

You will recall from GCE Mathematics AS Unit 2 that hypothesis testing involves three key steps:

- ◆ stating the null and alternative hypotheses, H_0 and H_1
- ◆ taking a sample of the population and applying a statistical test
- ◆ stating the conclusion of the test together with a comment on the population.

In A2 Unit 4, you will need to be able to carry out hypothesis testing on:

- ◆ correlation coefficients as measures of how close data points lie to a straight line
- ◆ the mean of a Normal distribution with known, given or assumed variance.

Hypothesis testing for correlation coefficients

Example

Average social interactions versus Average Self reported happiness for a random sample of 17 individuals



A well-being researcher is interested in the relationship between social interactions and happiness. A random sample of 17 participants record their happiness, on a scale of 1 to 10, and the number of social interactions they have over a period of one week, providing the researcher with an average of both numbers. The graph on the left shows the results. The product moment correlation coefficient (PMCC) is 0.885. Test at the 5% level of significance whether the correlation is significant.

$$H_0: \rho = 0 \quad H_1: \rho \neq 0$$

This is the PMCC

$$\text{Test statistic} = 0.885$$

$$\text{Critical Value} = 0.4821$$

This comes from the statistical tables with $n = 17$, two tail 5%

Since $TS > CV$ there is evidence to reject H_0 .

There is sufficient evidence to suggest that there is a correlation between the number of social interactions and perceived happiness.

Hypothesis testing for the mean of a Normal distribution

Example

In a certain test, student test scores, X , were found to be Normally distributed with mean 1100 and standard deviation 120, i.e. $X \sim N(1100, 120^2)$.

The distribution of the sample mean, \bar{X} , is given by $N(\mu, \frac{\sigma^2}{n})$.

A random sample of 50 students is selected and their mean test score is calculated and found to be 1050. Test at the 5% significance level whether the mean test score in this sample is less than expected.

Solution

$$H_0: \mu = 1100, H_1: \mu < 1100$$

$$P(\bar{X} < 1050 | H_0) = 0.0016$$

Since $0.0016 < 0.05$ there is sufficient evidence to reject H_0 . There is evidence to suggest that the mean test score is less than expected.

Note: The value of 0.0016 has been calculated using the Normal cumulative distribution function on the calculator with a lower limit of -9×10^{99} , an upper limit of 1050, $\sigma = \frac{120}{\sqrt{50}}$ and $\mu = 1100$.

Remember to consider the relevance of the correlation. Here it seems intuitive that there would be a causal factor, that is, more social interactions make people happier, but it may not always be the case that the correlations are meaningful!