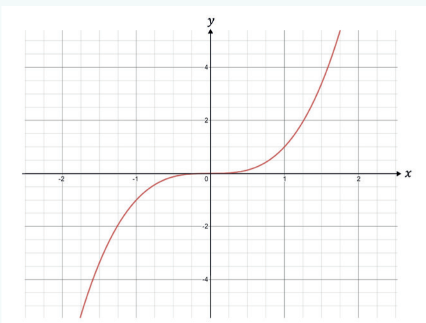
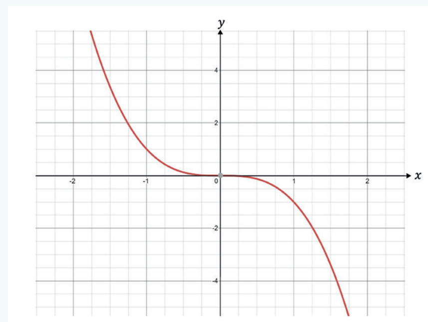
Recognising and drawing non-linear graphs
Cubic graphs, reciprocal graphs and exponential graphs.

CUBIC GRAPHS

A cubic equation is an equation where the highest power of the variable (usually 'x') is a cube (^{3}). In other words, the equation will contain an x^3 term and could contain, at most, three other terms; an x^2 term, an x term and a constant. It will not contain terms with any other powers such as x^4, x^5... or x^{-1}, x^{-2}...
The graph of a cubic equation is a curve.

When the coefficient of x^3 is positive, the curve will look something like this:





When the coefficient of x^3 is negative, the curve will look something like this:

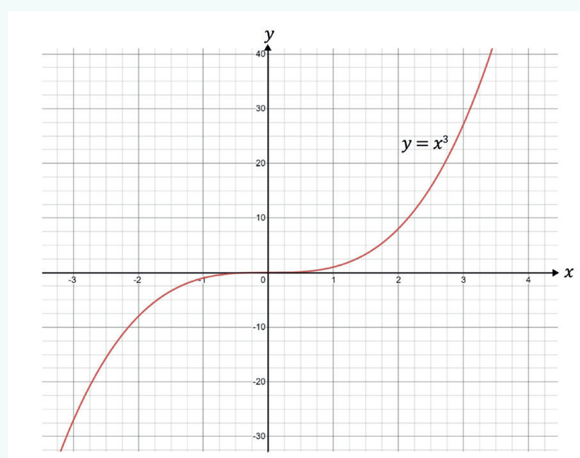
To draw a cubic graph, you would need to complete a table of values that satisfy your cubic equation.
Follow these steps to draw the graph of $y = x^3$.

1. First, complete a table of values.
2. To calculate the values for the table, substitute the values for x into the equation to find the values of y.

y	−3	−2	−1	0	1	2	3
$y = x^3$	−27	−8	−1	0	1	8	27

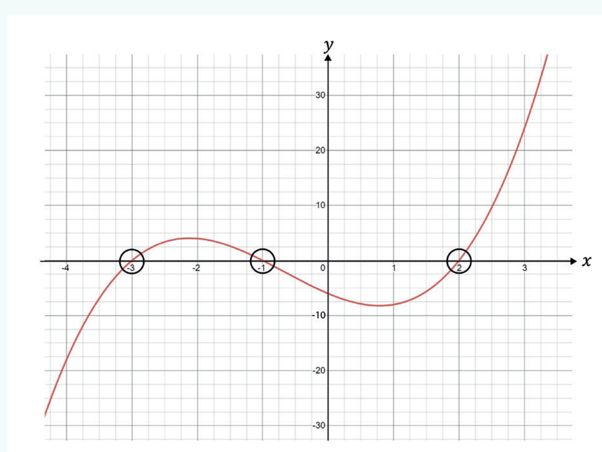
3. Then, draw the graph using these points.

Your graph should look like the one on the right.



