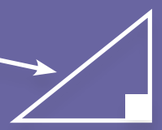
TRIGONOMETRY (right-angled triangles)

Trigonometry allows us to find missing sides and angles in right-angled triangles. It uses the sine, cosine and tangent ratios. These are special ratios connecting the length of two sides and an angle in a right-angled triangle. If we are given any two of these then we can find the third.

Check that you can recognise:

- a right-angled triangle
- the hypotenuse - the longest side in a right-angled triangle

Hypotenuse



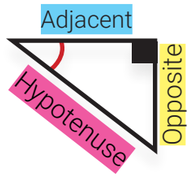
Labelling the right-angled triangle

Each side of the triangle must be labelled.

Hypotenuse (H) - This is the longest side in a right-angled triangle.

Opposite (O) – This is the side that lies directly opposite the marked angle.

Adjacent (A) – This is the side that lies next to the marked angle.

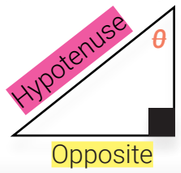


Once we know what sides and angles we know or are looking for we can then choose the correct ratio to use.

The sine ratio (SOH)

When working with the angle θ and the opposite and hypotenuse sides we use the sine ratio.

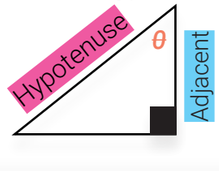
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$



The cosine ratio (CAH)

When working with the angle θ and the adjacent and hypotenuse sides we use the cosine ratio.

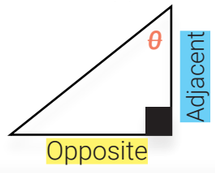
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$



The tangent ratio (TOA)

When working with the angle θ and the adjacent and opposite sides we use the tangent ratio.

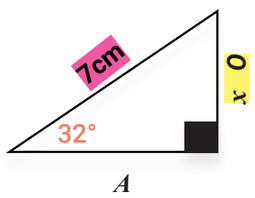
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



- SOH CAH TOA is used to help us remember which ratio connects which sides.
- θ ('theta') is the symbol used in trigonometry to represent the unknown angle.

Finding a side. To find the length of a side in a right-angled triangle you must know the length of one other side and the size of an angle (other than the right-angle!)

E.g.1) Find the length of x.



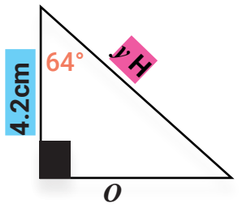
SOH CAH TOA

$\sin\theta = \frac{o}{h}$ $\quad \sin 32° = \frac{x}{7}$

$7 \times \sin 32° = x$

$x = 3.7\text{cm}$ (to 1 d.p.)

E.g. 2) Find the length of y.



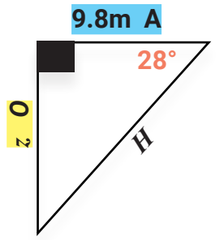
SOH CAH TOA

$\cos\theta = \frac{a}{h}$ $\quad \cos 64° = \frac{4.2}{y}$

$y = \frac{4.2}{\cos 64°}$

$y = 9.6\text{cm}$ (to 1 d.p.)

E.g. 3) Find the length of z.



SOH CAH TOA

$\tan\theta = \frac{o}{a}$ $\quad \tan 28° = \frac{z}{9.8}$

$9.8 \times \tan 28° = z$

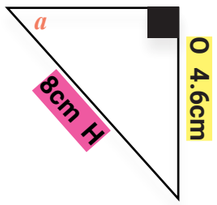
$z = 5.2\text{m}$ (to 1 d.p.)

Steps to success

1. Label the sides of the triangle H, O and A. What side do you know and what side do you need?
2. Use ***SOH CAH TOA*** to determine the correct trig ratio to use.
3. Substitute your values into the correct equation.
4. Solve the equation and give your answer to an appropriate degree of accuracy

Finding an angle. To find the size of an angle in a right-angled triangle you must know the length of two sides.

E.g.1) Find the size of the angle a.



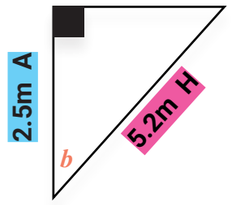
SOH CAH TOA

$\sin\theta = \frac{o}{h}$ $\quad \sin a = \frac{4.6}{8}$

$a = \sin^{-1}\frac{4.6}{8}$

$a = 35.1°$ (to 1 d.p.)

E.g. 2) Find the size of the angle b.



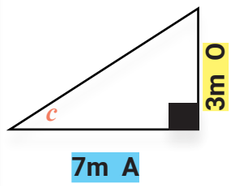
SOH CAH TOA

$\cos\theta = \frac{a}{h}$ $\quad \cos b = \frac{2.5}{5.2}$

$b = \cos^{-1}\frac{2.5}{5.2}$

$b = 61.3°$ (to 1 d.p.)

E.g. 3) Find the size of the angle c.



SOH CAH TOA

$\tan\theta = \frac{o}{a}$ $\quad \tan c = \frac{3}{7}$

$c = \tan^{-1}\frac{3}{7}$

$c = 23.2°$ (to 1 d.p.)

Steps to success

1. Label the sides of the triangle H, O and A. What side do you know and what side do you need?
2. Use ***SOH CAH TOA*** to determine the correct trig ratio to use.
3. Substitute your values into the correct equation.
4. Solve the equation and give your answer to an appropriate degree of accuracy.

Remember - SOH CAH TOA can only be used for right-angled triangles. Use Sin, Cos and Tan to find a missing side and their inverse $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ to find a missing angle.