ALGEBRAIC FRACTIONS

Simplifying algebraic fractions and understanding the arithmetic of algebraic fractions.

Check that you can:

- *factorise linear and quadratic equations*
- *add, subtract, multiply and divide numerical fractions*
- *find common factors.*

Simplifying algebraic fractions

Algebraic fractions have a numerator and a denominator just as an ordinary fraction does, but at least one of them is an algebraic expression involving an unknown.

Examples

$$\frac{1}{x} \qquad \frac{2}{3x+5} \qquad \frac{3x+9}{5} \qquad \frac{3x+9}{5x+11} \qquad \frac{x^2+2x+5}{2x^2-1}$$

Like numerical fractions, algebraic fractions can also be simplified if the numerator and the denominator have a common factor. In its simplest form, the numerator and denominator of an algebraic fraction have no common factors apart from 1.

To write an algebraic fraction in its simplest form:

- factorise the numerator and denominator
- divide the numerator and denominator by their highest common factor.

Examples

$$\frac{3x-6}{3} = \frac{3(x-2)}{3} = x-2$$

$$\frac{3x+9}{4x^2+12x} = \frac{3(x+3)}{4x\,(x+3)} = \frac{3}{4x}$$

$$\frac{x^2+5x+6}{x^2-2x-8} = \frac{(x+3)\,(x+2)}{(x-4)\,(x+2)} = \frac{x+3}{x-4}$$

REMEMBER!

Use the same methods for multiplying, dividing, adding and subtracting algebraic fractions as you would for numerical fractions. Multiplication: multiply numerators and denominators. Division: multiply the first fraction by the reciprocal of the second. Addition/subtraction: find a common denominator, add numerators/subtract second numerator from the first. Then for all four operations, simplify your answer.

The four operations with algebraic fractions

Take a look at the table below. Here, the same methods are being used for numerical fractions as for algebraic fractions.

	Numerical fractions	Algebraic fractions
Multiplication	$\frac{8}{9} \times \frac{3}{14} = \frac{\cancel{2} \times 4}{3 \times \cancel{3}} \times \frac{\cancel{3}}{\cancel{2} \times 7} = \frac{4}{21}$	$\frac{2x+2}{3x-6} \times \frac{2x-4}{4x+4} = \frac{2\cancel{(x+1)}}{3\cancel{(x-2)}} \times \frac{2\cancel{(x-2)}}{4\cancel{(x+1)}} = \frac{2 \times 2}{3 \times 4} = \frac{4}{12} = \frac{1}{3}$
Division	$\frac{3}{7} \div \frac{9}{35} = \frac{3}{7} \times \frac{35}{9} = \frac{\cancel{3}}{\cancel{7}} \times \frac{5 \times \cancel{7}}{\cancel{3} \times 3} = \frac{5}{3}$	$\frac{7x-14}{3x} \div \frac{4x-8}{x} = \frac{7x-14}{3x} \times \frac{x}{4x-8} = \frac{7\cancel{(x-2)}}{3\cancel{x}} \times \frac{\cancel{x}}{4\cancel{(x-2)}} = \frac{7}{12}$
Addition	$\frac{6}{11} + \frac{3}{5} = \frac{(6 \times 5) + (3 \times 11)}{11 \times 5} = \frac{30+33}{55} = \frac{63}{55}$	$\frac{4}{x-3} + \frac{2}{x+2} = \frac{4(x+2)+2(x-3)}{(x-3)(x+2)} = \frac{4x+8+2x-6}{(x-3)(x+2)} = \frac{6x+2}{(x-3)(x+2)}$
Subtraction	$\frac{7}{11} - \frac{2}{3} = \frac{(7 \times 3) - (2 \times 11)}{11 \times 3} = \frac{21-22}{33} = \frac{-1}{33}$	$\frac{4}{x-5} + \frac{2}{3x+1} = \frac{4(3x+1)-2(x-5)}{(x-5)(3x+1)} = \frac{12x+4-2x+10}{(x-5)(3x+1)} = \frac{10x+14}{(x-5)(3x+1)} = \frac{2(5x+7)}{(x-5)(3x+1)}$