GCE Mathematics, A2 Unit 3 Pure Mathematics B

Topic 3.2 - Algebra and functions B

The modulus function

The modulus of x is written as $|x|$ and is the absolute value of x, i.e. the positive value of x whether x itself is positive or negative. Examples: $|4| = 4$ and $|-7| = 7$.

Solving algebraic equations using the modulus function

$|x| = 5$ means $x = \pm 5$
$|y - 2| = 4$ means $y - 2 = \pm 4$, so $y = 6$ or $y = -2$

Consider $2|x - 5| = 6$.

Rearranging gives $|x - 5| = 3$, leading to $x = 8$ or $x = 2$.

To solve the following: $\dfrac{4|x| + 2}{|x| + 4} = 2$

Multiply both sides by the denominator $|x| + 4$ and then rearrange.

$4|x| + 2 = 2|x| + 8$
$2|x| = 6$
$|x| = 3$
$x = \pm 3$

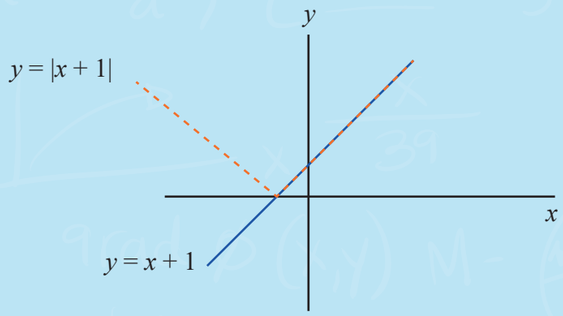
To solve the following: $|2x - 3| \geq 7$

We need to split the inequality into two separate inequalities.

$2x - 3 \geq 7$ or $2x - 3 \leq -7$
$2x \geq 10$ or $2x \leq -4$
$x \geq 5$ or $x \leq -2$

Graphs of modulus functions

The graph of $y = |f(x)|$ is obtained by reflecting any parts of the graph of $f(x)$ below the x-axis in the x-axis.



Graph transformations

You need to be able to apply combinations of the following transformations to a given function $f(x)$ and sketch the resulting graphs.

- $y = f(x) + a$ Translation of a units parallel to the y-axis.
- $y = f(x + a)$ Translation of a units parallel to the x-axis.
  If $a < 0$, the translation is in the positive direction of the x-axis.
  If $a > 0$, the translation is in the negative direction of the x-axis.
- $y = -f(x)$ Reflection in the x-axis.
- $y = af(x)$ One-way stretch by a scale factor a parallel to the y-axis.
- $y = f(ax)$ One-way stretch by a scale factor $\dfrac{1}{a}$ parallel to the x-axis.

Exponential and logarithmic functions

The domain of the exponential function $y = e^x$ is the set of all real numbers, and the function is a one-to-one function. Therefore, the inverse function of $f(x) = e^x$ exists and is given by $f^{-1}(x) = \ln x$. Both graphs are shown below – each is a reflection of the other in the line $y = x$.

$y = e^x$

Crosses the y-axis at $y = 1$.

x-axis is an asymptote.

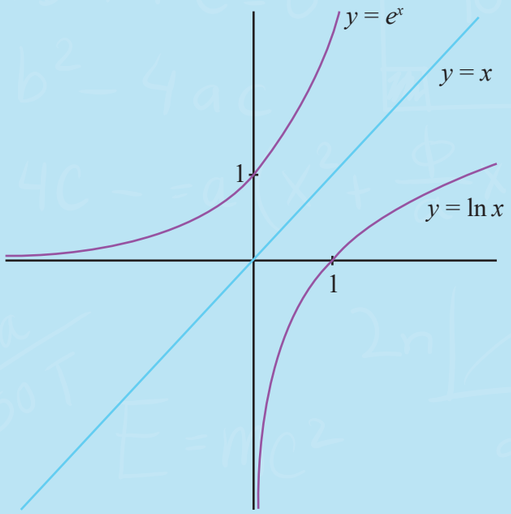
Domain of f is $(-\infty, \infty)$

Range of f is $(0, \infty)$

$y = \ln x$

Crosses the x-axis at $x = 1$.

y-axis is an asymptote.

Domain of f^{-1} is $(0, \infty)$

Range of f^{-1} is (∞, ∞)