Non-linear graphs: transformations

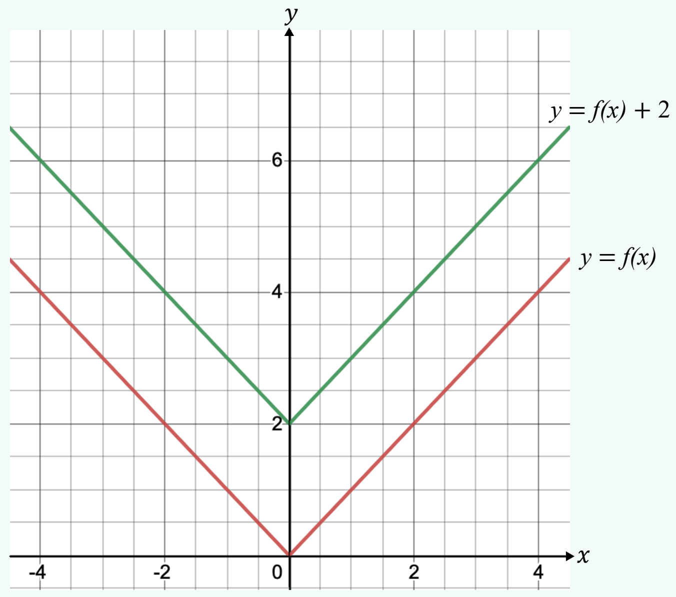
Applying the transformations to graphs of $f(x)$ and understanding how they change.

wjec cbac

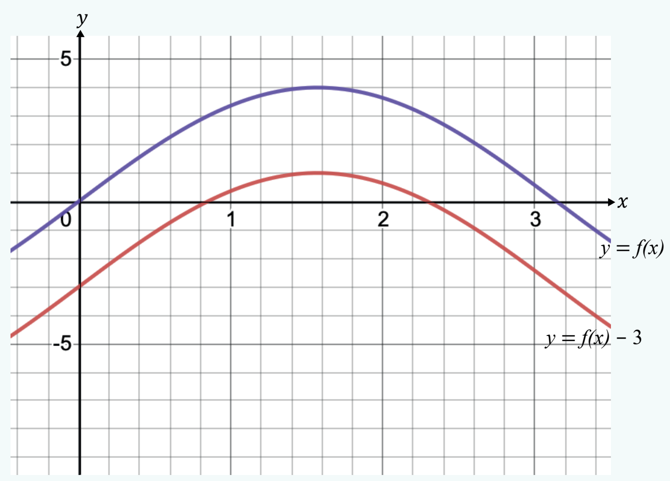
TRANSFORMING GRAPHS

$f(x)$ stands for 'a function of x'. Generally, $y = f(x)$.
For example, the equation $y = 3x + 8$ can also be written as $f(x) = 3x + 8$.
Letters other than f can also be used, with g and h being the most common alternatives.
For example, $y = x^2 + 2x + 6$ can be written as $g(x) = x^2 + 2x + 6$.
You need to be able to draw a transformed graph and recognise how the original graph has been transformed.

Example 1: Below are the graphs of $y = f(x)$ and $y = f(x) + 2$.
The shape is the same, but the position is different. The original graph has moved up the y-axis by 2 units.

