

There are mainly two types of hypothesis tests in this unit: hypothesis tests that involve the Normal distribution, and hypothesis tests that involve non-parametric distributions.

- a) Testing the mean from a large sample. In this case, use the Central Limit Theorem to estimate the variance.
- b) Testing the difference of two means for two independent Normal distributions with known variances. The difference may be 0, e.g. when testing the equality of means, or another specified value.

### Example

A farmer wants to know if the average masses of eggs produced by chickens of breed A are greater than those produced by breed B. He takes a random sample and records the mass,  $x$ , of 80 eggs from breed A and the mass,  $y$ , of 50 eggs from breed B, and calculates the following summary statistics:

$$\sum x = 4122, \sum x^2 = 214664, \sum y = 2453, \sum y^2 = 126240.$$

Carry out a hypothesis test, using a 10% significance level.

### Solution

$$\begin{array}{lll} H_0: \mu_A = \mu_B & H_1: \mu_A > \mu_B & \\ \bar{x} = 51.525 & \bar{y} = 49.06 & \bar{x} - \bar{y} = 2.465 \\ s_x^2 = 28.83... & s_y^2 = 120.32... & \text{(See KO 5.1)} \end{array}$$

$$ESE = \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}} = \sqrt{\frac{28.83...}{80} + \frac{120.32...}{50}} = 1.663...$$

Use your calculator to find  $P(\bar{X} - \bar{Y} > 0)$ .

$$P(\bar{X} - \bar{Y} > 0) = 0.93082 \Rightarrow p\text{-value} = 0.06918$$

Since  $0.069 < 0.1$ , there is sufficient evidence to reject  $H_0$  and conclude that there is evidence to suggest that breed A produce heavier eggs than breed B.

- c) Testing the mean of a Normal distribution with unknown variance. If the sample is large, refer to (a). If the sample is small, use the Student  $t$ -distribution.

### Example

A factory manufactures bricks that should have a mean mass of 2270 g. In order to test the quality of the product, 10 bricks are chosen at random and their masses, in grams, are recorded. The results are given below.

2258, 2332, 2317, 2287, 2287,  
2260, 2313, 2314, 2315, 2261

Test, at the 5% level of significance, whether there has been a change in the mean mass.

You may assume that this is a random sample from a Normal distribution with unknown mean  $\mu$  and unknown variance  $\sigma^2$ .

### Solution

$$\begin{array}{ll} H_0: \mu = 2270 & H_1: \mu \neq 2270 \\ \bar{x} = 2294.4 & s = 27.512 \text{ (see KO 5.1)} \end{array}$$

Test statistic

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{2294.4 - 2270}{27.512/\sqrt{10}} = 2.805$$

The critical value from the table with 9 degrees of freedom is 2.262.

Since  $TS > CV$ , there is sufficient evidence to reject  $H_0$  and suggest that there has been a change in mean mass of the bricks.

### Non-parametric tests

These are used when we do not have any information about the underlying distribution. If there are two independent samples, use Mann-Whitney. If there is one sample, or a paired sample, use Wilcoxon. The process is very similar for both tests.

#### Step 1: Define $\eta$ and form hypotheses.

Wilcoxon	Mann-Whitney
$\eta$ is the population median or $\eta_D$ is the population median difference for a paired sample.	$\eta_1$ and $\eta_2$ are the population medians.
$H_0: \eta = \eta_0$ $H_1: \eta < \eta_0$ $H_1: \eta > \eta_0$ $H_1: \eta \neq \eta_0$	$H_0: \eta_1 = \eta_2$ $H_1: \eta_1 < \eta_2$ $H_1: \eta_1 > \eta_2$ $H_1: \eta_1 \neq \eta_2$
$H_0: \eta_D = 0$ $H_1: \eta_D < 0$ $H_1: \eta_D > 0$ $H_1: \eta_D \neq 0$	

#### Step 2: Rank the data.

Remove any entries that have a difference of 0 and rank the **absolute** differences from lowest to highest.

If using the sum of ranks approach, rank the differences (both samples combined) from lowest to highest.

#### Step 3: Calculate the test statistic (TS).

$W$  is the sum of the ranks of the positive differences.

$U$  is the sum of the number of observations in the second sample that are greater than each observation in the first sample.

#### Step 4: Find the upper critical value (CV) from the statistical tables.

#### Step 5: Find the lower critical value, if needed, and state the critical region.

$$CV_L = \frac{1}{2}n(n+1) - w_c$$

where  $w_c$  is the upper CV.

$$CV_L = mn - u_c$$

where  $u_c$  is the upper CV.

#### Step 6: Check whether the TS lies in the critical region.

#### Step 7: State your conclusion in context.