AS Computer Science
Unit 1: Boolean algebra

Terminology

- TRUE will be represented by 1
- FALSE will be represented by 0
- Variables will be single letters e.g. A
- Logical operator OR will be +
  - A+B represents A OR B
- Logical operator AND will be •
  - A.B represents A AND B
- Logical operator NOT will be an overbar
  - $\bar{A}$ means NOT A
- Logical operator XOR will be ⊕
  - A⊕B means A XOR B

Order of precedence

There is an order of precedence for operations in Boolean algebra, just like BIDMAS is used in mathematical algebra.

The order of precedence is (highest first):
Brackets, NOT, XOR, AND, OR

Simplifying Boolean expressions

Using the OR operation

- Identity law – A + 0 = A
- Annulment law – A + 1 = 1
- Idempotent law – A + A = A
- Inverse law – A + $\bar{A}$ = 1

Using the AND operator:

- Identity law – A.1 = A
- Annulment law – A.0 = 0
- Idempotent law – A.A = A
- Complement law – A. $\bar{A}$ = 0

Commutative Law

The commutative laws of Boolean algebra are:

A.B = B.A

A+B = B+A

A⊕B = B⊕A

Associative Law

The associative laws of Boolean algebra are:

A.(B.C) = (A.B).C

A +(B+C) = (A+B) + C

A ⊕ (B⊕C) = (A⊕B) ⊕C

Absorptive law

A + (A.B) = A

A	B	A.B	A + (A.B)
1	1	1	1
1	0	0	1
0	1	0	0
0	0	0	0

The truth table shows that the final column is A
Similarly, it can be shown that A.(A+B) = A

Working with brackets

The distributive law is used to expand brackets.

A,(B + C) = A.B + A.C

Factorising expressions can be done if terms have a common factor.

A.B + A.C = A.(B + C)

Example question and solution

AS examinations include questions involving both truth tables and Boolean algebra. Here are two examples from the 2018 examination.

1. Complete the following truth table. [4]

A	B	C	A OR C	B AND C	(A OR C) XOR (B AND C)	NOT ((A OR C) XOR (B AND C))
0	0	0				
0	1	0				
1	0	0				
1	1	0				
0	0	1				
0	1	1				
1	0	1				
1	1	1				

4. Clearly showing each step, simplify the following expression using Boolean identities and rules:

$A.(\bar{A} + B) + \bar{C}.(A + B) + A.(\bar{B} + C) + \bar{B}.B$ [8]

Answers:

A OR C	B AND C	(A OR C) XOR (B AND C)	NOT ((A OR C) XOR (B AND C))
0	0	0	1
1	0	1	0
1	0	1	0
1	0	1	0
1	0	1	0
1	0	1	0
1	0	1	0
1	1	0	1

$A.(\bar{A} + B) + \bar{C}.(A + B) + A.(\bar{B} + C) + \bar{B}.B$
$A.(\bar{A} + B) + \bar{C}.(A + B) + A.(\bar{B} + C)$
$A.\bar{A} + A.B + \bar{C}.A + \bar{C}.B + A.\bar{B} + A.C$
$A.B + \bar{C}.A + \bar{C}.B + A.\bar{B} + A.C$
$A.(B + \bar{B}) + \bar{C}.A + \bar{C}.B + A.C$
$A.(B + \bar{B}) + A.(\bar{C} + C) + \bar{C}.B$
$A.(1) + A.(1) + \bar{C}.B$
$A + \bar{C}.B$
$A + B.\bar{C}$