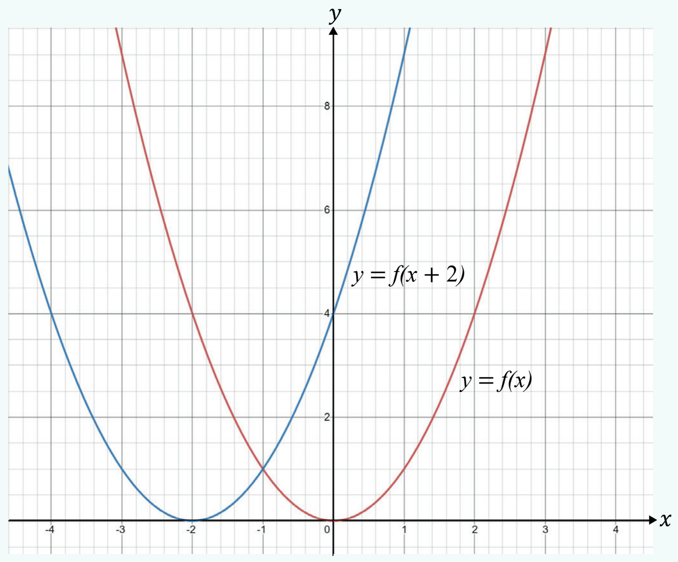


Example 2: Below are the graphs of $y = f(x)$ and $y = f(x) - 3$.
The shape is the same, but the position is different. The original graph has moved down the y-axis by 3 units.

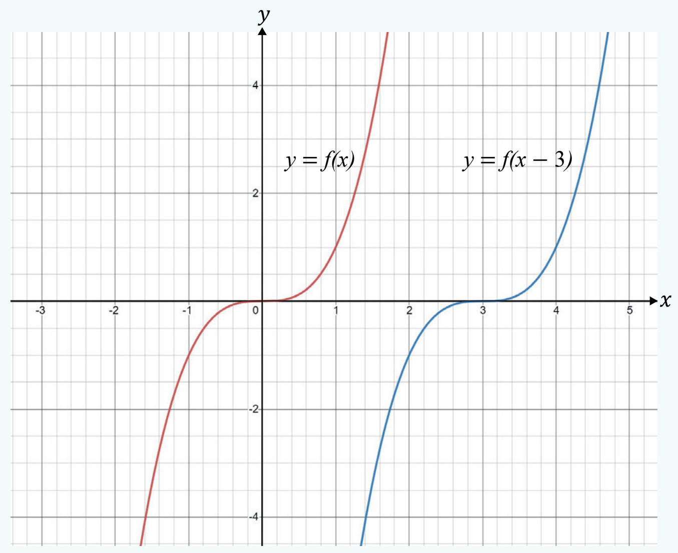


The transformation $\boldsymbol{y = f(x) + a}$ moves the original graph $y = f(x)$ **upwards by *a* units**.
The transformation $\boldsymbol{y = f(x) - a}$ moves the original graph $y = f(x)$ **downwards by *a* units**.

Example 3: Below are the graphs of $y = f(x)$ and $y = f(x + 2)$. The shape is the same, but the position is different. The original graph has moved to the left by 2 units. You can see that what used to happen at $x = 0$ now happens at $x = -2$.



Example 4: Below are the graphs of $y = f(x)$ and $y = f(x - 3)$. The shape is the same, but the position is different. The original graph has moved to the right by 3 units. What used to happen at $x = 0$ now happens at $x = 3$.

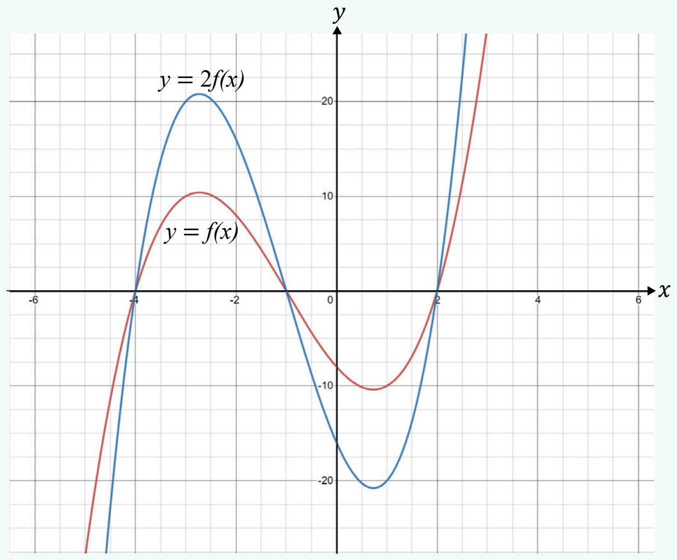


The transformation $\boldsymbol{y = f(x + a)}$ moves the original graph of $y = f(x)$ **to the left by *a* units**.
The transformation $\boldsymbol{y = f(x - a)}$ moves the original graph of $y = f(x)$ **to the right by *a* units**.

