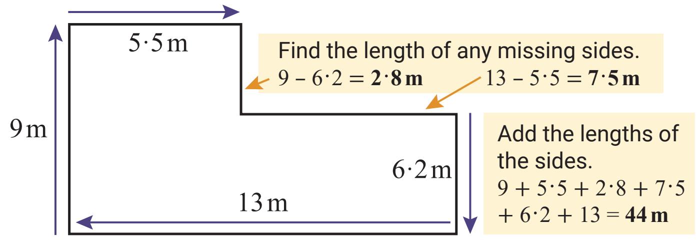
Area and perimeter, dimensions and volume

Understanding the difference between perimeter, area and volume. Calculating the perimeter, area and volume of different 2-D and 3-D shapes.

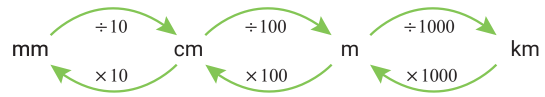
Perimeter

This measures the distance around the outline of a shape.



Perimeter is measured in units as mm, cm, m...

Converting between measurements of length

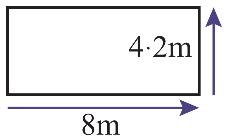


Area

Area This measures the amount of space within a 2-D shape.

Area of a Rectangle

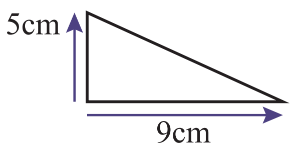
Area of Rectangle = length × width



$8 \times 4{\cdot}2 = 33{\cdot}6\,m^2$

Area of a Triangle

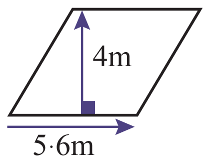
$\textit{Area of triangle} = \frac{1}{2} \times \textit{base} \times \textit{height}$



$\frac{1}{2} \times 9 \times 5 = 22{\cdot}5\,cm^2$

Area of a Parallelogram

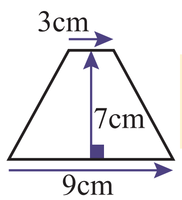
Area of a parallelogram = base × height



$5{\cdot}6 \times 4 = 22{\cdot}4\,m^2$

Area of a Trapezium

$\textit{Area of a trapezium} = \frac{1}{2}(a + b) \times \textit{height}$



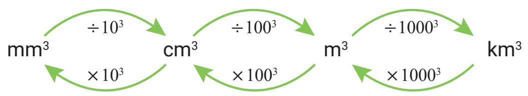
$\frac{1}{2} \times (3 + 9) \times 7$
$= \frac{1}{2} \times 12 \times 7$
$= 42\,cm^2$

Area is measured in units^2
e.g. mm^2, cm^2, m^2 ...

Converting between measurements of area

E.g. Convert 480 mm^2 to cm^2.

$480 \div 10^2 = 480 \div 100$
$= 4{\cdot}8\,cm^2$



Circumference of a circle

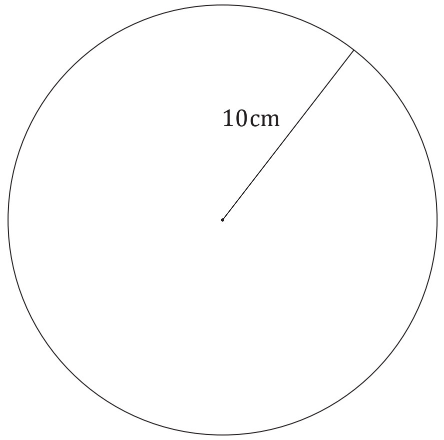
The perimeter of a circle is called the **circumference**.
Using C to represent circumference, and d to represent diameter, we write the formula for the circumference of a circle as:

$$C = \pi d$$

Since the diameter is twice as long as the radius ($d = 2r$), we can also write the formula for the circumference:

$$C = 2\pi r$$

We can use π in our calculators, or we can use an estimate of 3·14.



$C = 2\pi r$
$= 2 \times 3{\cdot}14 \times 10$
$= 62{\cdot}8\,cm$

Area of a Circle

Using A to represent area and r to represent radius, the formula for the area of a circle is:

$$A = \pi r^2$$

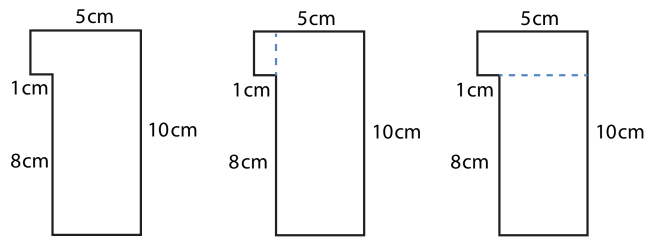
$A = \pi r^2$
$= 3{\cdot}14 \times 10^2$
$= 3{\cdot}14\,cm^2$

Area of compound shapes

To calculate the area of a compound shape, we need to cut the shape up into its different parts. The area of the shape then comes from adding the areas of these different parts.

Example

The shape on the left can be cut into two rectangles in two different ways. We can either make a vertical cut or a horizontal cut.

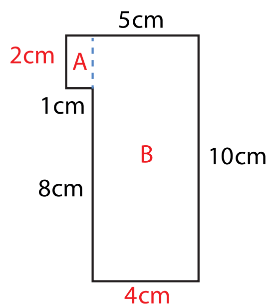


To calculate the area using the vertical cut we can label the two shapes A and B. Calculate any missing sides, then calculate the area.

