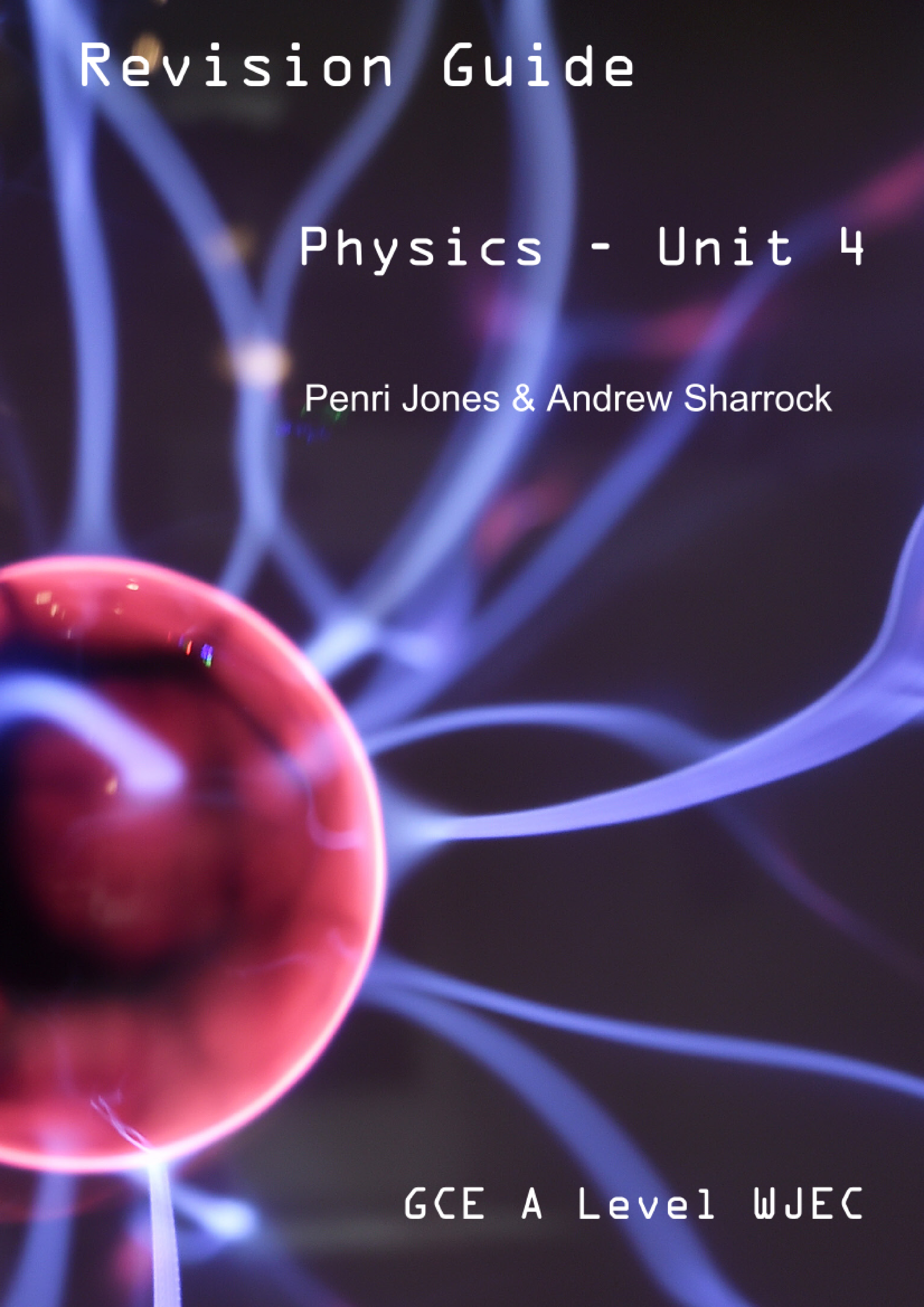


Revision Guide

Physics - Unit 4

Penri Jones & Andrew Sharrock

GCE A Level WJEC

These notes have been authored by experienced teachers and are provided as support to students revising for their GCE A level exams. Though the resources are comprehensive, they may not cover every aspect of the specification and do not represent the depth of knowledge required for each unit of work.