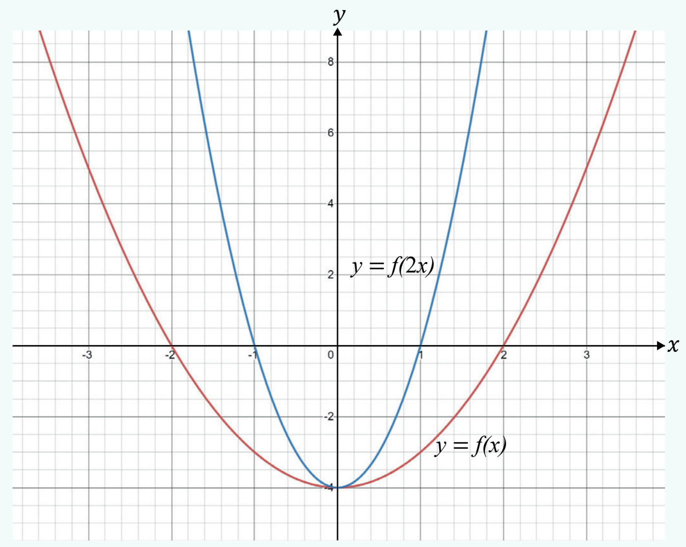
Check that you can:

- *recognise and draw non-linear graphs.*

Example 5: Below are the graphs of $y = f(x)$ and $y = 2f(x)$. Both the shape and the position are similar. The original graph has stretched parallel to the y-axis (vertically) by a scale factor of 2.

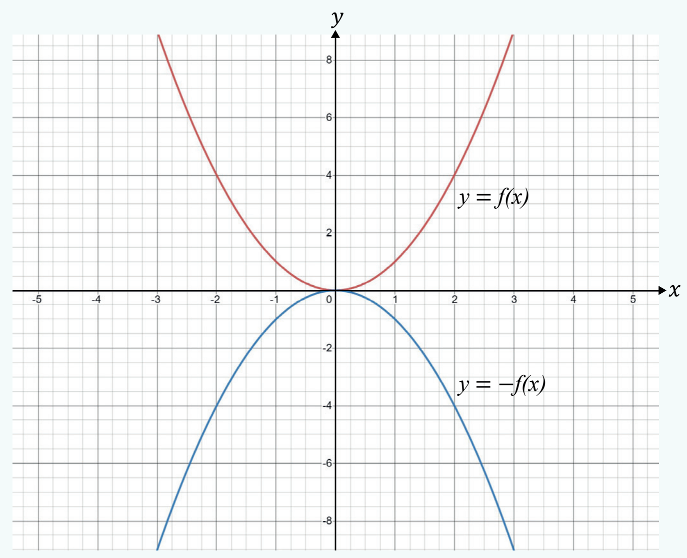


Example 6: Below are the graphs of $y = f(x)$ and $y = f(2x)$. The transformed graph is a similar shape to the original graph, but it has been stretched horizontally by a scale factor of $\frac{1}{2}$.

