SOLVING LINEAR EQUATIONS

Solving equations is about finding the unknown value.

How to solve simple linear equations (in one or two steps).

How to form linear equations from worded problems.

Remember: Inverse Operations

The inverse of adding is subtracting.

The inverse of multiplying is dividing.

Check that you:

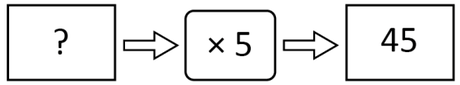
- *can solve calculations that involve negative numbers*
- *understand inverse operations*
- *can collect like terms*
- *understand how to add or subtract something to balance two sides of an equation.*

SOLVING ONE STEP EQUATIONS

We can use inverse operations to solve simple equations.

For example: $5x = 45$

Here, the information given has been put into a function machine:

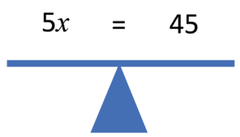


Now, we need to start from the answer and work backwards using inverse operations.

Therefore, $x = 45 \div 5$
$x = 9$

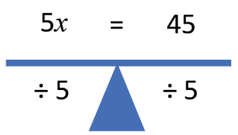
USING THE BALANCING METHOD TO SOLVE LINEAR EQUATIONS

We can also use the balancing method to solve simple equations. This is where we think of the equation as being on a balance, where both sides represent the same amount.



When working to find the value of x, we must perform the same operation on each side of the equation to ensure the equation remains balanced i.e. the left-hand side = the right-hand side.

In the case of $5x = 45$, to find the value of x, we need to divide $5x$ by 5. If we do this to the left-hand side, we must also do this on the right-hand side to ensure the equation remains balanced.



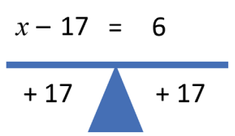
$x = 9$

Another way of showing the working is: $\frac{5x}{5} = \frac{45}{5}$
$x = 9$

Another example:

Solve $x - 17 = 6$.

Once you have identified that you need to add 17 to each side, your working could be as shown below. Many find that the first line is unnecessary.



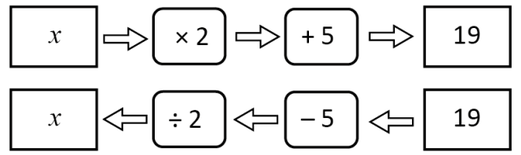
$x = 23$

$x - 17 + 17 = 6 + 17$
$x = 23$

SOLVING TWO STEP EQUATIONS

We can use function machines and inverse operations to solve two step equations.

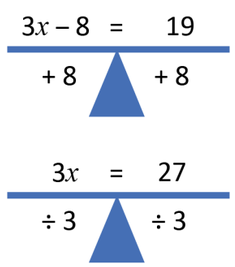
Example
Solve $2x + 5 = 19$



So $x = 7$.

USING THE BALANCING METHOD TO SOLVE TWO STEP EQUATIONS

Example 2
Solve $3x - 8 = 19$.

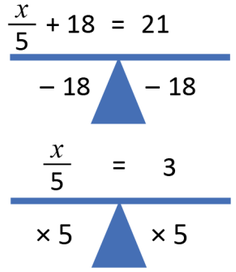


$x = 9$

Your working could be written as:

$3x - 8 + 8 = 19 + 8$
$\frac{3x}{3} = \frac{27}{3}$
$x = 9$

Example 2
Solve $\frac{x}{5} + 18 = 21$.



$x = 15$

Your working could be written as:

$\frac{x}{5} + 18 - 18 = 21 - 18$
$\frac{x}{5} \times 5 = 3 \times 5$
$x = 15$

Remember, it is important that you get to the correct answer. It does not matter which method you choose but, you should be able to write a clear method to demonstrate how you have got your answer.

FORMING AND SOLVING EQUATIONS

Example
Two friends went shopping for Christmas presents.

Between them they bought 24 presents.

Ayesha bought six more presents than Gerwin.

How many presents did they each buy?

Solution
Write expressions for each person's part:

Let the number of items Gerwin bought be represented by x.

Ayesha bought six more presents: $x + 6$

Total = 24

We can now write an equation to find how many presents each person bought:

$x + x + 6 = 24$

Collect like terms: $2x + 6 = 24$

Solve the equation:
$2x + 6 = 24$
$-6 \quad -6$
$2x = 18$
$\div 2 \quad \div 2$
$x = 9$

So, Gerwin bought nine presents.

Ayesha bought $(9 + 6)$ 15 presents.