Contents

Section 4.1 - Capacitance

Some basic definitions

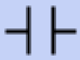
When a charge is placed on a conductor its potential changes. Capacitance is a conductor's ability to store this charge, and is defined thus:

$$\text{capacitance} = \frac{\text{charge on either plate}}{\text{potential difference between the plates}}$$

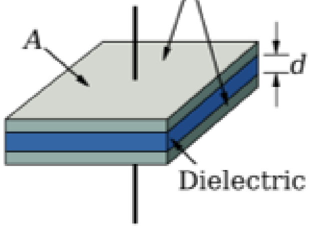
$$C = \frac{Q}{V}$$

where C = capacitance, measured in **Farads, F.**

A capacitor is a pair of conducting plates separated by an insulator. If a potential difference is placed across the plates, they acquire equal and opposite charges. The symbol for a capacitor is:



The parallel plate capacitor



The simplest capacitor is just two metal plates placed close together, separated by a vacuum or air.

The capacitance can be increased by inserting a layer of insulating material in between the plates, known as a **dielectric.**

The capacitance of a parallel plate capacitor can be calculated with this equation:

$$C = \frac{\varepsilon_0 A}{d}$$

where ε_0 = permittivity of a vacuum; units = **Fm^{-1}**,
= 8.85×10^{-12} Fm^{-1}
A = Area of one plate; units = **m^2,**
d = separation of the plates; units = **m.**

If a dielectric is introduced, the permittivity increases by an amount equal to the relative permittivity, ε_r. You will NOT need to incorporate this into the above equation for the exam.

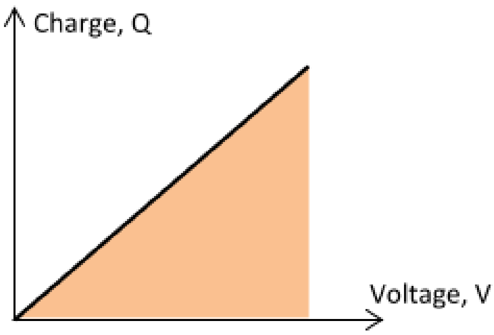
Example
A parallel plate capacitor is to be built where the separation of the plates is 1.5mm. Calculate the area needed to create a capacitance of 3μF.

A = C x d / ε_0 = $(3\times10^{-6}) \times (1.5\times10^{-3}) / (8.85\times10^{-12})$ = 508 m^2 !!

If the capacitor was square shaped, it would need a side length of about 23m! ☺

The energy stored by a capacitor

If the equation for capacitance, C = Q/V is re-arranged, we get: Q = V C. This shows that as 'C' is constant, Q α V. A graph of Q vs V is therefore a straight line, through the origin, as seen opposite:



It can be shown that the energy stored by a capacitor charged from 0V up to a voltage, V, is given by the area under the graph shown, hence,

$$U = \frac{1}{2} Q V$$

and since Q = VC, we can substitute to obtain 2 other forms of the same equation:

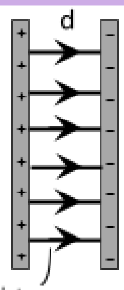
$$U = \frac{1}{2} C V^2$$

$$U = \frac{1}{2} \frac{Q^2}{C}$$

NOTE!
The energy stored is given the symbol 'U' here. This is because the energy is store inside the capacitor (as electrostatic potential energy), and hence is classed as 'internal energy'. This symbol is also used for internal energy in section 3.3 (thermal physics).

The electric field between the plates

The electric field is assumed to be **uniform** in between the plates of a parallel plate capacitor. This means that the electric field strength, E, will have a constant value everywhere between the plates. (See the next unit for details of electric fields). The value of E is therefore calculated by the following equation:



Electric field between the plates

$$E = \frac{V}{d}$$

where V = pd between the plates; units = Volts, V
d = separation of the plates; units = metres, m, hence…
units of electric field strength, E, are **Vm^{-1}**

The field lines are therefore all parallel, and equally spaced. Another consequence is that the potential changes steadily with distance across from one plate to the other.

