WRITING A NUMBER AS A PRODUCT OF ITS PRIME FACTORS

This is when a number needs to be broken down into its prime factors. There is more than one way to do this but in the following examples we will look at the factor tree method.

Check first that you:

- can use multiplication facts to find factors of a number e.g. 1 x 12 = 12, 2 x 6 = 12 and 3 x 4 = 12 therefore the factors of 12 are 1, 2, 3, 4, 6 and 12.
- can recognise prime numbers e.g. 2, 3, 5, 7, 11, 13… (numbers with only two factors, 1 and the number itself)
- understand the use of powers e.g. $5 \times 5 \times 5 = 5^3$

Steps to success

Find a pair of factors (not including 1 and the number itself) to make the branches of the tree.

Continue to find pairs of factors for the numbers at the end of each branch to make more branches of the tree.

Draw a circle around a prime number when it appears at the end of a branch. This branch is then complete and no further factors can be found.

The tree is complete when all of the branches end with a circled prime number.

Then write these prime numbers as a product (multiplication) and use powers to simplify.

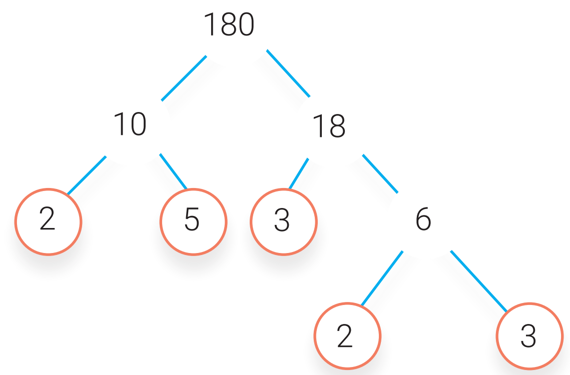
Examples

Express the following numbers as a product of their prime factors in index form

1) 180

180 = 2 x 5 x 3 x 2 x 3

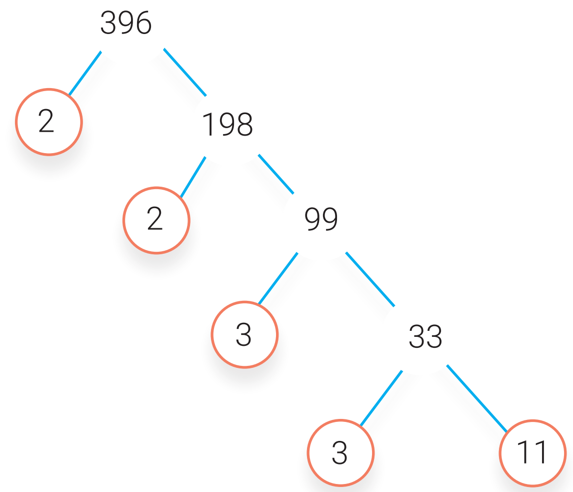
$180 = 2^2 \times 3^2 \times 5$



2) 396

396 = 2 x 2 x 3 x 3 x 11

$396 = 2^2 \times 3^2 \times 11$



Remember there is more than one way to complete a factor tree, as there are often a number of factors to choose from when breaking the number down. The prime factors will always be the same and so the final answer should always be the same.

Check your answer! When you multiply the prime factors do you get the original number?

A perfect square?

By writing a number as a product of its prime factors we can determine if a number is a perfect square. In perfect square numbers all prime factors appear an even number of times. For example:

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$ ✓ *Perfect square*

$500 = 2 \times 2 \times 5 \times 5 \times 5 = 2^2 \times 5^3$ ✗ *Not a perfect square*

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$ ✓ *Perfect square*

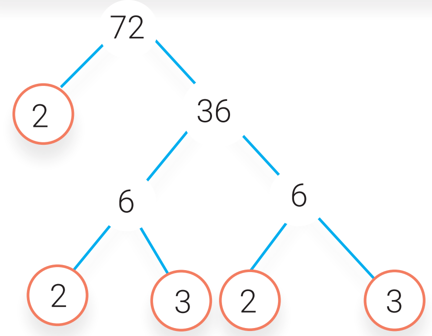
$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$ ✗ *Not a perfect square*

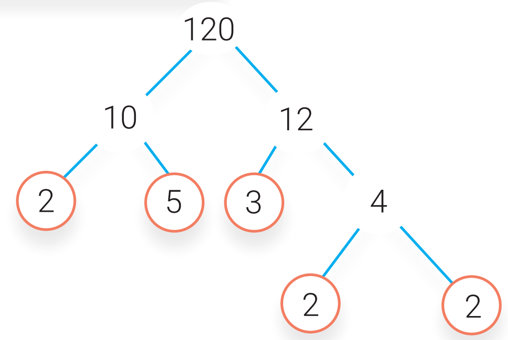
Take care not to mix up the method of finding the highest common factor (HCF) and the method of finding the lowest common multiple (LCM).

Highest common factor (HCF) and Lowest common multiple (LCM)

Example Find the highest common factor and the lowest common multiple of 72 and 120

STEP 1 Complete factor trees for 72 and 120





STEP 2 Write 72 and 120 as a product of their prime factors

72 = 2 x 2 x 3 x 2 x 3

$72 = 2^3 \times 3^2$

120 = 2 x 5 x 3 x 2 x 2

$120 = 2^3 \times 3 \times 5$

STEP 3 Find the HCF by multiplying the common prime factors of 72 and 120

72 = **2** x **2** x **2** x **3** x 3

120 = **2** x **2** x **2** x **3** x 5

HCF = **2** x **2** x **2** x **3** = 24

STEP 4 Find the LCM by multiplying all the listed prime factors of 72 and 120 remembering not to repeat any numbers that appear in both lists (those written in red).

72 = **2** x **2** x **2** x **3** x **3**

120 = **2** x **2** x **2** x **3** x **5**

LCM = **2** x **2** x **2** x **3** x **3** x **5** = 360