GCE Mathematics, A2 Unit 4, Applied Mathematics B, Statistics

Topic 4.1 Probability

Conditional probability

Conditional probability is the probability of an event happening, given that another event has already happened.

The formula used is

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

Or more commonly,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

We use probability models to try and describe real life situations. You should be prepared to critique assumptions made and comment on the likely effects of more realistic assumptions.

Example

Sioned observes that the number of buses, X, that drive past her on her 30-minute walk to school, follows a Poisson distribution with mean 1·4. She uses her calculator to calculate the probability that no buses will pass her on a randomly selected day in the future.

$P(X = 0) = 0{\cdot}2466$

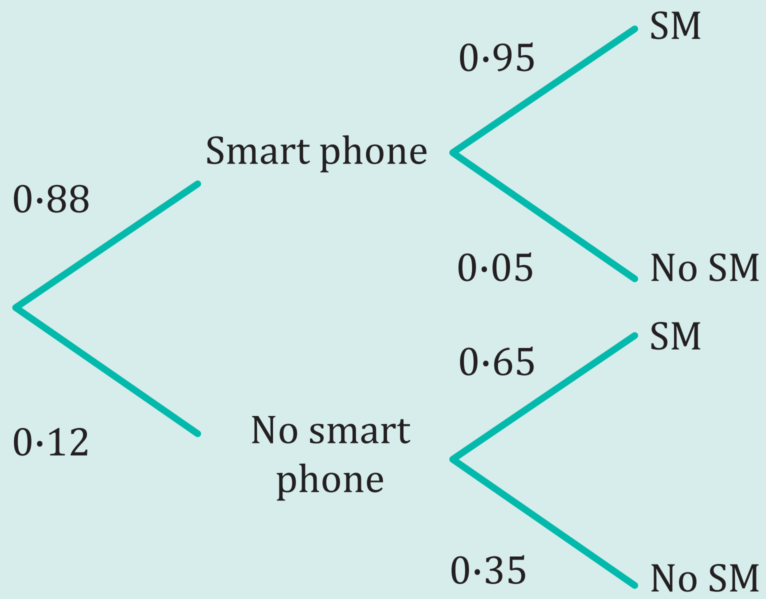
The following Sunday, she plans to walk to school and says that the probablity that no buses will drive past her is 0·2466. Explain why Sioned may not be right.

Answer: Sunday may be a quiter day and so it's realistically likely to be more than 0·2466.

Example using **tree diagrams**

It is known that in a certain population, 12% of the population do not have a smart phone (SP). Of those who do have a smart phone, 95% use social media (SM) regularly. Of those that do not have a smart phone, only 65% use social media regularly.

Given that a randomly chosen person uses social media regularly, find the probability that they do not have a smart phone.

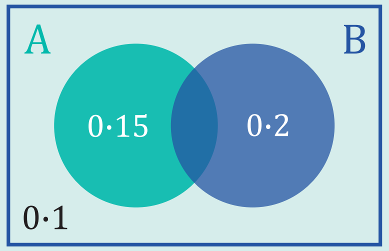


$$P(NoSP|SM) = \frac{P(NoSP \cap SM)}{P(SM)}$$

$$P(NoSP|SM) = \frac{0{\cdot}12 \times 0{\cdot}65}{0{\cdot}88 \times 0{\cdot}95 + 0{\cdot}12 \times 0{\cdot}65}$$

$$= \frac{39}{457} \approx 0{\cdot}0853$$

Example using **Venn diagrams**



A and B are two events. Using the Venn diagram, find $P(A|B)$.

$$P(A \cap B) = 1 - (0{\cdot}1 + 0{\cdot}15 + 0{\cdot}2)$$

$$P(A \cap B) = 0{\cdot}55$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{0{\cdot}55}{0{\cdot}2 + 0{\cdot}55}$$

$$P(A|B) = \frac{11}{15} \approx 0{\cdot}7333$$

Example using **two way tables**

The two way table below shows the Urdd activities in which 50 students participated.

	Football	Swimming	Dancing
Primary school	9	13	4
Secondary school	12	7	5

A child is selected at random. Given that they took part in swimming, find the probability that they are in primary school.

$$P(\text{Primary}|\text{Swimming}) = \frac{P(\text{Primary} \cap \text{Swimming})}{P(\text{Swimming})}$$

$$P(\text{Primary}|\text{Swimming}) = \frac{13/50}{20/50} = \frac{13}{20}$$