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Special graphs
Finding rates of change and finding the area under a curve.

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Check that you can:
◆ *recognise and use information from linear and non-linear graphs*
◆ *use a formula to calculate the area of simple 2-D shapes.*

Finding the gradient of linear functions

$$\text{Gradient} = \frac{\text{increase in } y}{\text{(corresponding) increase in } x}$$

For example, the gradient of the straight line in this graph can be found by drawing a triangle and finding the difference in the y coordinates and the difference in the x coordinates.

$$\text{Gradient} = \frac{40}{20} = 2$$

Remember, the right-angled triangle could be drawn anywhere on the line and the answer would always be 2 because it is a straight line.
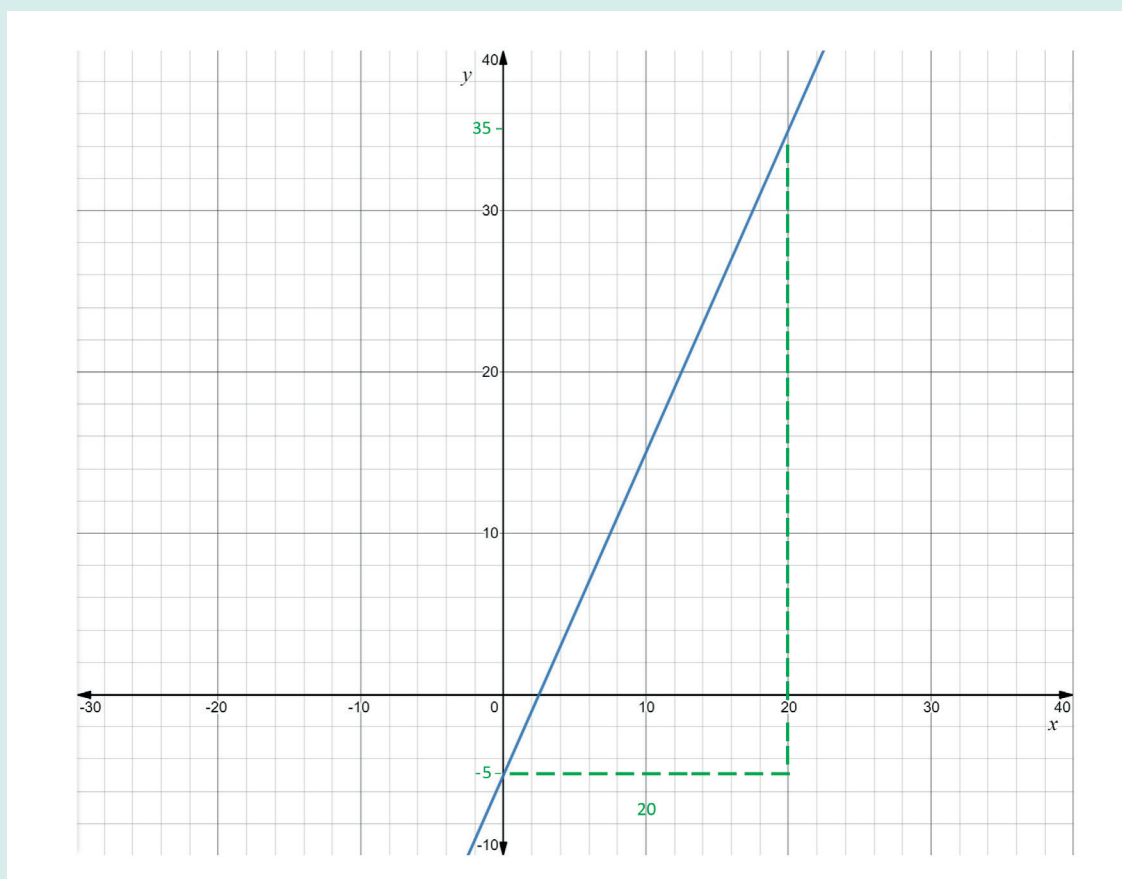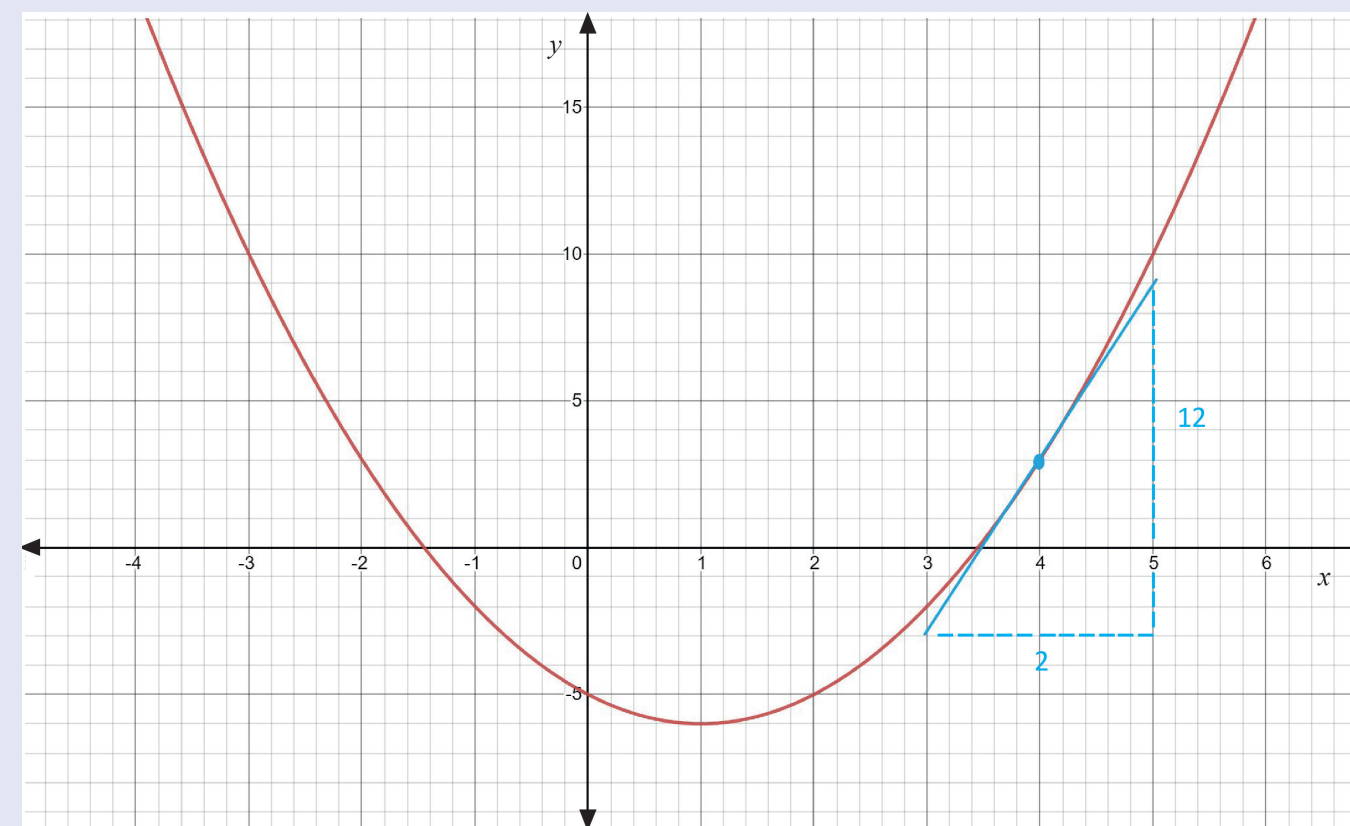
Finding the gradient of curves

The gradient of a curve is not constant as it is with a straight line – the gradient is different at each point on the curve. To find the gradient of a curve you need to use a tangent to the curve at a point to calculate an estimate of the gradient at that point. Remember, a tangent is a line that touches the curve at only one point.