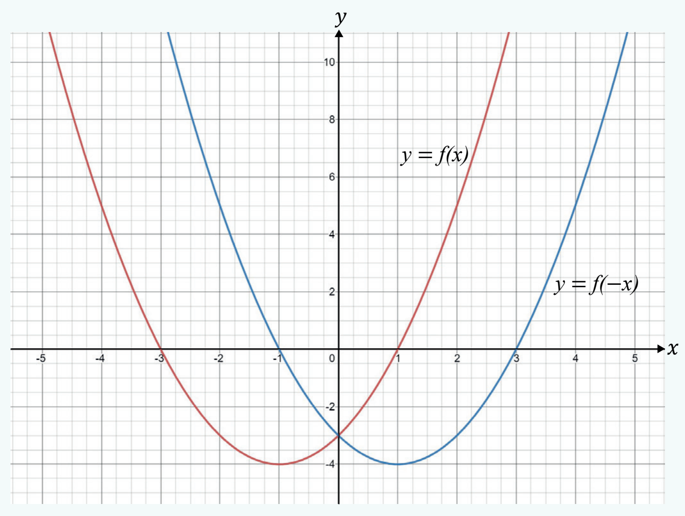


The transformation of $y = kf(x)$ **stretches the original graph of $\boldsymbol{y = f(x)}$ by a scale factor of $\boldsymbol{k}$ parallel to the $\boldsymbol{y}$-axis.**
The transformation of $y = f(kx)$ **stretches the original graph of $\boldsymbol{y = f(x)}$ by a scale factor of $\boldsymbol{\frac{1}{k}}$ parallel to the $\boldsymbol{x}$-axis.**

Example 7: Below are the graphs of $y = f(x)$ and $y = -f(x)$. The shape is similar, but the position is different. The original graph has been reflected in the x-axis.



Example 8: Below are the graphs of $y = f(x)$ and $y = f(-x)$. The shape is similar, but the position is different. The original graph has been reflected in the y-axis.

