SIMILAR SHAPES AND CONGRUENCE

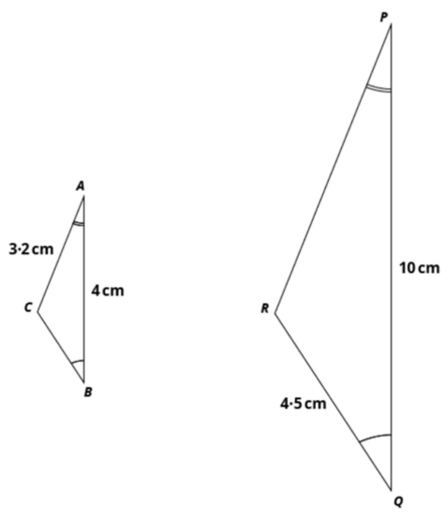
Two shapes are similar if they are exactly the same shape, but different in size. This means that one shape is an enlargement of the other. Two shapes are congruent if they are exactly the same shape and size.

Calculating unknown measurements for similar shapes

Lengths

Using the scale factor for the enlargement gives a method for calculating missing lengths.

Example
Triangles ABC and PQR are similar.
Calculate the lengths of PR and BC.



Answer
We are given the lengths of the **corresponding sides** AB and PQ. This means that the scale factor for the enlargement is $\frac{10}{4} = 2{\cdot}5$.

To calculate the length PR, we need to multiply the corresponding side by $2{\cdot}5$, as triangle PQR is the larger triangle.

Length of $PR = 3{\cdot}2 \times 2{\cdot}5 = 8\,\text{cm}$.

To calculate the length BC, we need to divide the corresponding side by $2{\cdot}5$, as triangle ABC is the smaller triangle.

Length of $BC = 4{\cdot}5 \div 2{\cdot}5 = 1{\cdot}8\,\text{cm}$

Another method is to use the **ratios** between the corresponding sides.

This can be written as:

$$\frac{PQ}{AB} = \frac{PR}{AC} = \frac{QR}{BC} \text{ or } \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

In this example, we have:

$$\frac{10}{4} = \frac{PR}{3{\cdot}2} \text{ and } \frac{4}{10} = \frac{BC}{4{\cdot}5}$$

Length of $PR = \frac{10}{4} \times 3{\cdot}2$

$= 8\,\text{cm}$

Length of $BC = 4{\cdot}5 \times \frac{4}{10}$

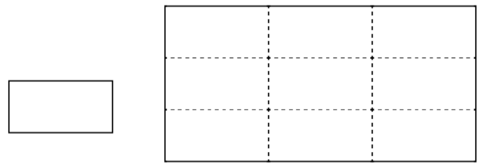
$= 1{\cdot}8\,\text{cm}$

We can also use ratios to find the unknown areas and volumes of similar shapes.

Areas

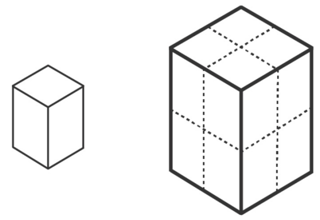
Example
A small rectangle has been enlarged by scale factor of three to create a large rectangle. What is the ratio of their areas?

Answer
Although the lengths have been increased by a ***linear*** scale factor of 3, it is obvious that the large rectangle has an area more than three times the area of the small rectangle. Since both the length and width of the rectangle have enlarged by a linear scale factor of 3, the area has been increased by a factor of 3 × 3 or 3^2, which is 9, as illustrated in this diagram.



Volumes
A small cuboid has been enlarged by a scale factor of 2 to create a large cuboid. What is the ratio of their volumes?

Answer
This diagram illustrates that, as each of the length, width and height have been increased by a linear scale factor of 2, the volume has been increased by a scale factor of 2 × 2 × 2 or 2^3, which is 8.



Using these two examples, we can obtain some general rules for similar shapes.

If the length scale factor is k, then:

- **the area scale factor is k^2**
- **the volume scale factor is k^3.**

Check that you can:

- *calculate the square, square root, cube and cube root of simple fractions, decimals and mixed numbers*
- *recognise and work with ratios*
- *rearrange equations to find missing values.*

Congruent triangles

For triangles to be congruent, they must satisfy one of four different conditions. These conditions for congruent triangles can then be used in proofs. In these conditions, the following notation is usually used:

S ≡ side, A ≡ angle, R ≡ right angle, H ≡ hypotenuse.

SSS: All three corresponding sides are equal.
SAS: Two sides and the included angle (the angle between the two given sides) are equal.
ASA: Two angles and one corresponding side are equal. Note that the corresponding side doesn't need to be the side included between the two angles.
RHS: Right angle, hypotenuse and side are equal on the corresponding triangle.

Conditions that do not guarantee congruent triangles

AAA: Three angles matching across triangles does not guarantee congruent triangles, only similar triangles.
SSA: Two sides and one angle that match across triangles does not guarantee they are congruent, as two different triangles are often possible.

You may come across examples stating that two angles and a side matching across triangles (AAS) is a condition for congruent triangles. However, although often correct, this is not correct as a general rule

REMEMBER!
For two shapes to be congruent, they must be exactly the same shape and size.