

A formula is an algebraic rule that contains a mixture of letters, numbers and mathematical symbols. It uses the letters to represent values that can be changed (variables). A formula is different to an expression as it always contains the '=' sign.

Writing a formula When we write a rule to describe the relationship between variables using letters instead of words.

E.g. Sioned is looking to change her mobile phone contract. The monthly cost is 4p a minute for calls plus £12 for unlimited texts and data plus vat at 5%. If m is the number of minutes used a month, write down the formula for the monthly cost C in terms of m .

$$\text{Cost} = (\text{£}0.04 \times \text{the number of minutes} + \text{£}12) \times 1.05$$

$$C = (0.04 \times m + 12) \times 1.05$$

$$C = 1.05(0.04m + 12)$$

Substituting into a formula When we replace the letters in a formula with numbers.

1 E.g. The monthly cost of a mobile phone contract is given by the formula $C = 1.05(0.04m + 12)$. If m is the number of minutes used a month, find C the monthly cost when 250 minutes are used.

$$C = 1.05(0.04m + 12)$$

$$C = 1.05 \times (0.04 \times 250 + 12)$$

$$C = 1.05 \times (10 + 12)$$

$$C = \text{£}23.10$$

2 E.g. The formula to convert Fahrenheit (F) to Celsius (C) is given by $C = \frac{5}{9}(F - 32)$.

(a) If $F = 86$ find C

$$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}(86 - 32)$$

$$C = \frac{5}{9}(54)$$

$$C = 30$$

(b) If $F = -13$ find C

$$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}(-13 - 32)$$

$$C = \frac{5}{9}(-45)$$

$$C = -25$$

The subject of the formula In $m = 3n - p$, m is the subject of the formula, it's the letter that is on its own on one side of the '='.

$$a = 2b + 3c \quad a \text{ is the subject of the formula} \quad 5r^2 - t = s \quad s \text{ is the subject of the formula}$$

We can change the subject of the formula by rearranging the formula using the balancing method and our knowledge of inverse operations.

Take care when substituting negative values, fractions or decimals into a formula.

Changing the subject of the formula when it only appears on one side

E.g. **1.** Make b the subject of the formula $a = 4b - 5$

$$\begin{aligned} a &= 4b - 5 \\ +5 & \quad +5 \\ a + 5 &= 4b \\ \div 4 & \quad \div 4 \\ \frac{a+5}{4} &= b \quad \text{or} \quad b = \frac{a+5}{4} \end{aligned}$$

E.g. **2.** Make y the subject of the formula $2y + 3x = c$

$$\begin{aligned} 2y + 3x &= c \\ -3x & \quad -3x \\ 2y &= c - 3x \\ -3x & \quad -3x \\ y &= \frac{c - 3x}{2} \end{aligned}$$

E.g. **3.** Make r the subject of the formula $V = \pi r^2 h$

$$\begin{aligned} V &= \pi r^2 h \\ \div \pi h & \quad \div \pi h \\ \frac{V}{\pi h} &= r^2 \\ \sqrt{\quad} & \quad \sqrt{\quad} \\ \sqrt{\frac{V}{\pi h}} &= r \end{aligned}$$

E.g. **4.** Make x the subject of the formula $y = \sqrt{5x + z}$

$$\begin{aligned} y &= \sqrt{5x + z} \\ \dots^2 & \quad \dots^2 \\ y^2 &= 5x + z \\ -z & \quad -z \\ y^2 - z &= 5x \\ \div 5 & \quad \div 5 \\ \frac{y^2 - z}{5} &= x \end{aligned}$$

Remember '=' means equal to so both sides must balance at all times, so make sure you do exactly the same to both sides of the formula

Changing the subject of the formula when it appears on both sides

We need to make sure that the subject of the formula only appears on one side. It can't appear on both.

E.g. **1.** Make a the subject of the formula $4a + 20b = 6c - 3a$

$$4(a + 5b) = 6c - 3a$$

$$+3a \quad +3a$$

$$7a + 20b = 6c$$

$$-20b \quad -20b$$

$$7a = 6c - 20b$$

$$\div 7 \quad \div 7$$

$$a = \frac{6c - 20b}{7}$$

Expand the bracket

Collect the a terms on the one side of the equation

Collect everything else on the other side of the equation

E.g. **2.** Make x the subject of the formula $6x + 7c = 4y - ax$

$$6x + 7c = 4y - ax$$

$$6x + 7c = 4y - ax$$

$$+ax \quad +ax$$

$$6x + ax + 7c = 4y$$

$$-7c \quad -7c$$

$$6x + ax = 4y - 7c$$

$$x(6+a) = 4y - 7c$$

$$\div (6+a) \quad \div (6+a)$$

$$x = \frac{4y - 7c}{6+a}$$

Collect the x terms on the one side of the

Collect everything else on the other side of the equation

Factorise to isolate the x