Area and perimeter, dimensions and volume

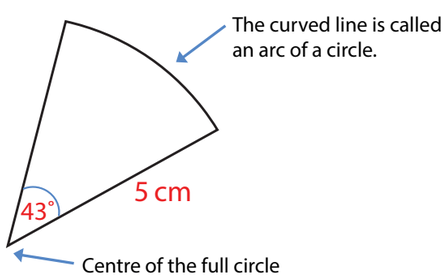
Building upon knowledge of circumference and area of a circle to consider arcs, sectors and segments. Calculating volume and surface area of more complex 3-D shapes.

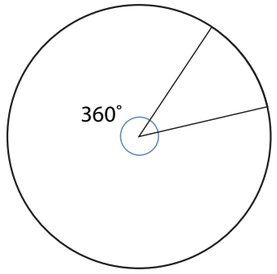
Check that you can:

- *calculate the area of more simple 2-D and 3-D shapes*
- *calculate the circumference and area of a circle.*

Length of an arc of a circle

A fraction of a circle, like this one, is called a **sector of a circle**. The curved line is called an **arc of a circle**. If we were looking at a full circle, this curved line would be the circumference.





This sector is a fraction of a whole circle. If the angle at the centre is 43°, and the angle at the centre of a circle is 360°, then the fraction of the circle is $\frac{43}{360^\circ}$.

To calculate an arc length, we simply need to calculate the relevant fraction of the circumference.

The formula for arc length is: $\text{Arc length} = \frac{\text{sector angle}}{360^\circ} \times \pi d$

Example

The arc length of this sector, given a radius of 5 cm ($d = 2r$) is:

$$\text{Arc length} = \frac{43}{360} \times \pi \times 10$$

$$= 3{\cdot}75\,\text{cm (to 2 d.p.)}$$

Area of a sector of a circle

We can work out the area of the sector in a similar manner, knowing that the sector is a fraction of the whole circle. If the sector is $\frac{43}{360}$ of a circle, the area of this sector is $\frac{43}{360}$ of the area of a full circle.

The formula for the area of a sector is: $\text{Sector Area} = \frac{\text{sector angle}}{360^\circ} \times \pi r^2$.

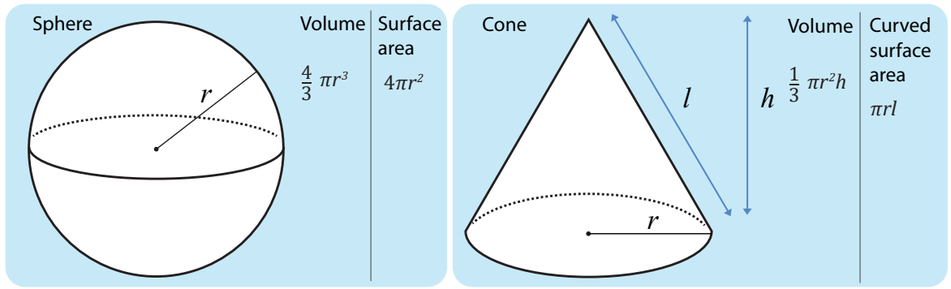
Example

The area of this sector, given a radius of 5 cm is:

$$\text{Sector Area} = \frac{43}{360} \times \pi \times 5^2$$

$$= 9{\cdot}38\,\text{cm}^2\text{ (to 2 d.p.)}$$

Volume and surface area of a sphere or cone



Example 1

Calculate the volume and surface area or a sphere with a radius of 5 cm.

Answer

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times 5^3$$

$$= 523{\cdot}6\text{ cm}^3\text{ to 1.d.p}$$

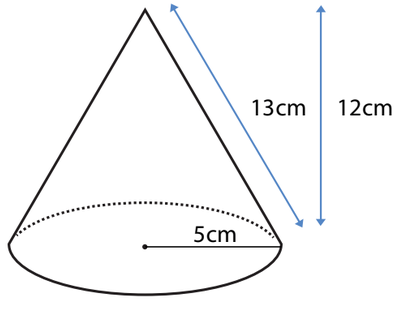
$$\text{Surface area} = 4\pi r^2$$

$$= 4 \times \pi \times 5^2$$

$$= 314\,\text{cm}^2\text{ to nearest cm}$$

Example 2

A solid cone is shown on the right. Calculate its volume and total surface area. Give your answers as multiples of π.



Answer

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 5^2 \times 12$$

$$= 100\pi\,\text{cm}^3$$

$$\text{Total surface area} = \text{curved surface} + \text{area of the base}$$

$$= \pi rl + \pi r^2$$

$$= \pi \times 5 \times 13 + \pi \times 5^2 = 90\pi\,\text{cm}^2$$

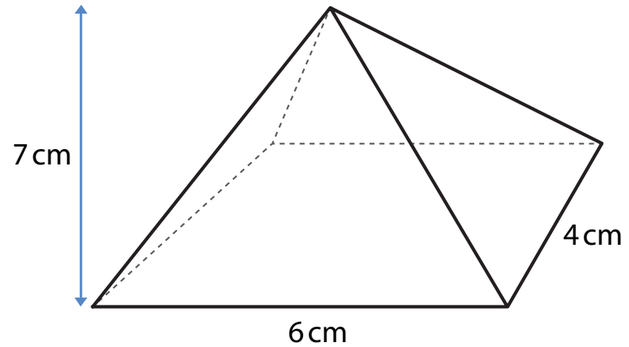
Volume of a Pyramid

The volume of **any** pyramid is given by the formula:

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$$

Example

Calculate the volume of this rectangular-based pyramid.



$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$$

$$= \frac{1}{3} \times (6 \times 4) \times 7$$

$$= 56\,\text{cm}^3$$

REMEMBER!
You can leave your answer in terms of π like in example 2.