Check by substituting your answer back into the original equation.

$9 + 15 = 24$ ✓

REMEMBER! You can check the solution to an equation by substituting the solution back into the original.

SOLVING LINEAR EQUATIONS WITH SINGLE BRACKETS

How to solve equations that have brackets.

How to solve equations with unknowns on both sides.

Check that you can:

- *use the balance method to solve linear equations*
- *collect like terms.*

SOLVING EQUATIONS WITH SINGLE BRACKETS

The best way to deal with equations containing brackets is to expand the brackets first.

The method is as follows:

1. Expand the brackets.
2. Write the resulting equation.
3. Solve the equation to find the unknown.
4. Substitute your solution back into the equation to check you are correct.

Example
Solve the equation:

$$5(3x - 6) = 12$$

You could use the grid method to multiply like so:

	$3x$	-6
5	$15x$	-30

Then solve the equation:

$$\begin{gathered} 15x - 30 = 12 \\ +30 \quad +30 \\ 15x = 42 \\ \div 15 \quad \div 15 \\ x = 2{\cdot}8 \end{gathered}$$

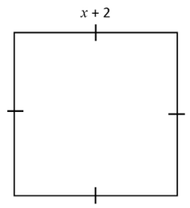
Remember you can check your answer by substituting it back into the original equation.

$$\begin{gathered} 5(3 \times \mathbf{2{\cdot}8} - 6) = \\ 5(\mathbf{8{\cdot}4} - 6) = \\ 5(\mathbf{2{\cdot}4}) = 12 \checkmark \end{gathered}$$

FORMING AND SOLVING EQUATIONS WITH SINGLE BRACKETS

Example
The perimeter of the square below is 20 cm.

What is the value of x?



Answer
Use the information to create an equation.

All four sides are the same length, and the total of all these lengths is 20 cm.

$$4(x + 2) = 20$$

Expand the brackets on the left and write the equation:

$$4x + 8 = 20$$

Solve it:

$$\begin{gathered} 4x + 8 = 20 \\ -8 \quad -8 \\ 4x = 12 \\ \div 4 \quad \div 4 \\ x = 3 \end{gathered}$$

Check it:

$$\begin{gathered} 4\,(\mathbf{3} + 2) = \\ 4(5) = 20 \checkmark \end{gathered}$$

You could start the question in a different way as each side is ¼ of the perimeter so:

$$x + 2 = \tfrac{20}{4} = 5$$

SOLVING EQUATIONS WITH UNKNOWNS ON BOTH SIDES

When the same unknown appears on both sides of the equation, we cannot use a function machine to solve it. You will need to use the balance method.

Example
Solve the following equation:

$$3x + 8 = 33 - 2x$$

To solve the equation, we should collect x terms on one side of the equation and numbers on the other.

Step one: Collect the x terms.

Always try to finish with a positive multiple of x. Adding $2x$ to each side is the best start since taking $3x$ from each side will leave us with $-5x$.

$$\begin{gathered} \qquad\qquad x \text{ terms} \quad \text{numbers} \\ 3x + 8 = 33 - 2x \\ +2x \qquad\quad +2x \\ \text{(Removing the } -2x \text{ from the right-hand side.)} \\ 5x + 8 = 33 \end{gathered}$$

Step two: Collect the number terms on the other side of the equation.

$$\begin{gathered} 5x + 8 = 33 \\ -8 \quad -8 \\ \text{(Removing 8 from the left-hand side.)} \\ 5x = 25 \end{gathered}$$

Step three: Solve the remaining equation.

$$\begin{gathered} 5x = 25 \\ \div 5 \quad \div 5 \\ x = 5 \end{gathered}$$

EQUATIONS WITH UNKNOWNS ON BOTH SIDES AND BRACKETS

Example
Solve the following equation:

$$8(x + 1) = 2(x + 16)$$

Answer

Step one: Expand the brackets.

$$8x + 8 = 2x + 32$$

Step two: Collect x terms on one side and number terms on the other and solve the equation to find the value of x.

$$\begin{gathered} 8x + 8 = 2x + 32 \\ -2x \qquad -2x \\ \text{(Removing } 2x \text{ from the right-hand side.)} \\ 6x + 8 = 32 \\ -8 \quad -8 \\ \text{(Removing 8 from the left-hand side.)} \\ 6x = 24 \\ \div 6 \quad \div 6 \\ x = 4 \end{gathered}$$

REMEMBER! Always lay out your working methodically and clearly to help prevent mistakes and to make it easier to check over your work.