

# SIMULTANEOUS EQUATIONS

Simultaneous equations are two equations that each contain two unknowns. When solving simultaneous equations we need to find the values of both these unknowns. We solve these equations using either algebraic or graphical methods.

**Solving simultaneous equations using the elimination method** This method eliminates one of the unknowns from the equation so that only one remains. This is done by adding or subtracting the equations.

## When to add and when to subtract the equations?

**SUBTRACT** when the terms to be eliminated are of the **SAME** sign i.e. + and + or - and - .

**ADD** when the terms to be eliminated are of **A DIFFERENT** sign i.e. + and -

## Same number of the unknown term

E.g. Solve the following simultaneous equations

$$\begin{aligned} 5x + y &= 13 \\ 3x + y &= 9 \end{aligned}$$

$$5x + y = 13 \dots (1)$$

$$3x + y = 9 \dots (2)$$

$$(1) - (2)$$

$$5x + y = 13$$

$$-3x + y = 9$$

$$\hline 2x = 4$$

$$x = 2$$

$$5 \times 2 + y = 13$$

$$10 + y = 13$$

$$y = 3$$

1) Label the equations (1) and (2)

2) Identify the unknown to eliminate, where we have an equal number of the unknown in each equation.

3) Subtract the equations (+y and +y have the same sign).

4) Solve the equation to find x.

5) Substitute the x value back into equation (1) to find the value of y.

**Remember to check your answers by substituting both values back into one of the original equations.**

## Worded problems

This is where the equations will need to be formed using the information given before solving them. E.g. Owain sells ice cream and ice lollies on his van. In one street he sells four ice creams and five ice lollies and takes £12. In another street he sells five ice creams and three lollies and takes £11.10. Form and solve simultaneous equations to find the price of an ice cream and the price of an ice lolly.

1) Assign a letter for each of the unknowns and use the information to form two equations.

x - the price of an ice cream    y - the price of an ice lolly

$$4x + 5y = 12 \dots (1) \times 3$$

$$5x + 3y = 11.10 \dots (2) \times 5$$

2) Label the equations (1) and (2)

3) Choose an unknown to eliminate. In this case, we'll choose y. To get the same number of y's we multiply (1) with 3 and (2) with 5 to give 15y.

4) Label the 'new' equations (3) and (4)

5) Subtract the equations (+15y and +15y have the same sign).

6) Solve the equation to find x.

7) Substitute the x value back into equation (1) to find the value of y.

$$12x + 15y = 36 \dots (3)$$

$$25x + 15y = 55.50 \dots (4)$$

$$(4) - (3)$$

$$25x + 15y = 55.50$$

$$-12x + 15y = 36.00$$

$$\hline 13x = 19.50$$

$$x = 1.5$$

$$4 \times 1.5 + 5y = 12$$

$$6 + 5y = 12$$

$$5y = 6$$

$$y = 1.2$$

Ice cream costs £1.50

Ice lolly costs £1.20

## Check that you:

- add, subtract, multiply and divide positive and negative numbers
- solve linear equations  
e.g.  $3x - 7 = 17$        $5(x + 3) = 35$   
 $3x = 24$                $5x + 15 = 35$   
 $x = 8$                    $5x = 20$   
                                  $x = 4$
- substitute values into an expression or equation
- draw straight line graphs knowing the equation of the line.

## Different number of the unknown term

E.g. Solve the following simultaneous equations

$$4x + 3y = 14$$

$$3x - 2y = 19$$

$$4x + 3y = 14 \dots (1) \times 2$$

$$3x - 2y = 19 \dots (2) \times 3$$

$$8x + 6y = 28 \dots (3)$$

$$9x - 6y = 57 \dots (4)$$

$$(3) + (4)$$

$$8x + 6y = 28$$

$$+ 9x - 6y = 57$$

$$\hline 17x = 85$$

$$x = 5$$

$$4 \times 5 + 3y = 14$$

$$20 + 3y = 14$$

$$3y = -6$$

$$y = -2$$

1) Label the equations (1) and (2)

2) Choose an unknown to eliminate. In this case, we'll choose y. To get the same number of y's we multiply (1) with 2 and (2) with 3 to give 6y and -6y

3) Label the 'new' equations (3) and (4)

4) Add the equations (+6y and -6y have a different sign).

5) Solve the equation to find x.

6) Substitute the x value back into equation (1) to find the value of y.

## Solving simultaneous equations using graphs

To do this we plot the two equations on the same graph. The coordinates of the point where the two lines meet are the x and y value that satisfy both equations.

E.g. Solve the following simultaneous equations  $x + y = 6$   
 $2x - y = -3$

using a graphical method.

1) i) Plot the line  $x + y = 6$  by drawing a table of values and substituting x values into  $x + y = 6$  to find the y values.

x	0	1	2
y	6	5	4

ii) Plot (0, 6) (1, 5) and (2, 4) and connect them with a straight line.

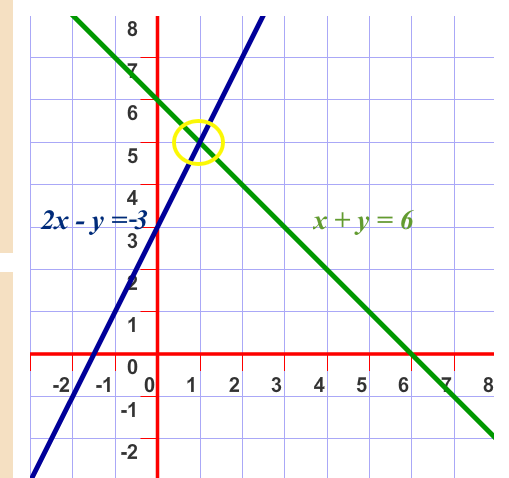
iii) Label the line  $x + y = 6$

2) i) Plot the line  $2x - y = -3$  by drawing a table of values and substituting x values into  $2x - y = -3$  to find the y values.

x	0	1	2
y	3	5	7

ii) Plot (0, 3) (1, 5) and (2, 7) and connect them with a straight line.

iii) Label the line  $2x - y = -3$



3) Find the point where both lines intersect

The lines  $2x - y = -3$  and  $x + y = 6$  intersect at the point (1, 5). Therefore  $x = 1$  and  $y = 5$ .