

Coordinates and linear graphs – regions described by inequalities

How to use straight lines to represent inequalities in 2 dimensions.
How to use inequalities to describe a region in 2 dimensions.

Check that you can:

- draw the graphs of the lines for linear equations
- recall the meaning of the inequality symbols $< \leq > \geq$.

Using an inequality to describe a region

We can use inequalities to describe regions in two dimensions.

Example 1

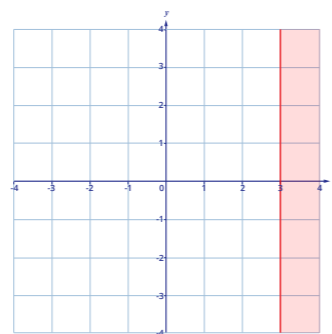
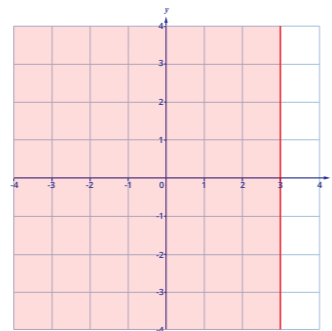
Consider the vertical line $x = 3$.

All points on the line or to the **left** of the line $x = 3$ satisfy the inequality $x \leq 3$.

The line itself **is included**, as indicated by it being a **solid** line.

(The inequality $x < 3$ would have a **dotted** line as a border.)

All points on the line or to the right of the line $x = 3$ satisfy the inequality $x \geq 3$.



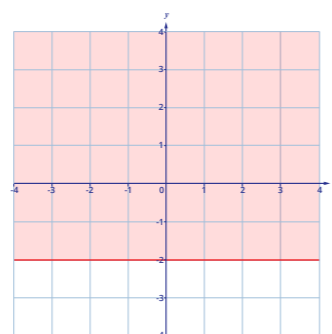
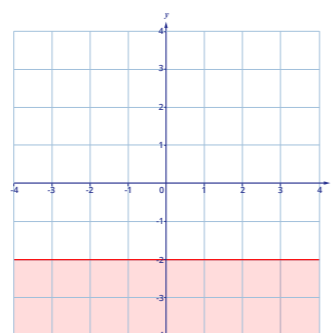
Example 2

Consider the horizontal line $y = -2$.

All points on the line or **below** the line $y = -2$ satisfy the inequality $y \leq -2$.

The line itself **is included**, as indicated by it being a **solid** line.

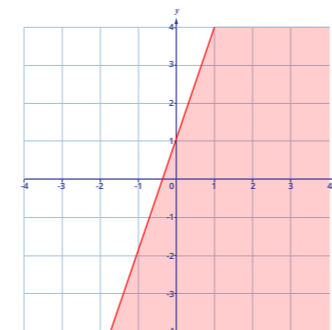
All points on the line or **above** the line $y = -2$ satisfy the inequality $y \geq -2$.



Example 3

Consider the sloping line $y = 3x + 1$.

All points on the line or **below** the line $y = 3x + 1$ satisfy the inequality $y \leq 3x + 1$.

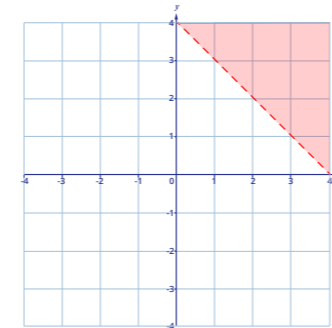


Example 4

Consider the sloping line $y + x = 4$.

All points **above** the line $y + x = 4$ satisfy the inequality $y + x > 4$.

The **dotted line** indicates that the line itself **is not** included in the region.



Example 5

Consider the sloping line $x - 2y = 6$.

All points on the line or **below** the line $x - 2y = 6$ satisfy the inequality $x - 2y \geq 6$.

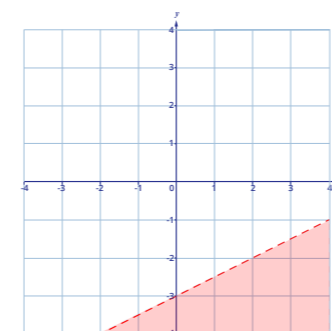
When it is not immediately clear which side of the line should be shaded, it can be easiest to start by making y the subject of the inequality.

$$\begin{aligned} x - 2y &\geq 6 \\ x &\geq 2y + 6 \\ x - 6 &\geq 2y \\ \frac{1}{2}x - 3 &\geq y \quad \text{OR} \quad y \leq \frac{1}{2}x - 3 \end{aligned}$$

Alternatively, you can check a pair of coordinates on either side of the line. e.g. test the point $(2, 2)$, in the original equation, $x - 2y \geq 6$

$$\begin{aligned} \text{L.H.S. (left-hand side)} &= x - 2y \\ &= 2 - 2 \times 2 \\ &= -2 \end{aligned}$$

Since -2 is NOT greater than or equal to 6 , the point with coordinates $(2, 2)$ is NOT in the shaded region.



Combining inequalities to describe a region

Example

Draw the region that satisfies all the following inequalities.

$$x \geq -2$$

$$y \leq 4$$

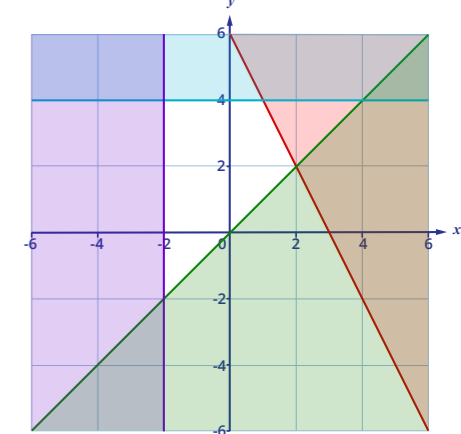
$$y \geq x$$

$$y \leq 4x + 6$$

Answer

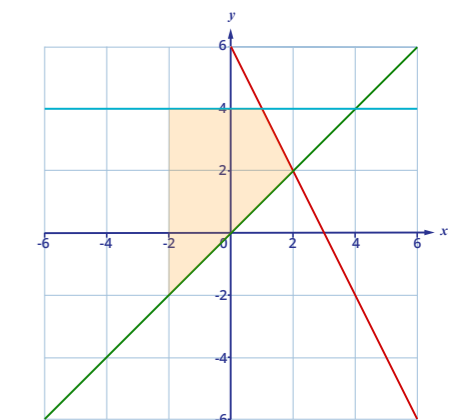
The required region is:

- to the right of the line $x = -2$
- below the line $y = 4$
- above the line $y = x$
- below the line $y = 4x + 6$.



Although it is common to only shade the required region, here the region that isn't required is shaded. Hence, the region which satisfies all these conditions is shown in white above.

Note that there are alternative ways of indicating the required region. The above example leaves the required region unshaded. It is more common to do the opposite and shade **only** the required region, as shown on the right.



Recall the symbols used to write inequalities

The symbol $<$ means 'less than'.

The symbol \leq means 'less than or equal to'.

The symbol $>$ means 'greater than'.

The symbol \geq means 'greater than or equal to'.

E.g. $x < 3$ states that 'x is less than 3'.

Remember!

When it is not immediately clear which side of the line should be shaded, it can be easiest to start by making y the subject of the inequality.

Alternatively, you can check a pair of coordinates on either side of the line.