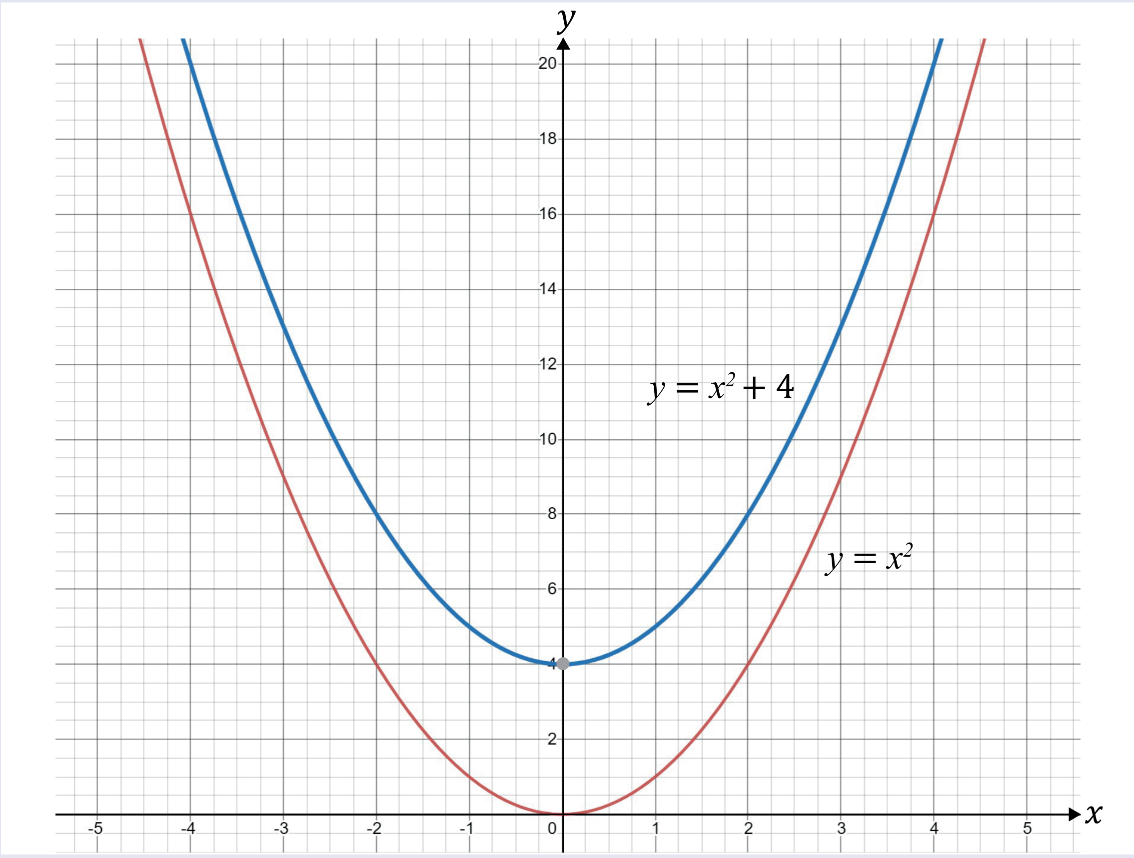


The transformation of $y = -f(x)$ **is a reflection of the original graph of $\boldsymbol{y = f(x)}$ in the $\boldsymbol{x}$-axis.**
The transformation of $y = f(-x)$ **is a reflection of the original graph of $\boldsymbol{y = f(x)}$ in the $\boldsymbol{y}$-axis.**

REMEMBER!
Take care to note if we are adding or subtracting from $f(x)$ or adding or subtracting from x.
Take care to note if we are multiplying $f(x)$ by a factor or multiplying x by a factor.

TRANSFORMING KNOWN FUNCTIONS

We need to be able to transform known functions.
For example, $y = x^2$ is a known function and looks like the red curve below.
If we consider $y = x^2$ to be $y = f(x)$, then $y = x^2 + 4$ is $y = f(x) + 4$.
Therefore, $y = x^2$ is moved 4 units upwards (blue curve).



We can apply this approach to similar transformations, for example:

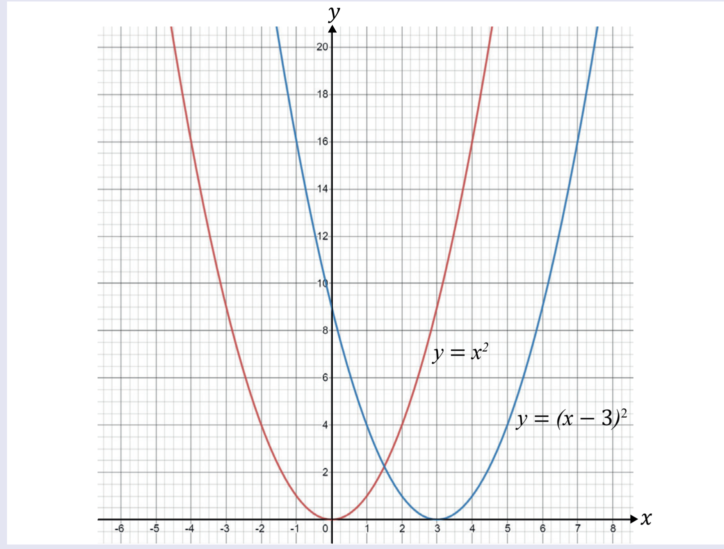
a. $y = (x - 3)^2$ is $y = f(x - 3)$.
Therefore, $y = f(x)$ is moved 3 units to the right.

b. $y = 3x^2$ is $y = 3f(x)$.
Therefore, $y = x^2$ is stretched parallel to the y-axis by a scale factor of 3.

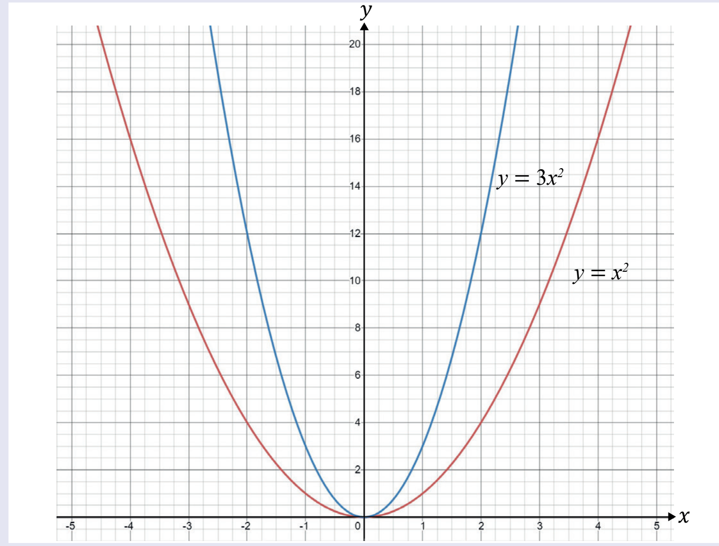
c. $y = 5x^2$ is $y = f(5x)$.
Therefore, $y = x^2$ is stretched parallel to the x-axis by a scale factor of $\frac{1}{5}$.
Here, the x values have been multiplied by $\frac{1}{5}$.

