TRANSFORMATIONS – CHANGING THE POSITION

Transformations is the name given to the group of rules that change the size or position of an object.

Translations, reflections and rotations change the object's position, but not the size.

Check that you can:

- *draw a reflection of a simple shape in a mirror line*
- *understand coordinates*
- *can draw simple straight-line graphs such as $x = 3$, $y = -2$, $y = x$ and $y = -x$*
- *understand the difference between clockwise and anti-clockwise*
- *understand that 90° = ¼ turn, 180° = ½ turn, 270° = ¾ turn and 360° = 1 whole turn.*

Translations

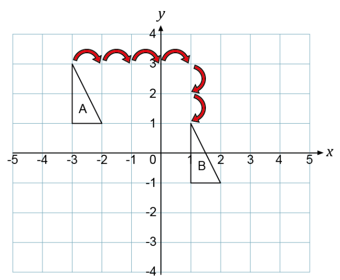
When a shape is translated, it is moved left or right, up or down, usually on a coordinate grid. The shape stays the same size and is not reflected or rotated in any way.

Example
Describe the translation that moves shape A to shape B.

Answer
Consider the vertex at the top of shape A and the vertex at the top of shape B.

From its position on shape A, it has moved **four units to the right and two units down**. This is the answer.
We always state the x-direction movement first, followed by the y-direction movement.

We would have arrived at the same answer had we considered either of the other two vertices of shape A. Moving four units to the right and two units down takes us to the corresponding vertex of shape B.

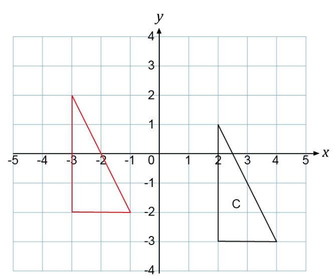


We can also translate a shape if we are given the x-direction movement and the y-direction movement.

Example
Translate shape C by five units left and one unit up.

Answer
The answer is shown in red in this diagram.
Either:

- translate each vertex of shape C by five units to the left and one up, **OR**
- translate one vertex of shape C by five units to the left and one up, and then draw a congruent shape to C, in the same orientation, from that point.

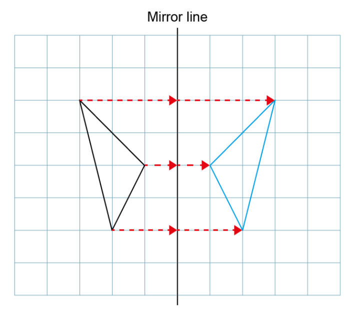


Reflections

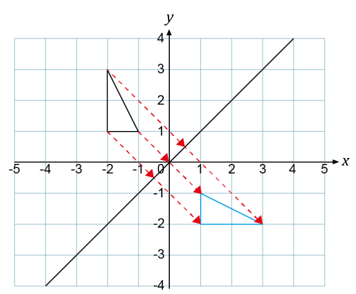
When a shape is reflected, a mirror image of the shape is drawn about a given mirror line. Sometimes, you will need to use your knowledge of the equations of simple horizontal, vertical and sloping straight lines to determine where on a diagram the mirror line is.

When reflecting a shape in any mirror line, the technique is the same as shown below.

First, consider each vertex of the shape. Count how many squares each vertex is away from the mirror line. Always count in the direction that is opposite to the direction of the mirror line. Here, the mirror line is a vertical line, so we count in a horizontal direction. The vertices of the reflection will be an equal number of squares away from the mirror line, but on the other side.



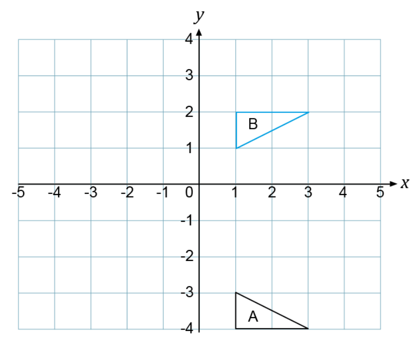
This technique also works for sloping mirror lines. Here, the black triangle is reflected in the line $y = x$ to give the blue triangle.



Another method of drawing the reflection is to use tracing paper (but this is probably more time consuming and could lead to slight errors).

Sometimes, you will need to describe a reflection that maps a shape onto another.

Example
Here we are asked to describe the reflection that maps shape A onto shape B.

