Solving Linear Equations

Equations use letters as symbols to represent unknown values.
The purpose of solving an equation is to find the unknown value.

Check that you can solve simple linear equations e.g.

$3x = 18$	$x - 7 = 12$
$\div 3 \quad \div 3$	$+7 \quad +7$
$x = 6$	$x = 19$

and that you are able to expand brackets e.g. $3(2x + 9) = 6x + 27$

1 Equations with brackets

In this case we must first expand the brackets then we use the balancing method to solve the equation e.g.

1) Solve $7(x + 3) = 49$

Expand the brackets	$7x + 21 = 49$
-21 from both sides	$7x = 28$
÷7 on both sides	$x = 4$

2) Solve $5(x - 6) = 15$

Expand the brackets	$5x - 30 = 15$
+30 to both sides	$5x = 45$
÷5 on both sides	$x = 9$

2 Equations with unknowns on both sides

In this case we collect terms containing x (or the unknown) on one side of the equation and the numbers (constant terms) on the other side e.g.

Solve $10x + 3 = 8x - 19$

$-8x$ from both sides	$2x + 3 = -19$
-3 from both sides	$2x = -22$
÷2 on both sides	$x = -11$

3 Equations with unknowns and brackets on both sides

In this case we must first expand the brackets. Then we can collect terms containing x on one side of the equation and the numbers on the other side. e.g.

Solve $9(x - 4) = 3(x + 2)$

expand the brackets	$9x - 36 = 3x + 6$
$-3x$ from both sides	$6x - 36 = 6$
+36 to both sides	$6x = 42$
÷6 on both sides	$x = 7$

Remember

- '=' means equal to so both sides must balance at all times. You must therefore make sure you do exactly the same to both sides of the equation.
- You can check that you have solved the equation correctly by substituting the value back into the original equation.

Solving quadratic equations

A quadratic equation takes the form $x^2 + ax + b = 0$, where a and b are numbers. You may be expected to re-arrange the equation to get it in this form.

Solving quadratic equations by factorising

e.g. 1) Solve $x^2 + 4x - 21 = 0$

Factorise the quadratic expression to get $(x - 3)(x + 7) = 0$
The next stage relies on the fact that if a product of two factors equals zero, then at least one of the factors themselves must equal zero. If the product equals anything else, you cannot proceed in this way!
So, in this example, either $x - 3 = 0$ or $x + 7 = 0$
so that $x = 3$ or $x = -7$

2) Solve $x^2 - 9x + 20 = 0$.

Factorising turns the equation into $(x - 4)(x - 5) = 0$
Remember if a product of two factors equals zero, then at least one of the factors themselves must equal zero.
So, in this example, either $x - 4 = 0$ or $x - 5 = 0$
so that $x = 4$ or $x = 5$

Check that you can factorise quadratic expressions e.g.

1) $x^2 + 9x + 20 = (x + 4)(x + 5)$ or $(x + 5)(x + 4)$
2) $x^2 - 5x - 6 = (x + 1)(x - 6)$ or $(x - 6)(x + 1)$
3) $x^2 - 7x + 12 = (x - 4)(x - 3)$ or $(x - 3)(x - 4)$

Remember that the word 'or' is an important part of the final answer.
Compare the signs of the numbers in the final answer with those which were in the brackets. What do you notice?
Avoid being caught out! If the question requires you only to factorise the expression, then you should not also solve an equation, e.g. 'Factorise $x^2 - 9x - 10$' should result in a final answer of $(x - 10)(x + 1)$ (or equivalent) without going on to give '$x = 10$ or $x = -1$'.