TRIGONOMETRY (non-right-angled triangles)

Check that you are familiar with how to label triangles for this:

- Each angle is named using a (single) capital letter.
- Each side takes the lower case of the letter of the opposite angle e.g. in triangle ABC, side AB is denoted by the single letter c.

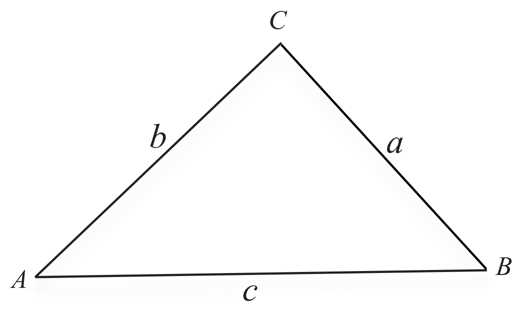
Extract from the formula page in the exam paper:

In any triangle *ABC*

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$



When you need to calculate a side or an angle in a non-right-angled triangle, remember that (including the side or angle you are trying to find):

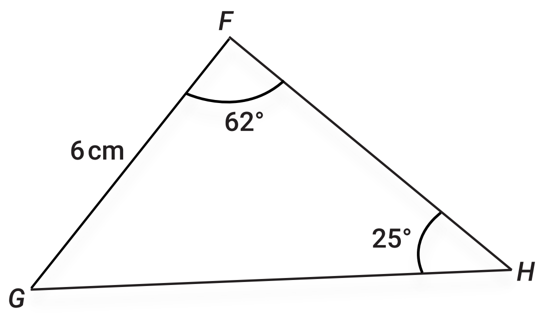
- the sine rule involves 2 sides and 2 angles
- the cosine rule involves 3 sides and 1 angle.

For the area formula, you need 2 sides and the angle between them.

For the triangle you are using, start by listing the sides and angles you know. If two angles are given in the triangle, it may be useful to get the third by subtracting from 180°.

Remember that for a right-angled triangle you should use SOH CAH TOA.

Example 1



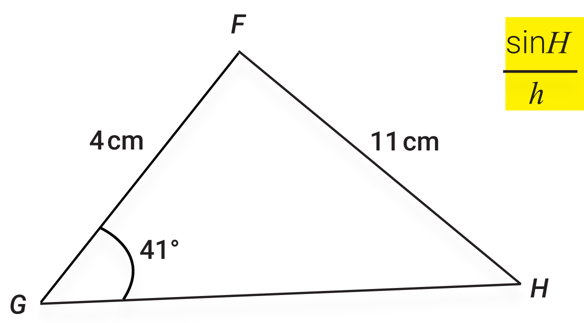
To find GH, we use f, h, F, H (using sine rule).

$\dfrac{f}{\sin F} = \dfrac{h}{\sin H}$ becomes $\dfrac{f}{\sin 62} = \dfrac{6}{\sin 25}$

then $f = \dfrac{6 \times \sin 62}{\sin 25}$

= 12·5cm (1 d.p.)

Example 2 To find $F\hat{H}G$, we use H, G, h, g (using sine rule).



$\dfrac{\sin H}{h} = \dfrac{\sin G}{g}$ becomes $\dfrac{\sin H}{4} = \dfrac{\sin 41}{11}$

then $\sin H = \dfrac{4 \times \sin 41}{11}$

$H = \sin^{-1}\left(\dfrac{4 \times \sin 41}{11}\right) = 13{\cdot}8°$(1 d.p.)

Example 3 To find GH, we use f, g, h, F (using cosine rule).

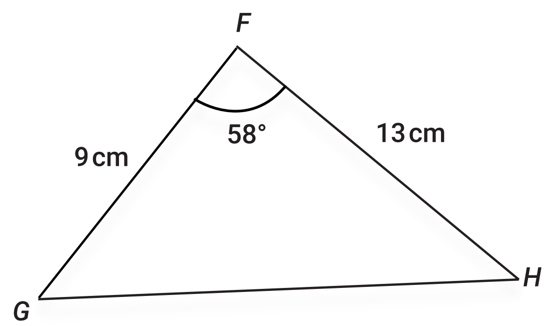
$f^2 = g^2 + h^2 - 2gh\cos F$

$f^2 = 13^2 + 9^2 - 2(13)(9)\cos 58$

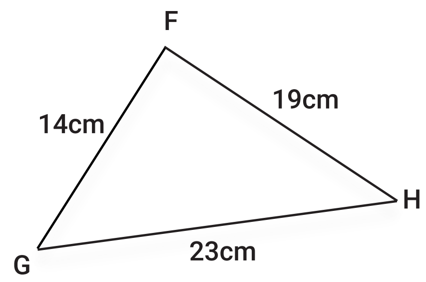
$f^2 = 125{\cdot}999$

$f = \sqrt{(125{\cdot}999)}$

$f = 11{\cdot}2$cm (1 d.p.)



Example 4 To find $F\hat{H}G$, we use H, f, h, g (using cosine rule).



$h^2 = f^2 + g^2 - 2fg\cos H$

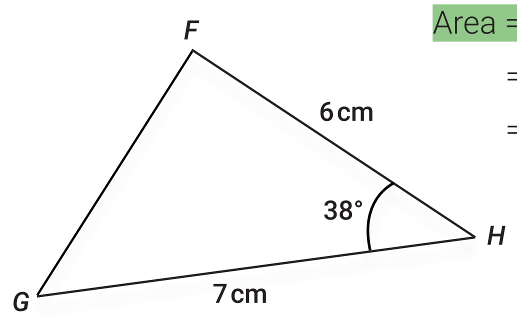
Needs re-arranging to become

$\cos H = \dfrac{f^2 + g^2 - h^2}{2fg} = \dfrac{23^2 + 19^2 + 14^2}{2(23)(19)}$

$= 0{\cdot}79405.....$

$H = \cos^{-1}(0{\cdot}79405) = 37{\cdot}4°$(1 d.p.)

Example 5 To find the area of triangle FGH, we use f, g, H.



Area = ½ $fg\sin H$

= ½ (7)(6)sin38

= 12·9cm² (1 d.p.)