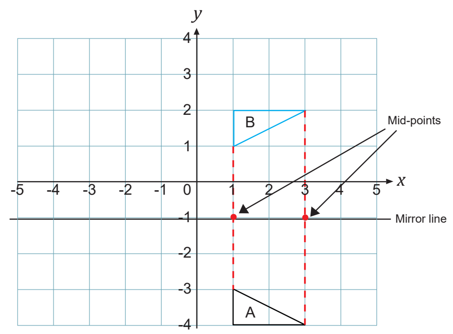


Answer
To describe a reflection, we need to find the mirror line and name it.

To do this you need to:

- connect two pairs of corresponding vertices of both shapes with straight lines
- find the mid-points of these two straight lines
- and the mirror line will be the line that connects these mid-points.

This is a reflection about the line $y = -1$.



REMEMBER! The origin is the point (0, 0) on a coordinate grid.

Rotations

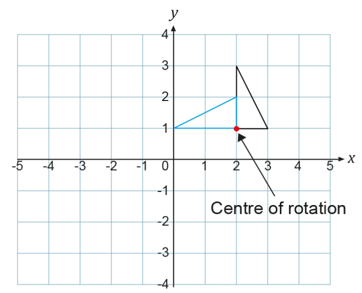
When you are asked to rotate a shape:

- the rotated shape will be the same size as the original shape
- the original shape will be rotated either clockwise or anti-clockwise by 90°, 180° or 270°
- the rotation will be about a point on the grid called the centre of rotation.

Here, the black triangle has been rotated 90° anti-clockwise about the point $(2, 1)$, to give the blue triangle.

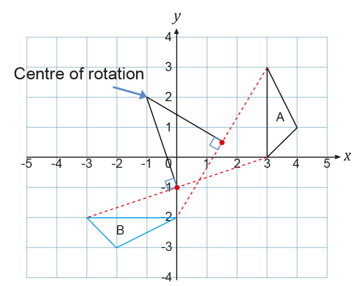
Tracing paper is usually used to perform a rotation. The steps are shown below.

- Trace over the shape and the centre of rotation.
- Fix the tracing paper at the centre of rotation using a pencil or the pin of a pair of compasses.
- Rotate the tracing paper by the desired number of degrees in the direction specified.
- The tracing paper will now show you the orientation and the position of the rotation required.



Describing rotations and finding the centre of rotation
How to find the centre of rotation:

- Draw the perpendicular bisector for two pairs of corresponding vertices.
- One method to draw these perpendicular bisectors is to start by drawing lines connecting two pairs of corresponding vertices, and then marking the mid-point of each of these lines.
- Now draw lines at 90° to these lines, going through the mid-points.
- Where these new lines cross will be the centre of rotation.



Shape A has been rotated 90° clockwise about (–1, 2). **You can check this is correct by using tracing paper to perform the rotation.**

TRANSFORMATIONS – CHANGING THE SIZE

Transformations is the name given to the group of rules that change the size or position of an object.

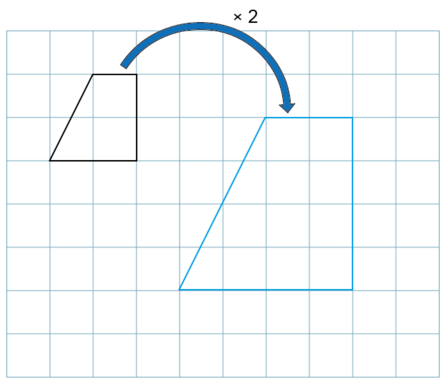
Enlargements change the size of an object. The enlarged object will usually be in a different position from the original object. This position will be determined by the centre of enlargement and the scale of the enlargement.

Enlargements

When a shape is enlarged, the resulting shape will be bigger or smaller than the original shape.

- The length of each side of the original shape is multiplied by a **scale factor** to give the length of each side of the enlargement.
- When the scale factor is **greater than** 1, the enlargement will be **bigger** than the original shape.
- When the scale factor is **less than** 1, the enlargement will be **smaller** than the original shape.

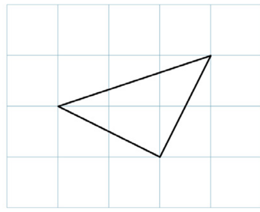
Example 1
The black shape below has been enlarged by a scale factor of 2 to make the blue shape.



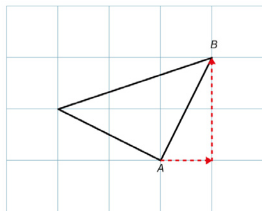
To achieve this, the length of each side of the shape was multiplied by 2.