For example, in this graph, a tangent has been drawn at the point (4, 3).

The gradient is found by using the same formula:

$$\text{Gradient} = \frac{\text{increase in speed}}{\text{increase in time}}$$
$$= \frac{12}{2}$$
$$= 6$$

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Speed-time graphs

A speed-time graph shows time on the horizontal axis and speed on the vertical axis. Acceleration is the rate of change of speed. It is measured in m/s^2 which can also be written as ms^{-2}.

Acceleration can be calculated from the gradient of a speed-time graph.

$$\text{Acceleration} = \frac{\text{increase in speed}}{\text{increase in time}}$$

Where there is a straight line on the graph, the acceleration is constant. When the graph is horizontal, the speed is constant, and the acceleration is zero. A positive gradient shows that the object is getting faster. Therefore, the object is accelerating.

A negative gradient shows that the object is slowing down. The object is decelerating. This means that the acceleration is negative. The area enclosed by a speed-time graph represents the distance travelled.
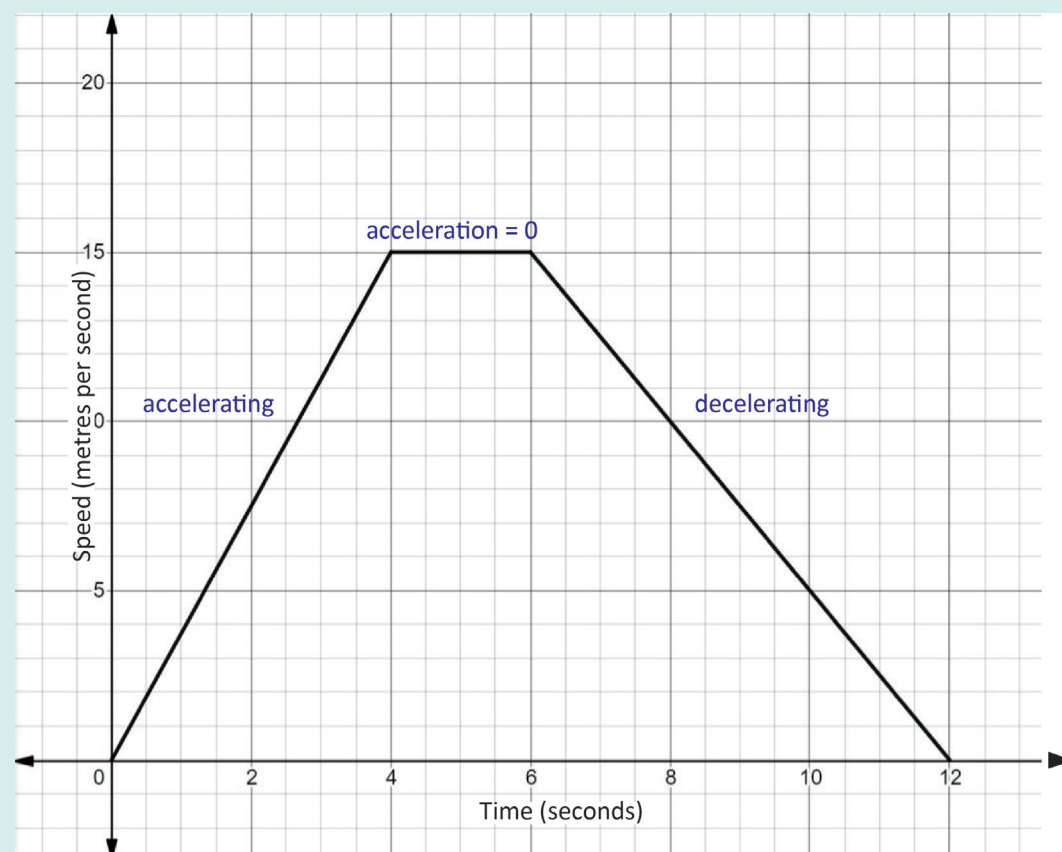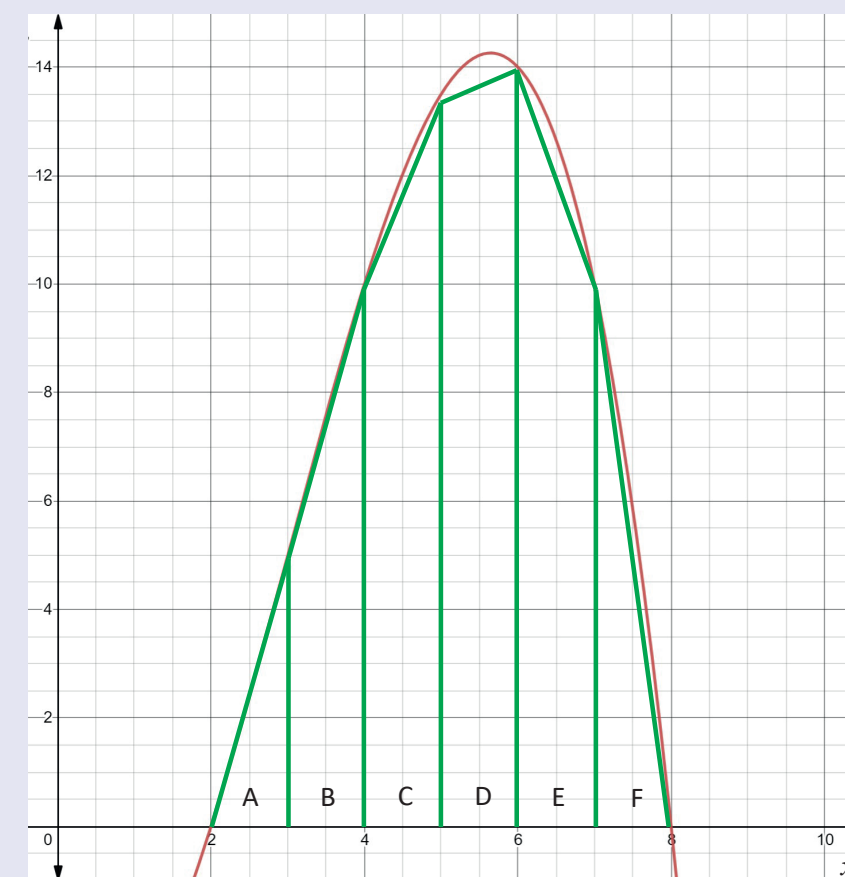

REMEMBER!
Using the trapezium rule will sometimes give an over-estimate and sometimes an under-estimate.

Finding the area under a curve: trapezium rule

To find the area under a curve, we must divide the area into very thin strips and calculate the individual areas. The thinner the strips, the more accurate your answer will be. Each strip is approximately the same shape as a trapezium, so we use the formula for the area of a trapezium to **estimate** the area under the curve.

Area of trapezium $= \dfrac{1}{2}(a + b)h$

An estimate for the area under the curve would be the total area of all the trapezia.

$2 \cdot 5 + 7 \cdot 5 + 11 \cdot 75 + 13 \cdot 75 + 12 + 5 = 52 \cdot 5$

This leads to the trapezium rule where h is the width of each strip (trapezium) and y_0 to y_n are the y-values:

$$\text{Area} = \frac{1}{2}(y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n) \times h$$

Many remember this as:

$$\text{Area} = \frac{1}{2}(\text{first} + 2(\text{sum of the middles}) + \text{last}) \times h$$

It is possible to use the trapezium rule to calculate the distance an object has travelled by finding the area of a speed-time graph between two points.