The nth term

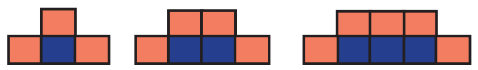
A sequence is a number pattern that follows a rule. We can find any term in a sequence using the nth term rule and knowing the position of the term we want within the sequence i.e 1st ($n = 1$), 2nd ($n = 2$), 3rd ($n = 3$)... 50th ($n = 50$).

Check that you can:

- recognise simple number patterns
- find the next term in a number pattern using the simple term-to-term rule

  e.g. 3, 7, 11, 15, 19, 23

  +4 +4 +4 +4 +4
- draw the next diagram in a pattern
- substitute values into expressions.



Finding the nth term rule – Linear sequences

Linear sequences The difference between the terms of a linear sequence is always the same.

Example Find the nth term for the following sequences.

a) 3, 6, 9, 12, 15...

n	1	2	3	4	5
term	3	6	9	12	15

+3 +3 +3 +3

The difference is +3.
Multiplying n (the position) by 3 gives the term. nth term = $3 \times n$

nth term = $3n$

b) 5, 10, 15, 20, 25...

n	1	2	3	4	5
term	5	10	14	20	25

+5 +5 +5 +5

The difference is +5.
Multiplying n (the position) by 5 gives the term. nth term = $5 \times n$

nth term = $5n$

c) −2, −4, −6, −8, −10...

n	1	2	3	4	5
term	−2	−4	−6	−8	−10

−2 −2 −2 −2

The difference is −2.
Multiplying n (the position) by −2 gives the term. nth term = $-2 \times n$

nth term = $-2n$

d) 7, 10, 13, 16, 19...

n	1	2	3	4	5
term	7	10	13	16	19

+3 +3 +3 +3

The difference is +3 (as in the sequence above).
The nth term rule starts with $3n$.
Comparing the sequence with $3n$ we see each term has moved up four places (+4).

nth term = $3n + 4$

e) 2, 7, 12, 17, 22...

n	1	2	3	4	5
term	2	7	12	17	22

+5 +5 +5 +5

The difference is +5 (as in the sequence above).
The nth term rule starts with $5n$.
Comparing the sequence with $5n$ we see each term has moved down three places (−3).

nth term = $5n - 3$

f) 4, 2, 0, −2, −4...

n	1	2	3	4	5
term	4	2	0	−2	−4

-2 -2 -2 -2

The difference is −2 (as in the sequence above). The nth term rule starts with $-2n$.
Comparing the sequence with $-2n$ we see each term has moved up six places (+6) from $-2n$.

nth term = $-2n + 6$

Using the nth term rule

Finding terms within a linear sequence

1) Find the first three terms of the sequence with nth term = $3n - 4$.

If n = 1 then $3 \times 1 - 4 = \mathbf{-1}$

If n = 2 then $3 \times 2 - 4 = \mathbf{2}$

If n = 3 then $3 \times 3 - 4 = \mathbf{5}$

−1, 2, 5, ...

2) Find the 100th term of the sequence with nth term $= -4n + 8$.

If n = 100 then $-4 \times 100 + 8 = \mathbf{-392}$

100th term = −392

Determining if a term is in a sequence

Determine if 254 is a term in the sequence nth term = $3n - 2$.

Write and solve an equation using the term and the nth term rule. If n (the term's position) is a whole number, then the term will be part of the sequence.

$3n - 2 = 254$

$3n = 256$

$n = \frac{256}{3} = 85\frac{1}{3}$ therefore 254 cannot be in the sequence

Finding terms within a quadratic sequence

1) Find the first three terms of the sequence with nth term = $n^2 + 5$.

If n = 1 then $1^2 + 5 = \mathbf{6}$

If n = 2 then $2^2 + 5 = \mathbf{9}$

If n = 3 then $3^2 + 5 = \mathbf{14}$

6, 9, 14, ...

2) Find the first three terms of the sequence with nth term = $2n^2 - 1$.

If n = 1 then $2 \times 1^2 - 1 = \mathbf{1}$

If n = 2 then $2 \times 2^2 - 4 = \mathbf{7}$

If n = 3 then $2 \times 3^2 - 1 = \mathbf{17}$

1, 7, 17, ...

Finding the nth term rule – Quadratic sequences

Quadratic sequence The nth term rule for a quadratic sequence will contain n^2 and this will be the highest power of n.

The first difference between each term of a quadratic sequence changes (although it will follow a pattern) so we the look at the second difference, which stays the same.

If the second difference is $2a$ then the sequence starts with an^2.
E.g. if the second difference is 2, the sequence starts with n^2. If the second difference is 4, the sequence starts with $2n^2$

Example Find the nth term for the following sequences.

a) 3, 6, 11, 18, 27...

+3 +5 +7 +9

+2 +2 +2

1) The second difference is +2 so the sequence starts with n^2.

2) Draw a table and compare n^2 with the sequence.

n	1	2	3	4	5
n^2	1	4	9	16	25
term	3	6	11	18	27

3) Each term has moved up two places (+2) from n^2.

nth term = $n^2 + 2$

b) −1, 8, 23, 44, 71...

+9 +15 +21 +27

+6 +6 +6

1) The second difference is +6 so the sequence starts with $3n^2$.

2) Draw a table and compare $3n^2$ with the sequence.

n	1	2	3	4	5
$3n^2$	3	12	27	48	75
term	−1	8	23	44	71

3) Each term has moved down four places (−4) from $3n^2$.

nth term = $3n^2 - 4$

c)

1	2	3	4	5
10	17	28	43	62

+7 +11 +15 +19

+4 +4 +4

1) The second difference is +4 so the sequence starts with $2n^2$.
2) Draw a table and compare $2n^2$ with the sequence (subtract $2n^2$ from the term).

n	1	2	3	4	5
$2n^2$	2	8	18	32	50
term	10	17	28	43	62
term − $2n^2$	8	9	10	11	12

3) The difference is a linear sequence (bn+c). Solve it the same way as the method on the left.

n	1	2	3	4	5
Linear term	8	9	10	11	12

+1 +1 +1 +1

4) The difference is +1. This is b.
Multiply n by +1.
5) Compare the linear term with $+1n$.
The term has moved +7 places from $+1n$.
c = +7.
The linear term is $n + 7$.
The final quadratic sequence is $2n^2 + n + 7$.

Don't forget to check your nth term rule by substituting the n values back into your rule in order to get the terms.

Remember that n is the position of the term within the sequence so it's this 'position' that we substitute into our rule in order to find the term.