Factorising Quadratic Expressions

A quadratic expression takes the form $ax^2 + bx + c$, where a, b, c are numbers, with $a \neq 0$.

'Factorising' means re-writing the expression using brackets.

Check first that you can expand double brackets (using any appropriate method, such as using a grid or 'FOIL') e.g.

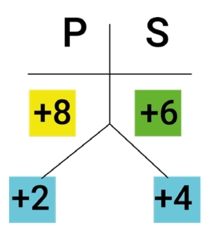
1. $(x + 1)(x - 6) = x^2 - 6x + x - 6$
   $= x^2 - 5x - 6$
2. $(2x - 3)(x + 4) = 2x^2 + 8x - 3x - 12$
   $= 2x^2 + 5x - 12$

1 Type 1 containing $1x^2$

There are many possible methods, only one of which is given here. e.g.

1) Factorise $x^2 + 6x + 8$.

Here you need two numbers with a product of +8 and a sum of +6.



(If you can't find the numbers immediately, try listing factor pairs of +8, namely 1×8, −1×−8, 2×4, −2×−4)
Place the two numbers in brackets like this

$(x + 2)(x + 4)$ or $(x + 4)(x + 2)$

(It's worth checking your answer by expanding your brackets.)

2) $x^2 - 5x + 4 = (x - 4)(x - 1)$ or $(x - 1)(x - 4)$

3) $x^2 - x - 12 = (x - 4)(x + 3)$ or $(x + 3)(x - 4)$

2 Type 2 containing ax^2 where $a \neq 1$

Again, there are many methods, only one of which is given here. e.g.

1) Factorise $2x^2 + 11x + 12$.

Here you need two numbers with a product of +24 (from +2 × +12) and a sum of +11.
The two numbers are +3 and +8.

Re-write the expression using your two numbers (in either order) to replace the middle term.
(Note that this time you don't put them straight into brackets!)

$2x^2 + 3x + 8x + 12$ or $2x^2 + 8x + 3x + 12$

Factorise a pair of terms at a time, by taking out common factors. (Make sure the 'introduced brackets' contain identical terms.)

$x(2x + 3) + 4(2x + 3)$ or $2x(x + 4) + 3(x + 4)$

Write down the 'repeated' bracket, then construct a second bracket using 'everything else'.
You then have

$(2x + 3)(x + 4)$ or $(x + 4)(2x + 3)$

(It's worth checking your answer by expanding your brackets.)

2) $3x^2 - 17x + 10 = (3x - 2)(x - 5)$ or $(x - 5)(3x - 2)$

3) $8x^2 - 2x - 1 = (4x + 1)(2x - 1)$ or $(2x - 1)(4x + 1)$

(For the last two examples, care is required with minus signs in order to ensure that the 'introduced brackets' contains identical terms.)

3 Type 3 "Difference of two squares"

Look out for this specific case where

$$a^2 - b^2 = (a + b)(a - b)$$

Remember that this won't work if it contains + instead of −. e.g.

1) Factorise $x^2 - 25 = (x + 5)(x - 5)$ or $(x - 5)(x + 5)$

(Expanding gives $x^2 - 5x + 5x - 25 = x^2 - 25$)

Sometimes there could be more than one variable (letter) in the expression.

2) $9x^2 - y^2 = (3x + y)(3x - y)$ or $(3x - y)(3x + y)$

3) $25c^2 - 16d^2 = (5c + 4d)(5c - 4d)$ or $(5c - 4d)(5c + 4d)$

Avoid being caught out!

- Sometimes a quadratic expression doesn't require double brackets e.g. $2x^2 - 7x = x(2x - 7)$
- Sometimes you can start by taking out a common factor
  e.g. $2x^2 - 72 = 2(x^2 - 36) = 2(x + 6)(x - 6)$