d. $y = -x^2$ is $y = -f(x)$.
Therefore, $y = x^2$ is reflected in the x-axis.

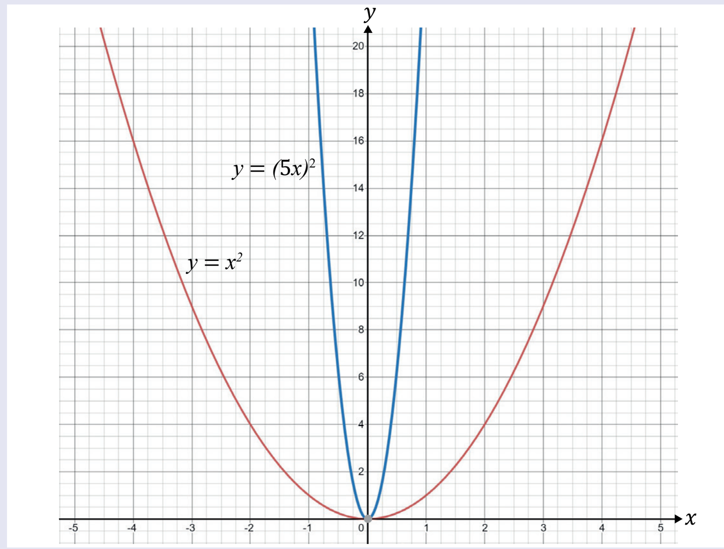
e. $y = (-x)^2$ is $y = f(-x)$.
Therefore, $y = x^2$ is reflected in the y-axis. Notice that this is exactly the same as the original because , $y = x^2$ is symmetrical about the y-axis.



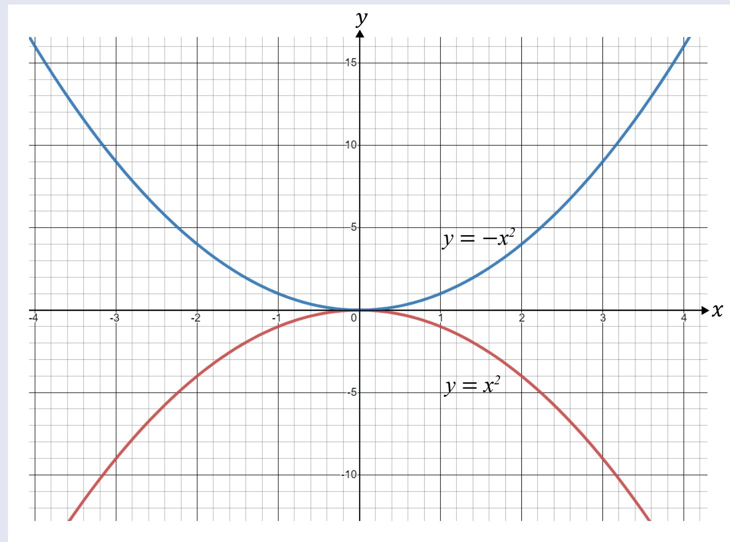
a.



