021; P_{\text{grad}} = \frac{0.0008}{0.0095} = 0.084$$

$$\therefore P_\alpha = 0.021 + 0.084 = 0.105$$

$$\text{Absolute uncertainty in } \alpha = 0.105 \times 3.96 \times 10^{-3} = 0.4 \times 10^{-3} \text{ K}^{-1} \text{ (to 1 s.f.)}$$

$$\alpha = (4.0 \pm 0.4 \times 10^{-3}) \text{ K}^{-1}$$

- (c) (i) The resistance in such materials decreases with temperature. As the temperature is reduced below the superconducting transition temperature (T_0), all electrical resistance disappears, as shown in the graph.



- (ii) The most common use for superconductors is in the coils of electromagnets, in MRI machines or in particle accelerators, such as the LHC in CERN.

The advantage of using superconductors is that large currents are possible without energy losses through ohmic heating in the coils. As well as the energy costs, the heat generated would necessitate additional resources to remove it.

2. (a) The principle of superposition states that, whenever two waves arrive at (or pass through) a point, the total displacement produced by the waves is equal to the vector sum of the displacements produced by the individual waves.

- (b) (i) The fringe separation, $\Delta y = \frac{16.0 \text{ mm}}{8}$.

$$\lambda = \frac{a\Delta y}{D} \therefore \alpha = \frac{\lambda D}{\Delta y} = \frac{650 \times 10^{-9} \times 2.00}{2.0 \times 10^{-3}} = 0.00065 \text{ m} = 0.65 \text{ mm}$$

- (ii) For the pattern to be observed, the beams of light from the two slits must overlap, so that the light received on the screen is from both slits. The slits need to be narrow so that significant diffraction occurs at the slits. If the slits are close together and the screen is distance the two diffracted beams overlap, allowing interference between the beams to occur.

The path difference between the light from the two slits and a point on the screen needs to be only a small number of wavelengths of light – and this path difference must change by a whole wavelength ($\sim 650 \text{ nm}$) if the point on the screen changes by $\sim 1 \text{ mm}$. This is only possible if the distance from the slits to the screen is large.

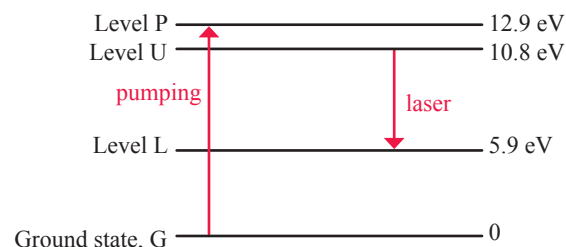
- (iii) The angle through which the light diffracts at the slits is proportional to the wavelength, so the width of the central part of the diffracted beam would be less. The separation between the fringes is also proportional to the wavelength, so the same number of fringes would be observed in this narrower pattern: the 9 fringes would occupy $\sim 12 \text{ mm}$ rather than 16 mm .

- (c) (i) Wavelength in mica = $\frac{650}{1.5} = 430 \text{ nm}$.

$$\text{Thickness of mica} = \frac{0.1 \text{ mm}}{430 \text{ nm}} \sim 230 \text{ wavelengths.}$$

- (ii) The results of the superposition at a point of the waves from the two slits depends upon their phase when they reach that point. The waves which pass through the mica have passed through more wavelengths (~ 230) than the ones which pass through air (~ 150). Because of this the pattern is shifted by ~ 80 fringes. It is unlikely that this difference will be a whole number of fringes so the pattern will be observed to move.

3. (a) (i)



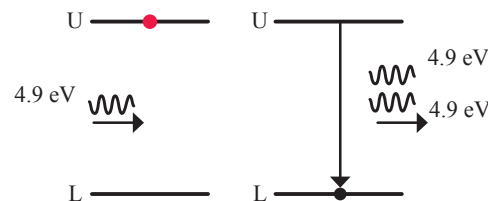
- (ii) The photon energy is 12.9 eV . To cause the upward transition, the photon energy must exactly match the difference in the energies of the two levels. This is 12.9 eV .

$$\text{(iii) Laser photon energy} = 10.8 - 5.9 \text{ eV} = 4.9 \text{ eV} = 7.84 \times 10^{-19} \text{ J.}$$

$$\therefore \lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{7.84 \times 10^{-19}} = 253 \text{ nm.}$$

- (iv) When electrons are pumped to level P, they drop down very quickly to level U, where they stay for a long time. Any that drop down to level L spontaneously, very quickly leave this level. Because of this the population of level U, caused by the pumping, is higher than level L, i.e. there exists a population inversion.

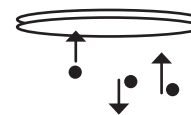
Once the population inversion is established, a 4.9 eV photon, e.g. from a spontaneous emission between U and L, which is incident on an atom in level U will cause the atom to drop to level L by stimulated emission (see diagram).



This process results in two photons, where previously there was one, i.e. amplification has occurred.

