Non-linear equations

How to recognise and solve a quadratic equation containing $1x^2$.
How to use a quadratic graph to solve related equations.

Check that you can:
- factorise quadratic expressions and equations
- solve linear equations.

Solving a quadratic equation

Expanding the following expression containing double brackets:

$(x + 8)(x + 1)$ gives the following: $x^2 + 9x + 8$.

Remember that factorising is the opposite process to expanding brackets.

Factorised expressions always contain at least one set of brackets.

Recall that a **quadratic expression** takes the form:

$ax^2 + bx + c$

where a, b, c are numbers, with $a \neq 0$.

Correspondingly, a **quadratic equation** takes the form:

$ax^2 + bx + c = 0$.

Here, we will look at solving a quadratic equation in the case where $a = 1$.

Example 1

Solve the equation $x^2 - 7x + 12 = 0$.

Answer:

The factorisation of the quadratic expression takes the form

$(x - 3)(x - 4)$

and the equation therefore becomes

$(x - 3)(x - 4) = 0$.

Remember: If pq = 0, then it must be the case that either p = 0 or q = 0.

Both $(x - 3)$ and $(x - 4)$ represent a number. If the product of these two numbers is zero, then at least one of the numbers must be equal to zero.

Either $x - 3 = 0$ or $x - 4 = 0$, then $x = 3$ or $x = 4$.

(Notice the use of the word 'or' as part of the answer. It is NOT correct to state '$x = 3$ and $x = 4$' as x cannot be both 3 and 4 at the same time.)

It may seem surprising that the quadratic equation has two answers!

To illustrate these answers, consider the graph of $y = x^2 - 7x + 12$ (on the right).

We can look at the points on the graph where $y = 0$ (on the x-axis).

The coordinates of these points are $(3, 0)$ and $(4, 0)$, again giving the solutions $x = 3$ or $x = 4$.

$y = x^2 - 7x + 12$
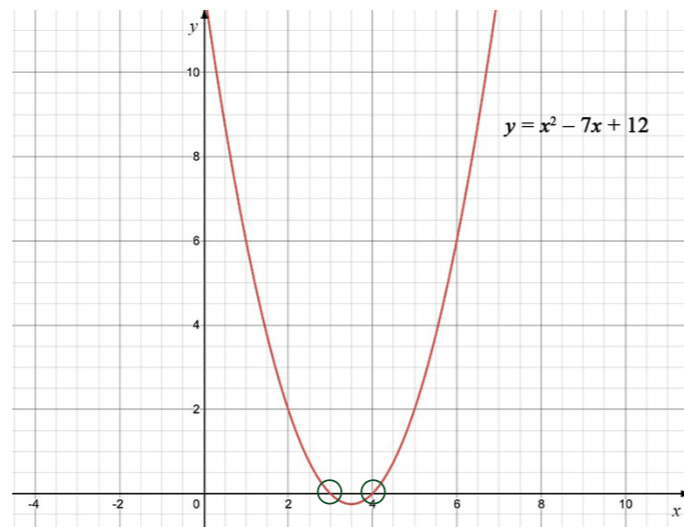
How to factorise a quadratic expression

If you need a recap on how to factorise a quadratic expression, have a look at lesson 5 of the WJEC Blended Learning resource on expanding and factorising:
https://d3kp6tphcrvm0s.cloudfront.net/el21-22_13-23/4/0

Using a quadratic graph to solve a quadratic equation

Remember that a quadratic equation can be solved by drawing a graph to represent the quadratic expression.

Solving an equation in this way will find the 'approximate' solutions because of the variation in the accuracy of sketched curves.

Look again at the graph of $y = x^2 - 7x + 12$ from **example 1**.

To solve the equation $x^2 - 7x + 12 = 0$, we can look at the points on the graph of $y = x^2 - 7x + 12$ where $y = 0$ (on the x-axis).

The x-coordinates of these points give the solutions $x = 3$ or $x = 4$.

Example 2

We can solve the equation $x^2 + 3x - 2 = 1$ in a similar way.

