

# CHANGING THE SUBJECT OF A FORMULA

We can make a variable the subject of a formula by rearranging the formula.

Check that you can:

- solve simple equations.

The subject of a formula should stand alone on one side of the formula.

For example, the subject of the formula  $y = mx + c$  is  $y$ .

To make  $x$  the subject of a formula, the formula should be rearranged so that  $x$  stands alone on one side of the formula:  $x = \dots$

The process of changing the subject of a formula is very similar to that of solving equations using the balancing method.

## Example 1:

Make  $w$  the subject of the following formula:  $f = w + 9m$

If we want  $w$  on its own, then  $9m$  must be removed from the right-hand side. **Remember**, we must do the same to both sides of the formula in order to keep them balanced.

$$\begin{array}{r} f = w + 9m \\ - 9m \quad - 9m \\ \hline f - 9m = w \end{array}$$

This can also be written as:

$$w = f - 9m$$

## Example 2:

Make  $a$  the subject of the following formula:  $c = 4a + b$

If we want  $a$  on its own, then  $b$  must be removed from the right-hand side. To keep the sides balanced, we must subtract  $b$  from each side of the formula.

$$\begin{array}{r} c = 4a + b \\ - b \quad - b \\ \hline c - b = 4a \end{array}$$

The second step is to remove the 4 from the right-hand side. We do this by dividing both sides by 4.

$$\begin{array}{r} c - b = 4a \\ \div 4 \quad \div 4 \\ \hline \frac{c - b}{4} = a \end{array}$$

which can be also written as:

$$a = \frac{c - b}{4}$$

## REMEMBER!

You must do the same to both sides of the formula in order to keep them balanced.

## Example 3:

Make  $q$  the subject of the following formula:  $t = pq^2 r$

The first step is to remove the  $p$  and  $r$  from the right-hand side. We do this by dividing both sides by  $pr$ .

$$\frac{t}{pr} = q^2$$

The next step is to take the square root of both sides.

$$\sqrt{\frac{t}{pr}} = q$$

## Example 4:

Make  $e$  the subject of the following formula:

$$d = \sqrt{3e - 5}$$

First, remove the root by squaring both sides.

$$d^2 = 3e - 5$$

Then, we add 5 to both sides.

$$d^2 + 5 = 3e$$

Finally, divide both sides by 3.

$$\frac{d^2 + 5}{3} = e$$

## Example 5:

Make  $s$  the subject of the following formula:

$$4r = 5(t + 2s)$$

First, expand the brackets.

$$4r = 5t + 10s$$

Then, subtract  $5t$  from both sides.

$$4r - 5t = 10s$$

Finally, divide both sides by 10.

$$\frac{4r - 5t}{10} = s$$

## Harder examples

### FORMULAE CONTAINING FRACTIONS

#### Example 6:

Make  $c$  the subject of the following formula:

$$a = \frac{5}{b + c}$$

First, remove the denominator by multiplying both sides by  $(b + c)$ .

$$a(b + c) = 5$$

Then, expand the brackets.

$$ab + ac = 5$$

Now, subtract  $ab$  from both sides.

$$ac = 5 - ab$$

Finally, divide both sides by  $a$ .

$$c = \frac{5 - ab}{a}$$

### WHEN THE VARIABLE APPEARS MORE THAN ONCE

When the variable that we want to make the subject of the formula appears more than once, rearranging the formula can be more challenging.

We need to collect these terms on one side of the formula and then factorise.

#### Example 7:

Make  $x$  the subject of the following formula:

$$ax + xy = b$$

First, we factorise so that there is a single variable ( $x$ ).

$$x(a + y) = b$$

Then, we divide both sides by  $a + y$ .

$$x = \frac{b}{(a + y)}$$

#### Example 8:

Make  $x$  the subject of the following formula:

$$a = \frac{b - x}{5x + c}$$

First, multiply both sides by  $5x + c$ .

$$a(5x + c) = b - x$$

Next, expand the brackets.

$$5ax + ac = b - x$$

Then, collect all terms containing  $x$  together and put all others on the other side of the equal sign.

$$5ax + x = b - ac$$

Now, take  $x$  as a common factor of the left-hand side and factorise.

$$x(5a + 1) = b - ac$$

Finally, divide both sides by  $(5a + 1)$ .

$$x = \frac{b - ac}{5a + 1}$$