THE NTH TERM

A sequence is a number pattern that follows a rule. We can find any term in a sequence using the *n*th term rule and knowing the position of the term we want within the sequence i.e 1st (*n*=1), 2nd (*n*=2), 3rd (*n*=3)... 50th (*n*=50).

Check that you can:

- recognise simple number patterns
- find the next term in a number pattern using the simple term-to-term rule
  e.g 3, 7, 11, 15, 19, 23 (+4 +4 +4 +4 +4)
- draw the next diagram in a pattern 
- substitute values into expressions.

Finding the *n*th term rule - Linear sequences

Linear sequences The difference between the terms of a linear sequence is always the same.

Example Find the *n*th term for the following sequences

a) 3, 6, 9, 12, 15...

n	1	2	3	4	5
term	3	6	9	12	15

(+3 between each pair)

The difference is +3
Multiply *n* (the position) by 3 gives the term.
*n*th term = 3 x *n*

*n*th term = $3n$

b) 5, 10, 15, 20, 25...

n	1	2	3	4	5
term	5	10	14	20	25

(+5 between each pair)

The difference is +5
Multiply *n* (the position) by 5 gives the term.
*n*th term = 5 x *n*

*n*th term = $5n$

c) -2, -4, -6, -8, -10...

n	1	2	3	4	5
term	-2	-4	-6	-8	-10

(-2 between each pair)

The difference is -2
Multiply *n* (the position) by -2 gives the term.
*n*th term = -2 x *n*

*n*th term = $-2n$

d) 7, 10, 13, 16, 19...

n	1	2	3	4	5
term	7	10	13	16	19

(+3 between each pair)

The difference is +3 (as in the sequence above).
The *n*th term rule starts with $3n$.
Comparing the sequence with $3n$ we see each term has moved up four places (+4).

*n*th term = $3n + 4$

e) 2, 7, 12, 17, 22...

n	1	2	3	4	5
term	2	7	12	17	22

(+5 between each pair)

The difference is +5 (as in the sequence above).
The *n*th term rule starts with $5n$.
Comparing the sequence with $5n$ we see each term has moved down three places (-3).

*n*th term = $5n - 3$

f) 4, 2, 0, -2, -4...

n	1	2	3	4	5
term	4	2	0	-2	-4

(-2 between each pair)

The difference is -2 (as in the sequence above).
The *n*th term rule starts with $-2n$.
Comparing the sequence with $-2n$ we see each term has moved up six places (+6) from $-2n$.

nth term = $-2n + 6$

Using the *n*th term rule

Finding terms within a linear sequence

1)Find the first three terms of the sequence with *n*th term = $3n - 4$.

If $n = 1$ then $3 \times 1 - 4 = \mathbf{-1}$
If $n = 2$ then $3 \times 2 - 4 = \mathbf{2}$
If $n = 3$ then $3 \times 3 - 4 = \mathbf{5}$

-1, 2, 5, ...

2) Find the 100th term of the sequence with *n*th term = $-4n + 8$.

If $n = 100$ then $-4 \times 100 + 8 = \mathbf{-392}$

100^{th} term = -392

Determining if a term is in a sequence

Determine if 254 is a term in the sequence *n*th term = $3n - 2$.

Write and solve an equation using the term and the *n*th term rule. If *n* (the terms position) is a whole number then the term will be part of the sequence.

$3n - 2 = 254$

$3n = 256$

$n = \frac{256}{3} = 85\frac{1}{3}$ therefore 254 cannot be in the sequence

Finding terms within a quadratic sequence

1) Find the first three terms of the sequence with *n*th term = $n^2 + 5$.

If $n = 1$ then $1^2 + 5 = \mathbf{6}$
If $n = 2$ then $2^2 + 5 = \mathbf{9}$
If $n = 3$ then $3^2 + 5 = \mathbf{14}$

6, 9, 14, …

2) Find the first three terms of the sequence with *n*th term = $2n^2 - 1$.

If $n = 1$ then $2 \times 1^2 - 1 = \mathbf{1}$
If $n = 2$ then $2 \times 2^2 - 4 = \mathbf{7}$
If $n = 3$ then $2 \times 3^2 - 1 = \mathbf{17}$

1, 7, 17, ...

Finding the *n*th term rule - Quadratic sequences

Quadratic sequence The *n*th term rule for a quadratic sequence will contain n^2 and this will be the highest power of n.

The first difference between each term of a quadratic sequence changes (although it will follow a pattern) so we look at the second difference, which stays the same.

If the second difference is $2a$ then the sequence starts with an^2
e.g If second difference is 2, the sequence starts with n^2. If the second difference is 4, the sequence starts with $2n^2$

Example Find the *n*th term for the following sequences

a) 3, 6,11,18, 27...

(+3 +5 +7 +9 / +2 +2 +2)

1)The second difference is +2 so the sequence starts with n^2

2) Draw a table and compare n^2 with the sequence

n	1	2	3	4	5
n^2	1	4	9	16	25
term	3	6	11	18	27

3) Each term has moved up two places (+2) from n^2

*n*th term = $n^2 + 2$

b) -1, 8, 23, 44, 71...

(+9 +15 +21 +27 / +6 +6 +6)

1)The second difference is +6 so the sequence starts with $3n^2$

2)Draw a table and compare n^2 with the sequence

n	1	2	3	4	5
$3n^2$	3	12	27	48	75
term	-1	8	23	44	71

3) Each term has moved down four places (-4) from $3n^2$

*n*th term = $3n^2 - 4$

Don't forget to check your *n*th term rule by substituting the *n* values back into your rule in order to get the terms.

Remember that *n* is the position of the term within the sequence so it's this 'position' that we substitute into our rule in order to find the term.