USING A GRAPH TO SOLVE A CUBIC EQUATION

The solutions to the cubic equation $ax^3 + bx^2 + cx + d = 0$ are called the roots of the equation. They are the values of x where the curve, $ax^3 + bx^2 + cx + d = 0$, crosses the x-axis, since at those points, $y = 0$.
The graph shows there are three solutions to $x^3 + 2x^2 - 5x - 6 = 0$ and these are $x = -3$, $x = -1$ and $x = 2$.

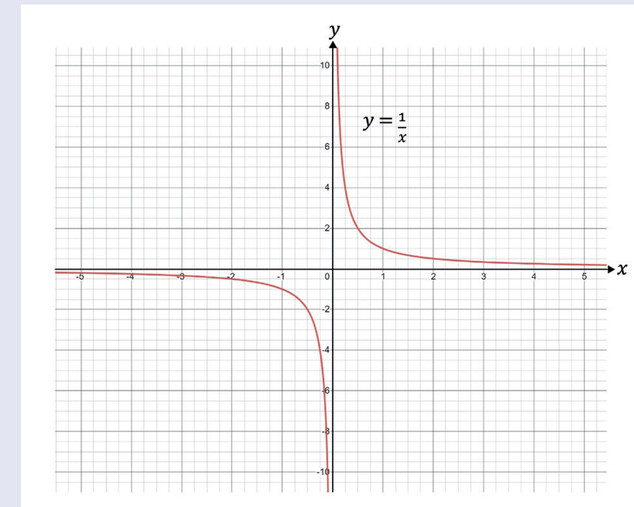


Check that you can:

- *draw quadratic graphs, (look at the intermediate knowledge organizer)*
- *substitute values into an equation to find an answer*
- *find the reciprocal of a number or a fraction.*

RECIPROCAL GRAPHS

The graph of $y = \frac{1}{x}$ is shown on the right. This is an example of a **reciprocal** function.
This type of graph is a smooth curve called a **hyperbola**.
When x is very small, y is very large.
When x is very large, y is very small.
The curve gets very close to both axes, but never touches them.
There is a break in the graph when $x = 0$ since division by zero is undefined.
The graph has rotational symmetry of order 2 about the origin.

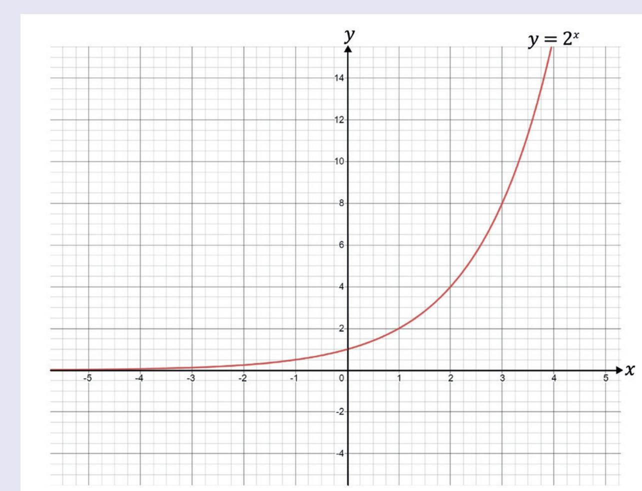


You can draw a reciprocal graph in the same way as you draw a quadratic or cubic graph:

- Complete a table of values, by substituting in values for x into the equation to find the values of y.
- Draw the graph using these points.

EXPONENTIAL GRAPHS

An exponential equation is an equation where the unknown is the exponent, e.g. $y = 2^x$, $y = 3^x$ and $y = 5^x$.
Here on the right is the graph of $\boldsymbol{y = 2^x}$:
The exponential graph is continuous (there are no breaks).
The curves all increase quite rapidly; as x increases, y increases.
The curves get very close to the x-axis, but never touch it.
You may have noticed that all the curves in the graphs crossed the y-axis at the same point (0,1).



You can draw an exponential graph in the same way as described earlier.

REMEMBER!
Study the shape and features of these three types of graphs so that you can learn to recognise them.