Combining capacitors in series and parallel

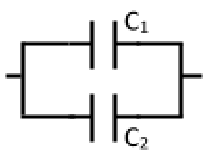
The equations below show how to calculate the combined, effective capacitance of 2 or more capacitors. (Note how the equations take a form that's opposite to the equations for combining resistors!).

In series

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

In parallel

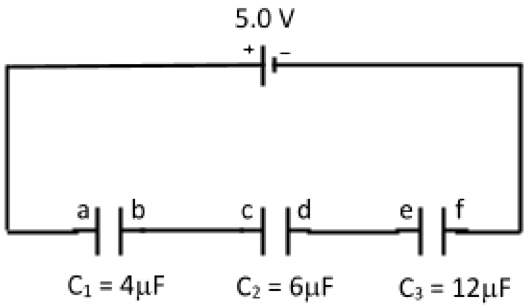
$$C_T = C_1 + C_2$$



Tackling questions on capacitors in series and parallel

Capacitors in series

There are a few pitfalls to look out for when answering questions on combining capacitors.



Taking the example on the right, if we need to calculate, say, the charge and energy stored by each capacitor, the first step is relatively easy:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{4\mu} + \frac{1}{6\mu} + \frac{1}{12\mu} = \frac{12}{24\mu} = \frac{1}{2\mu}$$

$\mathbf{C_T = 2\mu F}$

Ok, now we can calculate the total charge stored, $Q_T = V_T \times C_T = 5 \times 2\mu = 1\text{x}10^{-5}$C.

What next? Although we know that in a series circuit the voltage splits between each component ($V_T = V_1 + V_2 + V_3$), we can't calculate the individual voltages until we find the charge on each one.

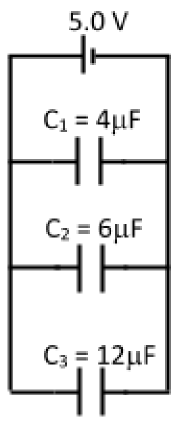
The trick comes from realising that the power supply pushes electrons onto plate '**f**' and removes electrons from plate '**a**' – the charges that build up on plates **b**, **c**, **d** and **e**, are all **induced** by the charges placed on **a** and **f**. This means that, no matter how different each capacitor value is, **the charge separated by each one is equal**! Hence,

$Q_T = Q_1 = Q_2 = Q_3 = 1\text{x}10^{-5}$C

We can now calculate the individual voltages by applying Q = VC, and from there apply U = ½ QV to calculate the energy stored by each capacitor.

Capacitors in parallel

This is slightly easier! Look at the example opposite. The first step is even easier than before – calculating the total capacitance is done as follows:



$C_T = C_1 + C_2 + C_3 = 4\mu + 6\mu + 12\mu = \mathbf{22\mu F}$

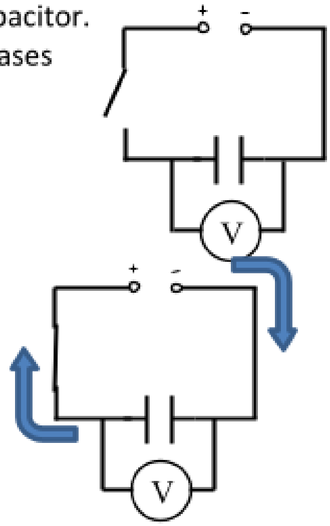
This time, however, we must apply the understanding that the voltage across components connected in parallel must be equal, i.e.

$\mathbf{V_T = V_1 = V_2 = V_3 = 5.0V}$

We can now use "Q = V C" to calculate the charge stored by each individual capacitor, and also, if needed, we can calculate the energy stored using U = ½ QV, as before.

Charging and discharging capacitors – exponential growth & decay

The circuit opposite shows a power supply connected to a switch and a capacitor. A resistor is usually placed in series to limit the flow of charge, which increases the charging time. This is so that measurements, e.g. of voltage across the capacitor, can be taken at set time intervals. (If there's no resistor present, charging and discharging occurs in a matter of microseconds!).

