Solving Quadratics Equations

A quadratic equation takes the form $ax^2 + bx + c = 0$, where a, b, c are numbers, with $a \neq 0$. Sometimes you will need to re-arrange the equation to get it in this form.

Check that you can factorise the different types of quadratic expressions e.g.

1) $x^2 - 5x - 6 = (x + 1)(x - 6)$ **or** $(x - 6)(x + 1)$
2) $2x^2 + 5x - 12 = (2x - 3)(x + 4)$ **or** $(x + 4)(2x - 3)$
3) $9x^2 - 49 = (3x - 7)(3x + 7)$ **or** $(3x + 7)(3x - 7)$

without forgetting easier versions e.g.

$$4x^2 + 9x = x(4x + 9)$$

1 Solving by factorising

e.g. 1) Solve the equation $x^2 + 4x - 21 = 0$ (containing $1x^2$)

Factorise the quadratic expression to get $(x - 3)(x + 7) = 0$ (or equivalent, swapping the brackets round).

It's worth checking your factorising by expanding your brackets.

The next stage relies on the fact that if a product of two factors equals zero, then at least one of the factors themselves must equal zero. If the product equals anything else, you cannot proceed in this way!

So, in this example, either $x - 3 = 0$ or $x + 7 = 0$
so that $x = 3$ or $x = -7$

e.g. 2) Solve the equation $2x^2 + 3x - 20 = 0$.

Factorising turns the equation into $(x + 4)(2x - 5) = 0$ (or equivalent)

Either $x + 4 = 0$ or $2x - 5 = 0$
so that $x = -4$ or $2x = 5$
$x = 5/2$ **(= 2½ = 2·5)**

e.g. 3) Solve the equation $4x^2 - 9 = 0$

If you factorise, you get **$(2x - 3)(2x + 3) = 0$,** leading to

$2x - 3 = 0$ or $2x + 3 = 0$
so that $x = 3/2$ or $x = -3/2$

However, in example 3), instead of using the difference of two squares to factorise, it is probably easier to isolate x^2 to get $x^2 = 9/4$. Taking the square root then gives $x = \pm 3/2$ (but

Remember that the word 'or' is an important part of the final answer.

Compare the signs of the numbers in the final answer with those which were in the brackets. What do you notice?

2 Solving by using the quadratic formula

This should be used when factorising doesn't work!

Extract from the formula page in the exam paper:

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

e.g. Solve the equation $3x^2 - 5x - 1 = 0$, giving your answers correct to 2 decimal places.

First, identify a, b, c.

Here we have $a = 3$, $b = -5$, $c = -1$.

Substitute into the formula, being particularly careful with minus signs and also with the order of operations (BIDMAS) under the square root.

(Notice that if b is negative, then $-b$ and b^2 are both positive.)

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)} = \frac{5 \pm \sqrt{25 + 12}}{6} = \frac{5 \pm \sqrt{37}}{6} = -0{\cdot}18 \text{ or } 1{\cdot}85 \text{ (to 2 d.p.)}$$

Avoid being caught out!

- **If the question requires you only to factorise the expression, then you should not also solve an equation, e.g. 'Factorise $x^2 - 9x - 10$' should result in a final answer of $(x - 10)(x + 1)$ (or equivalent) without going on to give '$x = 10$ or $x = -1$'.**
- **If the whole equation contains a common factor then you can simply divide everything by it, making the question easier. (Note, however, that if you are only factorising the expression, then the common factor must be kept.)**

e.g. $2x^2 - 14x + 24 = 0$ can all be divided by 2 to become $x^2 - 7x + 12 = 0$.