On the right is a graph of the quadratic equation, $y = x^2 + 3x - 2$.

To solve the equation $x^2 + 3x - 2 = 1$, we can look at the points on the graph of $y = x^2 + 3x - 2$ where $y = 1$ (by drawing the horizontal line with equation $y = 1$ together with the quadratic curve).

The x-coordinates of the points of intersection of the two graphs give the solutions: $x = -3{\cdot}8$ or $x = 0{\cdot}8$.

$y = x^2 + 3x - 2$

$y = 1$
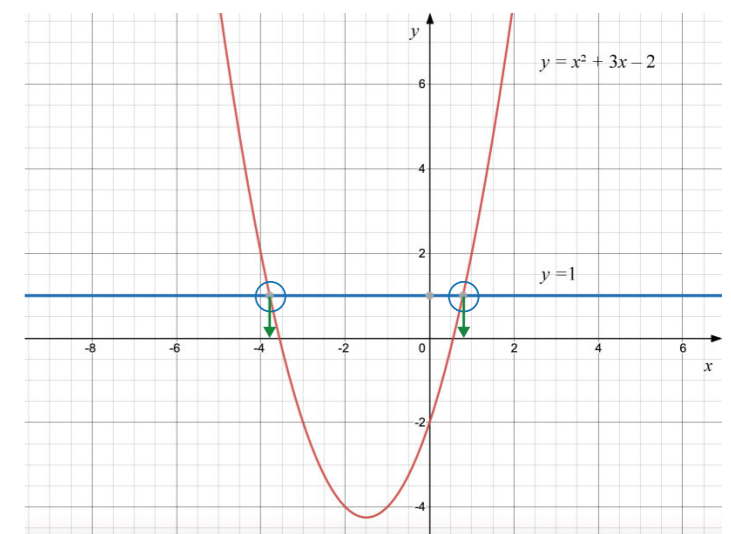
Example 3

Let's look at how we can solve $x^2 + 2x - 2 = 0$ using the graph of $y = x^2 + 3x - 2$.

In order to use the original graph, we need to isolate the quadratic expression $x^2 + 3x - 2$ within the equation which is to be solved.
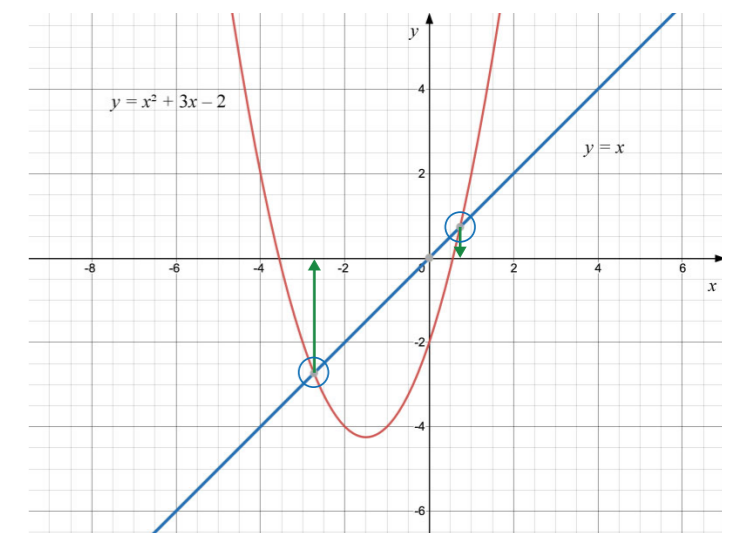
$x^2 + 2x - 2 = 0$ becomes $[x^2 + 3x - 2] - x = 0$

then $x^2 + 3x - 2 = x$.

The graphs of $y = x^2 + 3x - 2$ and $y = x$ are shown on the right.

The x-coordinates of the points of intersection of the two graphs give the following solutions:

$x = -2{\cdot}7$ or $x = 0{\cdot}7$.

$y = x^2 + 3x - 2$

$y = x$

Remember!

We can check both solutions to a quadratic equation by substituting them one by one into the original quadratic expression and ensuring that both sides of the equation agree.