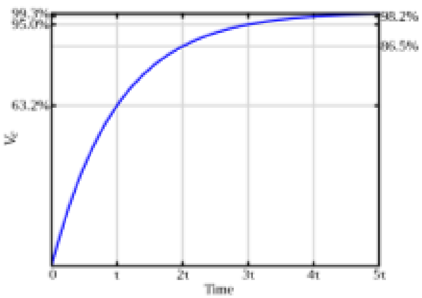


As soon as the switch is closed, electrons flow **onto** the right hand plate, and flow **from** the left hand plate. As opposite charges build up on the plates, the voltage across the capacitor gradually increases. This means that the supply voltage has an increasing 'opposing' voltage building up in the circuit, which means the current gradually decreases. This is typical of exponential behaviour – the rate of flow of charge is linked to the amount of charge present. The capacitor therefore charges with exponential growth, given by this equation:

$$Q = Q_0\,(1 - e^{-t/RC})$$

where,
Q = charge at a time, t (s)
Q_0 = charge at t=∞ (Coulombs, C)
R = resistance (Ω)
C = capacitance (F)

The graph opposite shows how the charge (or voltage) builds up on a capacitor. Notice that in a certain time interval, the charge increases to half of what's left to be fully charged (i.e. Q_0). This works with any fraction or %. The time intervals shown are "τ" (this is known as the **time constant**), where τ = RC.
This makes the function in the bracket in the equation:

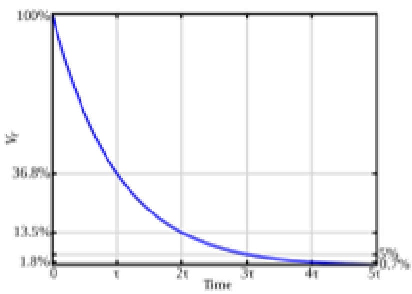


$$(1 - e^{-t/RC}) = (1 - e^{-RC/RC}) = (1 - e^{-1}) = 0.632$$

i.e. 63.2% of Q_0 . It takes another time interval of "τ" to reach 63.2% of what's left, i.e. 83.5% of Q_0, and so on.

Discharging capacitors

Discharging a capacitor is an example of exponential decay. When a capacitor discharges through a resistor, electrons flow from the negative plate, through the resistor and back to the positive plate, thus gradually discharging the capacitor. The relevant equation this time is:

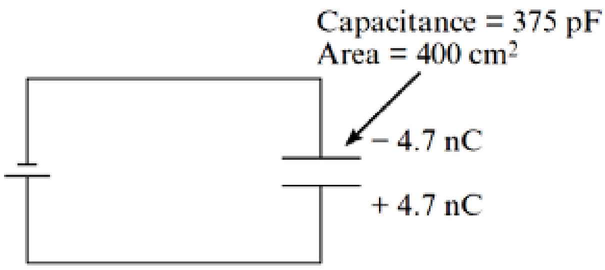


$$Q = Q_0\, e^{-t/RC}$$

The only difference is that Q_0 this time represents the charge at time, t = 0, i.e. the initial (full) charge on the capacitor. This equation also works on the principle that it takes the same amount of time for the charge (or voltage) to drop a certain percentage of the charge that's left, e.g.
If a capacitor takes 5s to discharge to 10% of its initial charge, in the next 5s it will drop to 10% of the 10% that's left, namely 1%; in the next 5s it will be 0.1% of Q_0, etc.

Example (Taken from the PH5, June 2011 paper)

A parallel plate capacitor with no dielectric between the plates is charged by a cell as shown in the diagram.

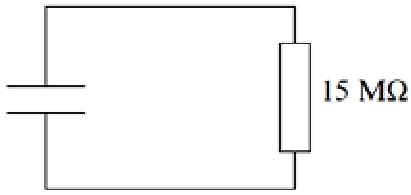


(a) (i) Calculate the emf of the cell. [2]

(ii) Calculate the separation of the plates of the capacitor. [2]

(b) The capacitor is now discharged through a 15 MΩ resistor.

Calculate the time for the charge on the capacitor to drop from 4.7 nC to 0.7 nC.



[4]

(a) (i) $V = Q / C$ = $4.7\times10^{-9} / 375\times10^{-12}$ = 12.5V (to 3 s.f.)