In practice, it is easier to start by enlarging the horizontal and vertical lines. Once these have been drawn, simply closing the shape will ensure that the sloping line has been doubled in size correctly.

Example 2
This shape needs to be enlarged by a scale factor of 3.

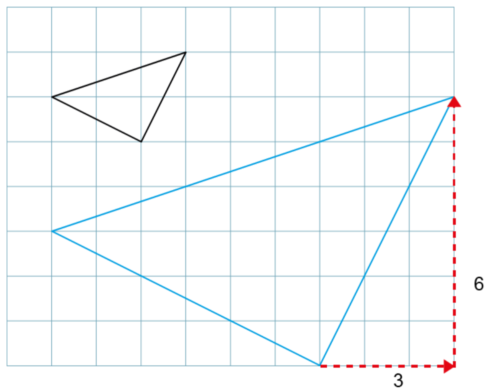


This is a more difficult shape to enlarge as we do not have any horizontal or vertical lines. We can use the grid on which the shape has been drawn to help us enlarge the length of each of the sides.



Two of the vertices have been labelled *A* and *B*. From *A*, if you move one square to the right and two squares up you arrive at *B*. On the enlargement, the corresponding movements will need to be three times as much as these, i.e. three squares to the right and six squares up.

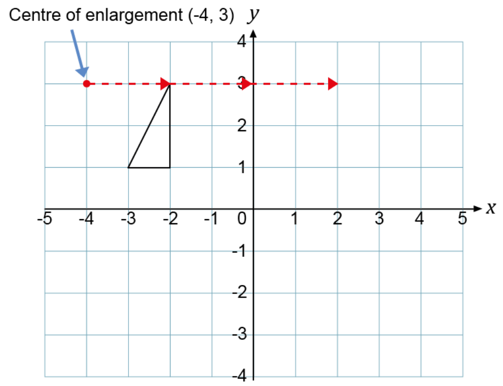
Using a similar method for the third vertex will ensure a correct enlargement of the lengths of all the sides.



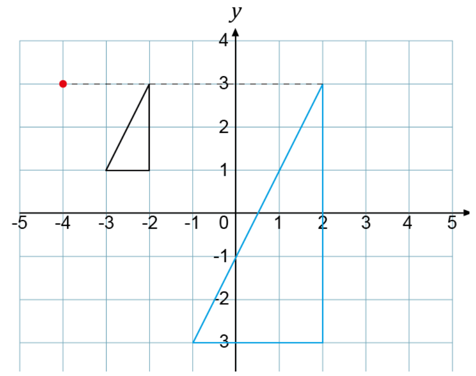
Example 3
Enlarge the shape by a scale factor of 3 about the centre of enlargement (-4,3).

When a centre of enlargement is included in an enlargement question, the enlargement needs to be drawn at the correct position on the grid relative to this centre of enlargement. In this example, the centre of enlargement is (-4, 3).

1. Start by identifying the centre of enlargement and then drawing a line (called a **ray**) from the centre of enlargement to one of the vertices of the shape. If possible, choose a vertex that is in line with the centre of enlargement, either horizontally or vertically.
2. The length of this ray needs to be multiplied by the scale factor. For this example, we need to triple the length of this line.
3. The end point of this extended ray will now be the corresponding vertex on the enlargement.

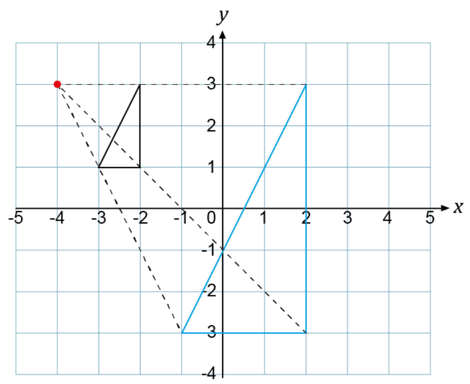


To complete the enlargement, **either:**
draw a correctly sized enlargement of the original shape from this end point



OR

- repeat the process of drawing rays from the centre of enlargement through the other two vertices, using the same scale factor, 3, to find the positions of these vertices on the enlargement.



REMEMBER! The origin is the point (0, 0) on a coordinate grid. When enlarging a shape, you can make use of the grid on which the shape has been drawn to help you enlarge the length of each side of the shape.