Topic 4.2 Statistical distributions

Continuous uniform distribution

In this distribution, any intervals of equal length are equally likely to occur between a minimum value a and a maximum value b.

Example

A cylindrical glass with radius 3 cm and height 12 cm is filled with some water. Find the probability that the volume of water will be more than 200 ml.

Solution

The height of water, W, has the distribution U(0,12).
The height of water for 200 ml will be $\frac{200}{\pi \times 3^2} = 7{\cdot}07$ cm.

We need $\mathbf{P}(W > 7{\cdot}07) = \frac{12 - 7{\cdot}07}{12 - 0} = 0{\cdot}411$.

Normal distribution

Many real-life situations can be modelled using a Normal distribution $N(\mu,\sigma^2)$, where μ is the mean and σ^2 is the variance. You will need to be able to use your calculator to evaluate probabilities from a Normal distribution.

Example

In a certain test, student scores, X, were found to be Normally distributed, i.e. $X \sim N(1100, 120^2)$.

Find the proportion of students who scored more than 1250 in the test.

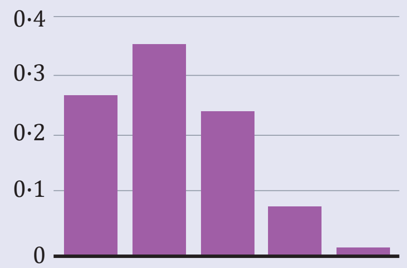
Solution

$\mathbf{P(X > 1250) = 0{\cdot}1056}$, by using your calculator with a lower limit of 1250, an upper limit of 9×10^{99}, $\sigma = 120$, $\mu = 1100$.

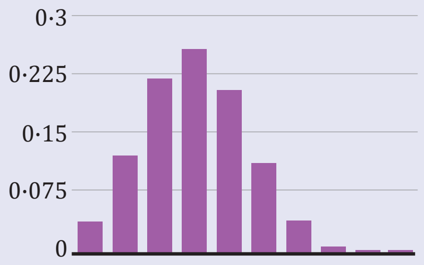
Some supplementary considerations

For a large enough value of n, the **binomial distribution** can be modelled by the Normal distribution.

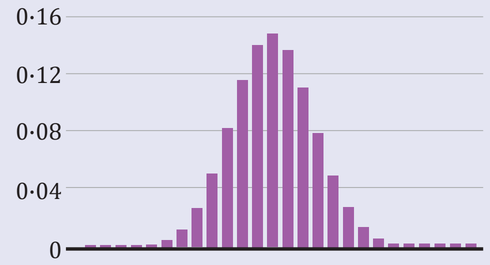
As n increases, the distribution is beginning to look like a Normal distribution.



$n = 5, p = 0{\cdot}4$



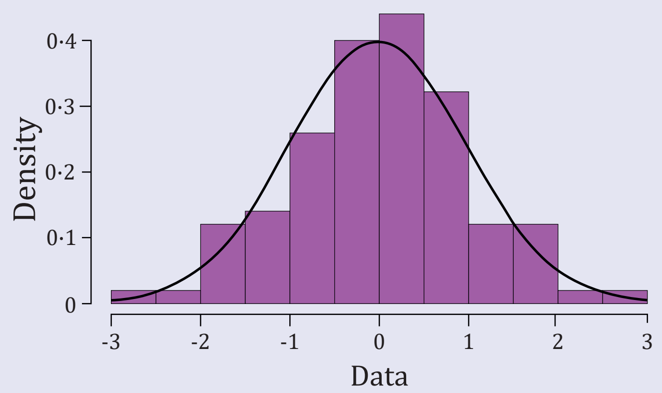
$n = 10, p = 0{\cdot}4$



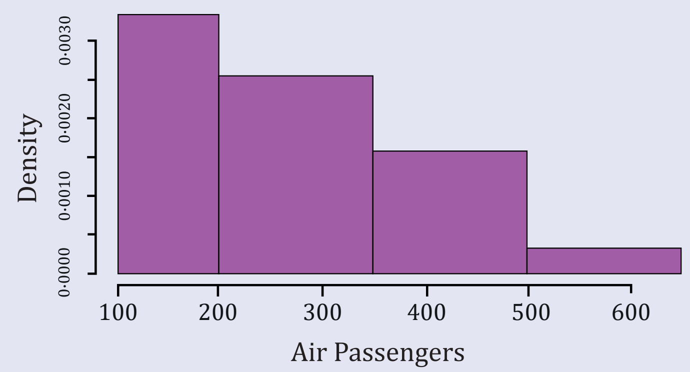
$n = 30, p = 0{\cdot}4$

Histograms

Histograms can also be modelled using the Normal distribution.



Although, it is not always appropriate to model using a Normal distribution, as seen in the data below, which is skewed.



Remember:

For a binomial distribution:

- there must be a fixed number of trials
- each observation must be independent
- each observation must represent two outcomes, success or failure
- the probability of success must be the same for each trial.

An example of modelling a real-life situation using a binomial distribution could be:

A treatment for foot pain is successful in 60% of people. In a random sample of 10 people, the number of people whose treatment was successful, X, can be modelled by $X \sim B(10,0{\cdot}6)$.

However, if there are other factors which affect the treatment, then the binomial distribution may not be an appropriate model. In this case, the probability of success may not be the same for each person.

Remember:

For a Poisson distribution:

- each event must occur singularly and be independent of any other event
- each event must occur at random, at a constant rate in time or space.

An example of modelling a real-life situation with a Poisson distribution could be:

The number of customers that enter a coffee shop occurs at a rate of 5 per hour. To find the number of customers that arrive in a randomly selected 2-hour time frame, you could use a Poisson distribution, $Y \sim Po(10)$.

However, if customers arrive together, then this may not be a good model, as the events may not be independent.

Selecting an appropriate distribution including the Normal distribution, could include the Poisson, binomial and uniform distributions.