(ii) $C = \varepsilon_0 A / d$ ∴ $d = \varepsilon_0 A / C$ = $(8.85\times10^{-12})(0.04) / 375\times10^{-12}$ = 9.44×10^{-4} m

NOTE !
Be careful when converting cm^2 to m^2. There are 100x100 cm^2 in a square metre, hence 400cm^2 = 400/10 000 m^2 = 0.04m^2

(b) $Q = Q_0\, e^{-t/RC}$ ∴ $\frac{Q}{Q_0} = e^{-t/RC}$

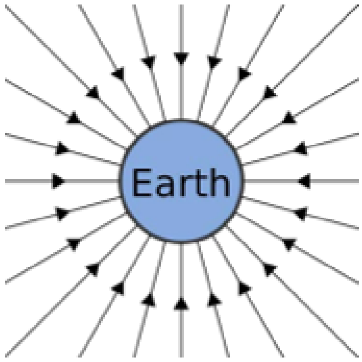
Taking logs : $\ln \frac{Q}{Q_0} = -t / RC$

∴ **Re-arranging:** $t = -\ln(Q/Q_0) \times RC = -\ln(0.7n/4.7n) \times (15\times10^{6}) \times (375\times10^{-12})$

∴ **t = 0.0107s**

Section 4.2 - Electrostatic and Gravitational fields of Force

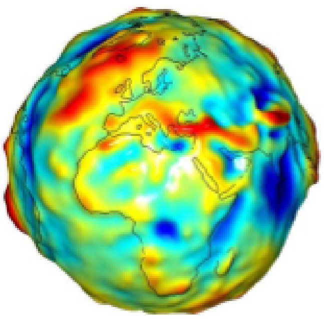
Gravitational fields



A **field** is a region in which a force is felt. **Gravity** is a weak force and occurs between any objects with mass. The force of gravity is always **attractive** and never repulsive. This attraction occurs between any object with mass, however small. Gravitational fields always start at infinity and end on matter.

Important points about the diagram:

1. The field surround the Earth not just where the lines are.
2. The field is always directed towards the Earth.
3. The field is zero at the centre of the Earth.
4. The gravitational field outside spherical bodies such as the Earth is essentially the same as if the **whole mass were concentrated at the centre**



The gravitational field of the Earth is not uniform because the matter is not evenly distributed across the Earth due to rocks have different densities and mountain ranges.

Gravitational field strength 'g'.
The force experienced per unit mass by a mass placed in the field.

Gravitational field strength 'g' $= \frac{force}{mass}$ Units **N kg^{-1} or ms^{-2}.**

On the Earth the gravitational field strength = $\frac{\text{weight of 1 kilogram mass}}{\text{1 kilogram}}$

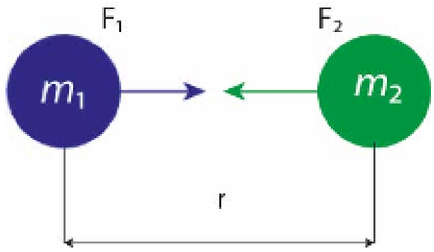
Gravitational field strength of the Earth at surface = $\frac{9.81}{1.00}$ = 9.81 Nkg^{-1}

Example
An astronaut of mass 120kg in his spacesuit would weigh 3.2**kN** at the surface of Jupiter if he/she could get there. What is the gravitational field strength at the surface of Jupiter?

Gravitational field strength 'g' $= \frac{force}{mass} = \frac{3200}{120} = 27$ Nkg^{-1}

Newton's law of gravitation: the gravitational force between two particles is proportional to the product of their masses, m_1 and m_2, and inversely proportional to their separation squared, r^2.

$$F \propto \frac{m_1 m_2}{r^2} \qquad F = \frac{G\,m_1 m_2}{r^2}$$



Units: F - (N), mass- (kg) and distance - (m).

The value of the gravitational constant G = 6.67×10^{-11} N m^2 kg^{-2}

Example
The Earth has a mass of 6x 10^{24}kg, radius of 6.4×10^3km. Calculate the force between the Earth and a 60kg mass on the surface?