Area A = 1 × 2
= 2 cm^2

Area B = 4 × 10
= 40 cm^2

Total area = A + B
= 2 + 40
= 42 cm^2



Check that you can:

- *recognize 2-D shapes and their properties*
- *recognize 3-D shapes and their properties*
- *draw nets of 3-D shapes*
- *substitute values into a formula*
- *understand the relationship between units of the metric system.*

REMEMBER!

Don't forget to use the information you are given to calculate the lengths of any missing sides.

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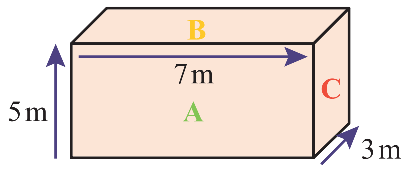
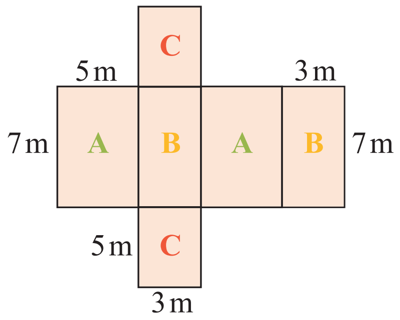
Check that you can:

- *calculate the area of different 2-D shapes.*

Surface Area This is the total area of every surface (face) of a 3-D shape.



It is useful to consider the net of the 3-D shape, to find the area of each face, in order to calculate the total surface area.



Area of A $= 7 \times 5 = 35\,\text{m}^2$

Area of B $= 7 \times 3 = 21\,\text{m}^2$

Area of C $= 3 \times 5 = 15\,\text{m}^2$

Total surface area
$= 2 \times (35 + 21 + 15)$
$= 142\,\text{m}^2$

REMEMBER!

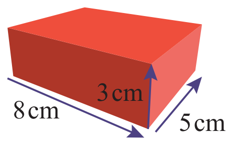
- Take care to include every side of the shape when calculating surface area.
- Draw a sketch of the sides you are using for your calculation, and draw on the dimensions you need.

Volume This measures the amount of space within a 3D shape.

Volume of a Cuboid

Volume of a cuboid = length × width × height

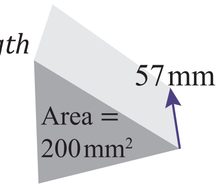
$= 8 \times 5 \times 3$
$= 120\,\text{cm}^2$



Volume of a Prism

Volume of a prism = area of the cross-section × length

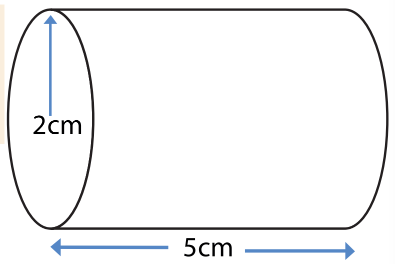
$= 200 \times 57$
$= 11400\,\text{mm}^3$



Volume of a Cylinder

Volume of a cylinder = area of the cross-section × length

$= \pi r^2 \times l$
$= 3{\cdot}14 \times 2^2 \times 5$
$= 62{\cdot}8\,\text{cm}^2$

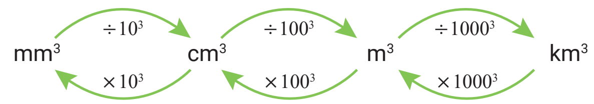


Volume is measured in units³ e.g. mm³, cm³, m³ ...

Converting between measurements of volume

E.g. Convert 5·5 mm³ to cm³.

$5{\cdot}5 \times 100^3 = 5{\cdot}5 \times 1\,000\,000$
$= 5\,500\,000\,\text{cm}^3$



Remember to check the measurements are in the same units before adding or multiplying to find perimeter, area or volume.

Dimensions and formulae

We can decide if a formula is for a perimeter, area or volume by looking at the dimensions involved.
We know that:

- **perimeter** is a measurement of length (L): 1 dimension
- **area** is given by a length × length (L^2): 2 dimensions
- **volume** is given by a length × length × length (L^3): 3 dimensions.

Deciding if a formula is length, area or volume formula

Example 1:

Perimeter of a rectangle $= 2 \times \text{length} + 2 \times \text{width}$
$= 2l + 2w$

Remember, numbers have no dimension.
So, the expression $2l + 2w$ is simply a length and has one dimension.

Example 2:

Volume of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

Here we have number × length × length, so the expression $\frac{1}{2}bh$ is an area and has two dimensions.

Example 3:

Volume of a cylinder $= \pi r^2 h$

Here we have a number, (π), multiplying an area, (r^2), and a length, (h). Now number × area × length is the same as number × length³ and so is a volume. The expression $\pi r^2 h$ represents a volume and so has three dimensions.

We don't need to recognise the formula to decide if it is used to calculate a length, an area or a volume. We just need to work out how many dimensions it has.

Example 4: $d^2 + hr$

length × length   length × length

So, we have area + area, which means this is an **area formula**.

Mixed dimensions

We can add a length to a length or an area to an area but it's not possible to mix dimensions. We can't add a length to an area or an area to a volume, therefore a formula will never contain a mixture of dimensions.

Example 5: $6rh + 5h^2r$

length × length   length × length × length

So, we have area + volume which means this is not a formula for length, area or volume (none of these).