# a <br> CONSTRUCTIONS AND LOCI - CONSTRUCTIONS WITH COMPASSES 

UJeC A pair of compasses can be used to carry out several constructions.

## cbac

Bisecting a line by constructing a perpendicular bisector

The word bisect means to divide in half.
So, to bisect a line means to divide it into two equal halves.

## Example

If we are asked to bisect a given line, we will start by opening a pair of
compasses so that the opening is greater than half the length of the line.


Next, place the pin at one end of the line (the left end has been chosen here), draw arcs above and below the line.
$\qquad$

Then, place the pin at the other end of the line, and repeat the process of drawing arcs above and below the line. It is important that the opening between the pin and pencil is not altered.



## Bisecting an angle

## Example

In order to bisect this angle, we would first place the pin of the compasses at the vertex and draw arcs that cross the two lines. The size of the opening between the pin and pencil needs to be kept the same here.


Maintaining the same opening of the compasses, place th compasses, place th pin at the other poin of intersection and draw a second arc that crosses the previous arc.


Finish by drawing a line from the vertex of the two vertex of the two lines through the intersection of the final two arcs. This line bisects the angle.

REMEMBER! The construction of angles of $60^{\circ}$ and $30^{\circ}$ are related, as 30 is half of 60 . When an angle $30^{\circ}$ are related, as 30 is half of 60 . When an angle
of $60^{\circ}$ has been constructed, bisecting the angle of $60^{\circ}$ has been constructed, bisecting the angle
will result in an angle of $30^{\circ}$. It is the same once will result in an angle of $30^{\circ}$. It is the same once
we've constructed an angle of $90^{\circ}$, an angle of $45^{\circ}$ is constructed by bisecting it.

## Check that you can:

## use a pair of compasses to draw arcs

understand the term 'perpendicular', which means 'at a right angle to understand how angles are measured using degrees ( ${ }^{\circ}$ ).

## Constructing angles of $60^{\circ}$

Start by drawing a straigh line.


Constructing angles of $30^{\circ}$

- Construct an angle of $60^{\circ}$

Place the pin of a pair of compasses at any point on the line (one end of the line chosen here) and draw a long arc that must cross the line. Then, maintaining the same opening of the compasses, place the pin at the
intersection of the arc and the line and draw another arc that crosses the first.

Finish by drawing a line from the starting point (in this case the end of the line) through the intersection of the two arcs. The angle formed will be $60^{\circ}$

- Construct the angle bisector
- Carefully label your angle of $30^{\circ}$



## Constructing angles of $90^{\circ}$

The easiest way to construct an angle of $90^{\circ}$ is to construct a perpendicular bisector


## Constructing angles of $45^{\circ}$

Start by constructing an angle of $90^{\circ}$, and then bisect it.


Loci is the plural of the word locus. A locus is a set of points that satisfy a given rule. These points may form a line, a curve or a region.

## Check that you can:

- recognise a perpendicular bisector and an angle bisector and carry out these constructions
- use a pair of compasses to draw arcs and circles
- use a protractor to measure angles.


## The four loci

There are 4 loci that we need to learn for GCSE Mathematics. Once you understand the diagram required for each locus, you will need to know how to draw them accurately, using construction echniques if required.

## 1. The locus of points that are a given distance from one point

For example, if we were asked to construct the locus of points that are 3 cm from a point $A$, this means that we need the set of points that are always 3 cm from A. Remember that when there are enough of these points, such that all the points are very close to each other, they form a line. This ine is a circle, centered at $A$, with
 a radius of 3 cm .

## 2. The locus of points that are equidistant from two fixed points

For example, the set of points that are equidistant (equal distance) from two points $A$ and $B$ form the perpendicula bisector of the line from $A$ to $B$. We do not need to draw the line $A B$ to construc the perpendicular bisector although to do so is not incorrect).

## 3. The locus of the points that are a given distance from a line

For example, to draw the locus of the points that are 4 cm from the line $A B$, we need to draw lines on either side of $A B$ that are parallel to it and are also 4 cm away from it.

We could do this by placing a ruler at $90^{\circ}$ to the line and marking points 4 cm away. If we do this at both ends of $A B$, then by connecting the points, we will end up with parallel lines. Repeating the process on the other side of $A B$ results in another parallel line, 4 cm away from $A B$.



At the ends of the line, we have the points $A$ and $B$. At each of these points, we draw a semicircle of radius 4 cm . Every point on the semicircles will be 4 cm away from either point $A$ or point $B$.

4. The locus of points that are equidistant from two non-parallel lines
For example, in order to construct the locus of points that are equidistant from $A B$ and $C D$, we would need to find the angle bisector of the two lines.


To construct the locus, we first need to extend the lines back until we form a vertex. From this vertex, we now construct the angle bisector.


If the question had not asked us to construct this line, we could use a ruler and protractor instead.

REMEMBER! You will need to use your knowledge of perpendicular bisectors and angle bisectors, and determine when to use these construction techniques to indicate a specified locus from lines and points.