- (b) (i) The weight of the spheres

$$\begin{aligned} &= \frac{4}{3}\pi r^2 \rho g \\ &= \frac{4}{3}\pi (0.05 \times 10^{-3})^3 \times 1.5 \times 10^3 \times 9.81 \\ &= 7.70 \times 10^{-9} \text{ N} \end{aligned}$$



Photon momentum = $\frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c}$, where E is the photon energy.

$$\therefore \text{Momentum change per photon on collision} = -2 \frac{E}{c}$$

If intensity of beam = I , then the number of collision per second on a disc of area A is $\frac{IA}{E}$.

$$\therefore \text{The momentum change of the photons per second} = -2 \frac{IA}{c}$$

i.e. by N2 and N3 the force exerted on the disc = $2 \frac{IA}{c}$

$$\therefore 2 \frac{I \times \pi (0.05 \times 10^{-3})^2}{3.00 \times 10^8} = 7.70 \times 10^{-9}$$

$$\therefore I = 1.5 \times 10^8 \text{ W m}^{-2}.$$

- (ii) Intensity of sunlight at the Sun's surface =
 $\sigma T^4 = 5.67 \times 10^{-8} \times 6000^4 = 0.73 \times 10^8 \text{ W m}^{-2}$

i.e. the laser intensity must be twice the intensity of sunlight at the Sun's surface.

- (iii) Even a small percentage absorption of the laser photons will cause massive heating of the spheres and therefore lead to their evaporation.

4. (a) (i) The potential difference, V , between two points is the energy transfer per unit charge when a charge is moved between the two points.

(ii) $I = \frac{Q}{\Delta t} = \frac{1.2 \times 10^{-6}}{0.8 \times 10^{-3}} = 0.0015 \text{ A (1.5 mA)}.$

(iii) Mean pd = $\frac{1}{2} \times 120 \text{ kV} = 60 \text{ kV}$

$$\therefore \text{Energy transfer} = 60 \text{ kV} \times 1.2 \mu\text{C} = 72 \text{ mJ (0.072 J)}$$

- (b) (i) The pd across the 10Ω resistor = $0.3 \times 10 = 3.0 \text{ V}.$

$$\therefore \text{The pd across the } 15 \Omega \text{ resistor} = 3.0 \text{ V}$$

$$\therefore \text{The current in the } 15 \Omega \text{ resistor} = \frac{3.0}{15} = 0.20 \text{ A}$$

$$\therefore \text{Current in the } 12 \Omega \text{ resistor} = 0.30 + 0.20 = 0.50 \text{ A}$$

$$\therefore \text{pd across the } 12 \Omega \text{ resistor} = 0.50 \times 12 = 6.0 \text{ V}$$

$$\therefore \text{pd across the battery} = \text{pd across } 12 \Omega \text{ resistor} \\ + \text{pd across } 10 \Omega \text{ resistor} = 6.0 + 3.0 = 9.0 \text{ V}.$$

Note:

There are other ways of tackling this, e.g.

- the pd across the 15Ω resistor = 3.0 V (as above)
- the effective resistance of the $10 \Omega / 15 \Omega$ combination = 6Ω
- by ratios the pd across the 12Ω resistor = $2 \times 3.0 = 6.0 \text{ V}$, etc.

- (ii) Every second the energy transferred to the external circuit = $IV = 0.5 \text{ A} \times 9.0 \text{ V} = 4.5 \text{ W}.$

$\frac{2}{3}$ of this is transferred in the 12Ω resistor, i.e.

3.0 W . Of the remaining 1.5 W , $\frac{2}{5}$ is transferred in the 15Ω resistor (0.6 W) and the remainder (0.9 W) in the 10Ω resistor. This power is transferred as heat to the surroundings.

The energy transferred in the internal resistance per second = $I^2 r = 0.5^2 \times 2 = 0.5 \text{ W}.$

The energy transferred from chemical energy in the battery per second is $4.5 \text{ W} + 0.5 \text{ W} = 5.0 \text{ W}$

- (iii) The total resistance in the circuit increases because of the removal of an additional path, so the total current in the circuit decreases. The pd across the cell increases as a consequence of $V = E - Ir$ and the pd across the 12Ω resistor also decreases because of the reduced current. The pd across the 10Ω resistor increases because the terminal pd of the battery increases and the pd across the 12Ω resistor decreases.

Specification map

Book			Specification	
page	Section	Section title		
10	1.1.1	Quantities and units	1.1	a b
12	1.1.2	Checking equations for homogeneity	1.1	c
12	1.1.3	Scalar and vector quantities	1.1	d e
15	1.1.4	Working with vectors	1.1	f
16	1.1.5	Density	1.1	g
17	1.1.6	Moments of forces	1.1	h i j
20	1.1.7	Conditions for equilibrium	1.1	k
24	1.2.1	Speed and velocity	1.2	a b
26	1.2.2	Acceleration	1.2	a b c
28	1.2.3	Uniform acceleration equations	1.2	d e
31	1.2.4	Projectiles	1.2	e f g
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38	1.3.2	Elastic and inelastic collisions	1.3	f
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43	1.3.7	Force and acceleration	1.3	c
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