Number patterns and sequences

A sequence is a number pattern that follows a rule. We can find any term in a sequence using the *n*th term rule and knowing the position of the term we want within the sequence, i.e. 1st ($n = 1$), 2nd ($n = 2$), 3rd ($n = 3$)... 50th ($n = 50$).

Check that you can:

- *recognise simple patterns of numbers and pictures*
- *add, subtract, multiply and divide positive and negative whole numbers*
- *remember your times tables.*

The term-to-term rule

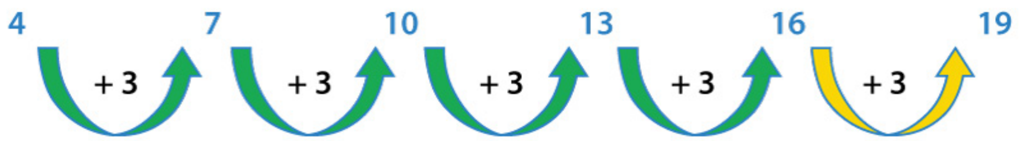
4	7	10	13	16

This list of numbers is called a **sequence**. A sequence is a set of numbers or symbols which follows a pattern or rule.

Each member of the sequence is called a **term**.

Example

Look again at the same number sequence. In this sequence, to go from one term to the next you add 3 to the previous term. 'Adding 3' to the previous term is the term-to-term rule for this sequence. You can use the term-to-term rule to continue the sequence. Here, $16 + 3 = 19$. So, the next term is 19.



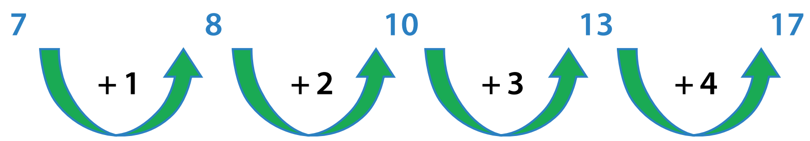
Example

Here, the term-to-term rule is 'subtract 6' from the previous term.

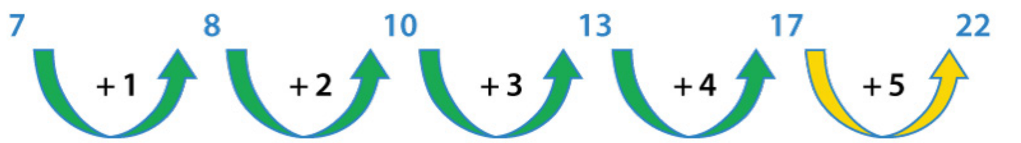
The term-to-term rule can involve lots of different operations including **adding**, **subtracting**, **multiplying** or **dividing**.

Beware – you should check more than one pair of terms to be sure of the term-to-term rule!

Example



For the term-to-term rule, the number being added increases by 1 each time.



Here, $17 + 5 = 22$. So, the next term is 22.

The *n*th term rule

The term-to-term rule is useful for listing the terms in a sequence in order. But it could mean listing a lot of terms if you require the term in a particular position, such as the 200th term.

The *n*th term rule takes you straight to the term in any position, without having to list all the previous terms.

n is the position number.

You can substitute a value for n to find the term in any position.

The *n*th term rule is also known as the **position-to-term rule.**

Example

We start with the rule of a sequence, e.g. $3n + 1$

The n here denotes the position in the sequence. For example, if we substitute 1 in as n, this will give us the value of the first number in the sequence.

We can generate the sequence using this rule. For the first 5 terms of the sequence, we substitute the numbers 1 to 5 into the rule.

$3(1) + 1 = 4$
$3(2) + 1 = 7$
$3(3) + 1 = 10$
$3(4) + 1 = 13$
$3(5) + 1 = 16$

So, the sequence looks like this: 4, 7, 10, 13, 16...

This sequence can be continued by adding 3 to the previous number.

If we wanted to find the 30th term of this sequence, we don't need to list all 30 terms by manually adding 3 to the previous term until we have 30 terms. We can just substitute in 30 in place of n in the rule of the sequence.

$3(30) + 1 = 91$

The 30th term is 91.

We would do the same to find the 100th term:

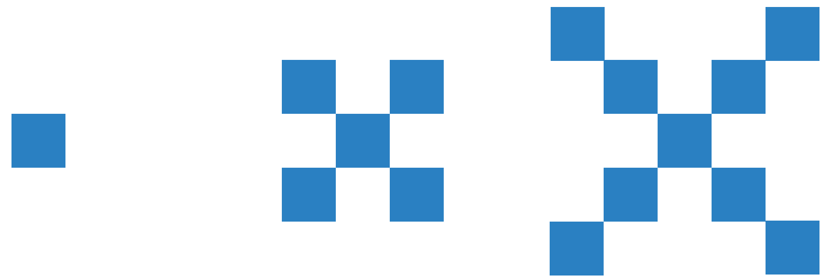
$3(100) + 1 = 301.$

Picture sequences

Sometimes a sequence is the result of drawing patterns which follow rules. You can use the sequence of patterns to write a sequence of numbers. Then, you can consider the term-to-term rule.

Example

For the sequence of patterns shown below:

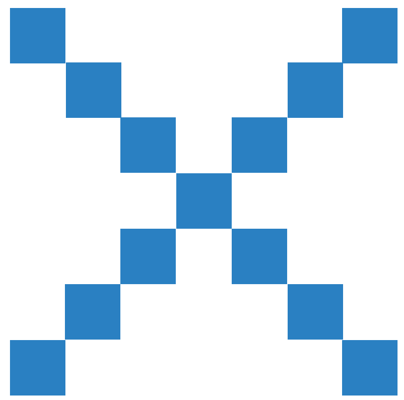


Pattern 1 Pattern 2 Pattern 3

a) Draw pattern 4.
b) Write the number of tiles in each pattern as a sequence.
c) Write down the rule for finding the next term in the sequence.

Answer

a)



b) 1, 5, 9, 13
c) Add four squares, (one at each corner) to the previous pattern.

REMEMBER! When moving between positive and negative numbers, it might help to use a number line.