EXPRESSIONS AND FORMULAE

What are expressions and formulae?

What are number machines and how can we use them?

FORMULAE AND EXPRESSIONS

Expressions, equations and formulae can look similar. These definitions help to understand the differences.

Name	Definition	Examples
Expression	An expression uses symbols, variables or a combination of both in order to represent the relationship between different values.	$x + 4$
Equation	An equation shows that two expressions are equal by using mathematical symbols. An equation usually has a left side, a right side and an equal sign separating the two sides.	$4x + 2 = 16$
Formula (plural formulae)	A formula is made up of numbers and symbols which show how to work something out. It shows the relationship between two or more variables. A formula is often written so that one variable is on the left-hand side and the way of calculating that variable is shown on the right-hand side.	$V = l \times wh$ (Volume = length × width × height)
Inequality	An inequality contains two expressions separated by one of the following symbols: $<$, $>$, $\leqslant$ or $\geqslant$.	$2x + 5 < 7$

SUBSTITUTION

Algebraically, substitution means replacing a letter with a numerical value in order to evaluate an expression.

For example

When $a = 2$ When $a = 6$ When $a = 9$

$a + 3$ $2 + 3 = 5$ $6 + 3 = 9$ $9 + 3 = 12$

Take care when substituting negative numbers. Remember the following:

- Positive × Positive = Positive
- Positive × Negative = Negative
- Negative × Positive = Negative
- Negative × Negative = Positive

Example
If $a = -2$, $b = 3$ and $c = -10$, find the value of the following expressions:

i. $3a = 3 \times (-2) = -6$
ii. $abc = (-2) \times 3 \times (-10) = 60$
iii. $a^2 + 1 = (-2)^2 + 1 = 5$

Notice that, although it isn't necessary, it is useful to place negative numbers within brackets when substituting negative values into an expression.

FORMULAE EXPRESSED IN WORDS

You will sometimes be required to substitute into a formula expressed in words.

Example
The cost to hire a van is calculated using the formula:

Cost = £10 × number of hours + £25

Calculate the cost of hiring a van for:

i. $8\frac{1}{2}$ hours
ii. A full day

Solution
We can think of it like this:

$C = 10 \times \square + 25$ or $C = 10 \times h + 25$

i. $C = 10 \times 8 \cdot 5 + 25 = £110$
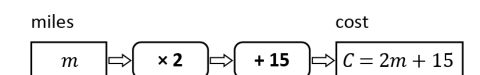ii. $C = 10 \times 24 + 25 = £265$

FORMULAE AND FUNCTION MACHINES

Example
The cost C (in pounds) of hiring a mini-bus and driving m miles is given by the formula:

$$C = 2m + 15$$

This can be written as a function machine.

So, the cost of hiring the mini-bus and driving 60 miles, i.e. $m = 60$, is given by:

The cost is £135.

EXPRESSIONS AND FORMULAE

What are expressions and formulae?

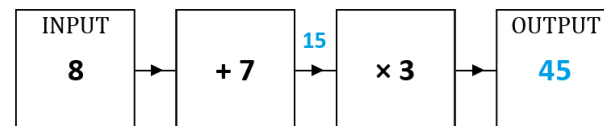**What are number machines and how can we use them?**

Check that you can:
- *use the four operations ($+ - \times \div$) to solve numerical calculations*
- *multiply negative numbers.*

NUMBER MACHINES

A number machine takes a number, the input, then applies some mathematical operations to it in order to produce an output.

Example 1
Look at the following number machine.

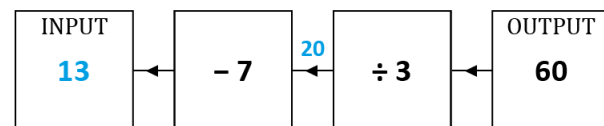| INPUT				OUTPUT
8	→	**+ 7**	15 → **× 3**	→	**45**

The input is 8.

First, we add 7 to this to give 15, then we multiply by 3 to give the output 45.

Example 2
When you are given the output but need to find the input, you can work backwards through the number machine using inverse operations.

Consider the same number machine as the previous example, but with an output of 60:

To find the input, we work backwards using inverse operations:

INPUT				OUTPUT
13	←	**– 7**	20 ← **÷ 3**	←	**60**

The output is 60.

First, we divide this by 3 to give 20, then we subtract 7 to give the input 13.

SIMPLIFYING EXPRESSIONS

We can simplify expressions by collecting similar terms (also called like terms) together.

Example
Simplify $4x + 2 + 2x + 7$.

We can collect the x terms together, which gives $4x + 2x = 6x$, and we can collect the numbers together, which gives $2 + 7 = 9$. The simplified expression is $6x + 9$.

Further examples
1. $a + a + a + a = 4a$
2. $3a + a + b + 5b = 4a + 6b$
3. $9t + 6w - 4t + w = 5t + 7w$

In the first example, there are 4 lots of a, or $4 \times a$. In algebraic terms, we can just write $4a$.

These examples demonstrate how multiplication and division work with algebraic terms.

4. $a \times a = a^2$
5. $a \times b \times c = abc$
6. $3a \times 5b = 15ab$
7. $60a \div 3b = \frac{60a}{3b} = \frac{20a}{b}$

If an expression has a set of brackets, we need to expand them first.

Example
$$6a + 3\,(a + 7)$$

First expand the brackets and then collect like terms.

$$6a + 3\,(a + 7) = 6a + 3a + 21$$
$$= 9a + 21$$

REMEMBER! Take care with negative terms. Remember the rules when multiplying or dividing them.