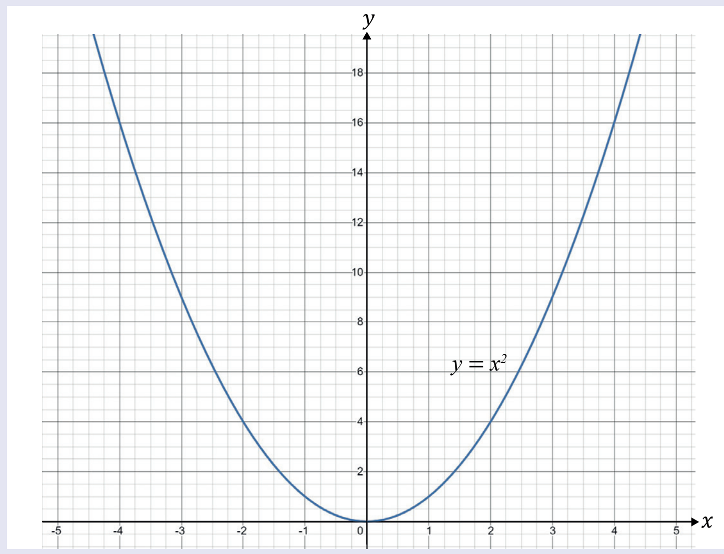
b.



c.



d.

