GCSE

Mathematics: Rules of Indices

Rules of indices can help us to simplify algebraic expressions but we can also use them to help us to simplify or calculate answers to some numerical problems involving indices.

Check first that you can:

- square and cube numbers e.g. $7^2 = 49$, $5^3 = 125$
- find square roots and cubed roots e.g. $\sqrt{81} = 9$, $\sqrt[3]{64} = 4$
- understand/use indices e.g. $3^4 = 3 \times 3 \times 3 \times 3$
- find the reciprocal of a number e.g. the reciprocal of 9 is $\frac{1}{9}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ (or $1\frac{1}{2}$)

Multiplying numbers with powers

When we multiply the same base number the powers are added.

$a^m \times a^n = a^{m+n}$

e.g. Simplify

1) $3^4 \times 3^3 = 3^{3+4} = 3^7$

2) $2^{-2} \times 2^6 = 2^{-2+6} = 2^4$

Dividing numbers with powers

When we divide the same base number the powers are subtracted

$a^m \div a^n = a^{m-n}$

e.g. Simplify

1) $8^7 \div 8^5 = 8^{7-5} = 8^2$

2) $5^3 \div 5^{-6} = 5^{3-(-6)} = 5^9$

Raising a power to a power

When we raise a power to another power the powers are multiplied.

$(a^m)^n = a^{mn}$

e.g. Simplify

1) $(4^5)^2 = 4^{5\times2} = 4^{10}$

2) $(9^3)^4 = 9^{3\times4} = 9^{12}$

Using more than one rule

e.g. Simplify

1) $\frac{3^5 \times 3^6}{3^7} = \frac{3^{11}}{3^7} = 3^4$

2) $\frac{(8^3)^4}{8^9 \times 8^{-3}} = \frac{8^{12}}{8^6} = 8^6$

Remember

- The rules for multiplication and division only work when the base number is the same.
- That $a^1 = a$ and $a^0 = 1$.

Negative powers

Negative powers are used to represent reciprocals i.e. 1 ÷ by the number.

The reciprocal of 2 can be written as $2^{-1} = \frac{1}{2}$

The reciprocal of 2^2 can be written as $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

The reciprocal of 2^3 can be written as $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Therefore, for negative powers: $a^{-n} = \frac{1}{a^n}$

E.g. Calculate the following

1) $7^{-1} = \frac{1}{7}$

2) $\frac{1}{5}^{-1} = \frac{5}{2} = 2\frac{1}{2}$

3) $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

4) $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

5) $5^{-4} = \frac{1}{5^4} = \frac{1}{625}$

6) $10^{-5} = \frac{1}{10^5} = \frac{1}{100\,000}$

Fractional powers and roots

Fractional powers are used to represent roots.

The square root of 4 can be written as $4^{\frac{1}{2}} = \sqrt{4} = 2$

The cubed root of 8 can be written as $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

Therefore, for fractional powers: $a^{\frac{1}{n}} = \sqrt[n]{a}$

Fractional powers are also used to show a root raised to a power: $a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$

E.g. Evaluate the following

1) $144^{\frac{1}{2}} = \sqrt{144} = 12$

2) $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

3) $81^{\frac{1}{4}} = \sqrt[4]{81} = 3$

4) $125^{\frac{2}{3}} = (\sqrt[3]{125})^2 = 5^2 = 25$

5) $32^{\frac{3}{5}} = (\sqrt[5]{32})^3 = 2^3 = 8$

6) $36^{\frac{3}{2}} = (\sqrt{36})^3 = 6^3 = 216$

Numbers with negative and fractional powers

We can combine the rules for negative and fractional powers to give $a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$ and also $a^{-\frac{m}{n}} = \frac{1}{(\sqrt[n]{a})^m}$

E.g. Calculate the following

1) $25^{\frac{-1}{2}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$

2) $1000^{\frac{-1}{3}} = \frac{1}{\sqrt[3]{1000}} = \frac{1}{10}$

3) $64^{\frac{-1}{6}} = \frac{1}{\sqrt[6]{64}} = \frac{1}{2}$

4) $27^{\frac{-2}{3}} = \frac{1}{(\sqrt[3]{27})^2} = \frac{1}{3^2} = \frac{1}{9}$

5) $32^{\frac{-4}{5}} = \frac{1}{(\sqrt[5]{32})^4} = \frac{1}{2^4} = \frac{1}{16}$

6) $1000^{\frac{-4}{3}} = \frac{1}{(\sqrt[3]{1000})^4} = \frac{1}{10^4} = \frac{1}{10\,000}$

Remember to check the question. Does it ask you to *simplify* and write your answer in index form or does it ask you to *evaluate* or *calculate* the answer?