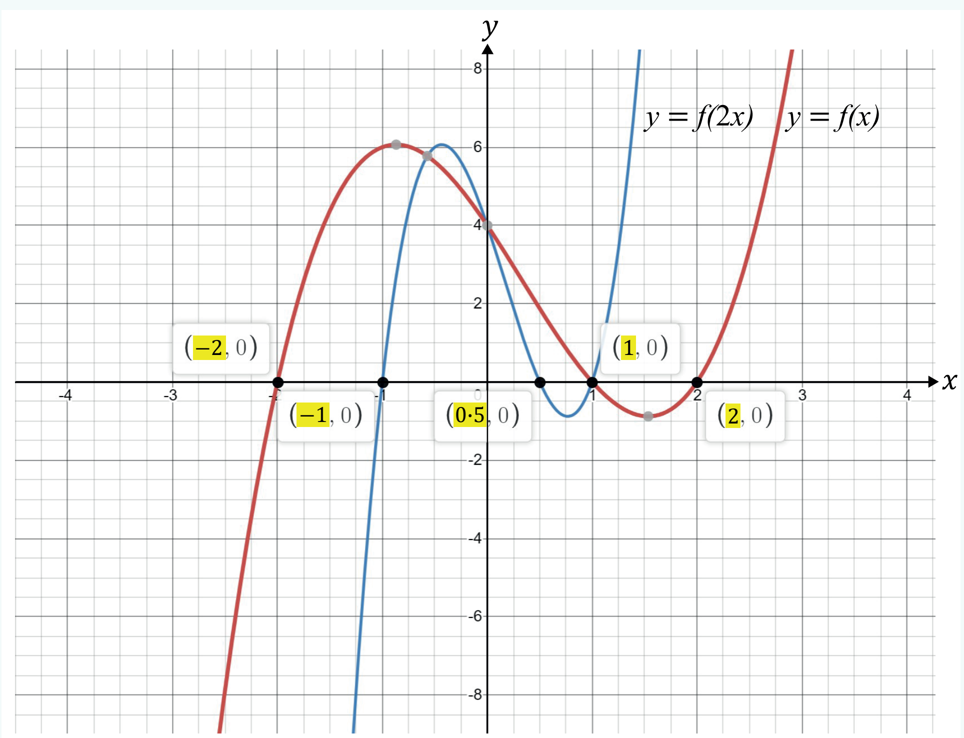


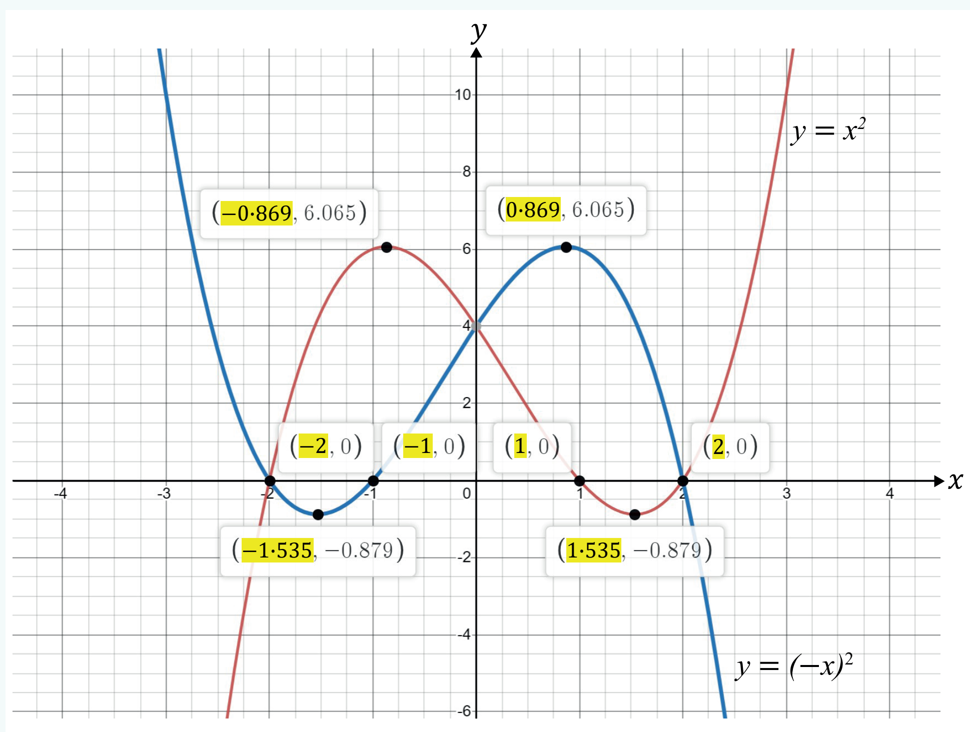
e.

THE EFFECT OF TRANSFORMATION ON THE COORDINATES OF A POINT ON THE ORIGINAL GRAPH (x, y).

- For $y = f(x) + a$, the x coordinate stays the same and a is added to the y coordinate of each point, e.g. $(x, y + a)$.
- For $y = f(x) - a$, the x coordinate stays the same and a is subtracted from the y coordinate of each point, e.g. $(x, y - a)$.
- For $y = f(x + a)$, a is subtracted from the x coordinate of each point and the y coordinate stays the same, e.g. $(x - a, y)$.
- For $y = f(x - a)$, a is added to the x coordinate of each point and the y coordinate stays the same, e.g. $(x + a, y)$.
- For $y = kf(x)$, the x coordinate stays the same and the y coordinate of each point is multiplied by k, e.g. (x, ky).
- For $y = f(kx)$, the x coordinate of each point is multiplied by $\frac{1}{k}$ and the y coordinate stays the same, e.g. $(\frac{x}{k}, y)$.
- For $y = -f(x)$, the x coordinate stays the same and the y coordinate of each point is multiplied by –1, e.g. $(x \ -y)$.
- For $y = f(-x)$, the x coordinate of each point is multiplied by –1 and the y coordinate stays the same, e.g. $(-x, y)$.

Below are some examples. Look closely at these graphs, taking particular notice of the coordinates shown.
Here you can see $y = f(x)$ in red and $y = f(2x)$ in blue:





HINTS FOR IDENTIFYING TRANSFORMATIONS

To identify transformations, you should look at the transformed graph and ask yourself the following questions:

- Is the transformed graph exactly the same shape as the original graph but in a different position? If yes, it must be $y = f(x) + a$ or $y = f(x) - a$.
- Are the points where the curve intersects an axis the same? If they intersect the x-axis at the same points, it must be $y = kf(x)$; if they intersect the y-axis at the same points, it must be $y = f(kx)$.
- Is it a reflection in the x-axis? If yes, then it is $y = -f(x)$.
- Is it a reflection in the y-axis? If yes, then it is $y = f(-x)$.

REMEMBER!
Use the guidance above as a checklist to help you identify transformations.