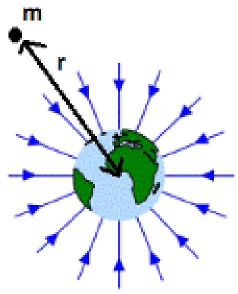
1st step is to change km to m. 6.4×10^3km x 1000 = $6.4 \times 10^6 m$

$$F = \frac{G\,m_1 m_2}{r^2} = \frac{6.67\times10^{-11}\times 60 \times 6\times10^{24}}{(6.4\times10^6)^2} = 589\ N$$

Resultant gravitational field strength

The gravitational field strength at any point in a gravitational field is equal to the acceleration due to gravity at that point. Consider the gravitational force on a mass m at a distance r from the Earth's centre.

Gravitational force = $\frac{G\,M\,m}{r^2}$ and weight = m g



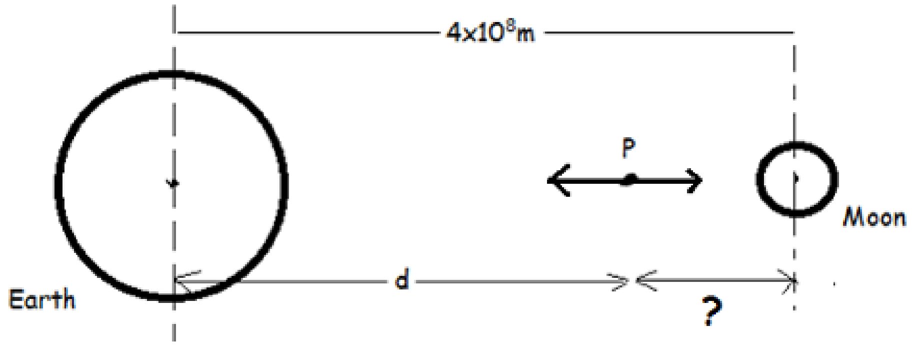
They are equal, so $m\,g = \frac{G\,M\,m}{r^2} \rightarrow \not{m}\,g = \frac{G\,M\,\not{m}}{r^2}$

$$g = \frac{G\,M}{r^2} \text{ (given in data book)}$$

Gravitational field strength 'g' is a vector and as such has direction: the direction in which a mass moves in the gravitational field.

The point of zero resultant field strength between the Earth and the Moon.

At P the gravitational field strength due to the Earth is the same as that due to the moon. We want to **calculate the distance 'd'** where this point P occurs. (diagram not to scale)



Mass of Earth = 6.00×10^{24} kg

Mass of Moon = 7.34×10^{22} kg

Field strength at P due to the Earth: $g = \frac{G\,M_E}{d^2}$

Field strength at P due to the Moon: $g = \frac{GM_m}{(4x10^8 - d)^2}$

At p $\quad g_E = g_m \quad \rightarrow \quad \frac{G\,M_E}{d^2} = \frac{GM_m}{(4x10^8-d)^2}$

Square root both sides $\quad \frac{G\,M_E}{d^2} = \frac{G\,M_m}{(4x10^8-d)^2} \quad \rightarrow \quad \frac{\sqrt{M_E}}{d} = \frac{\sqrt{M_m}}{(4x10^8-d)}$

Input numbers and rearrange $\quad \sqrt{M_E}\,.\,(4x10^8 - d) = \sqrt{M_m}\,.\;d$

$$2.45\text{x}10^{12}(4x10^8 - d) = 2.71\text{x}10^{11}\,d$$

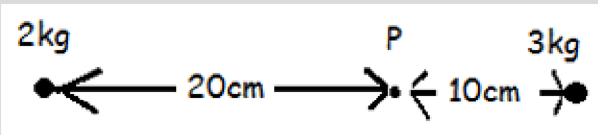
$$9.8\text{x}10^{20} - 2.45\text{x}10^{12}\text{d} = 2.71\text{x}10^{11}\,d$$

$$9.8\text{x}10^{20} = 2.71\text{x}10^{11}\,d + 2.45\text{x}10^{12}\text{d} \quad \rightarrow \quad 9.8\text{x}10^{20} = 2.721\text{x}10^{12}\text{d}$$

$$\text{d} = \frac{9.8\text{x}10^{20}}{2.721\text{x}10^{12}} = 3.60\text{x}10^8 \text{ m}$$

Calculating resultant gravitational field strength

Example
Calculate the resultant field strength at P.
(vector arrows not shown to scale)



Field strength due to 2kg mass 3.34×10^{-9} N kg^{-1} due to 3kg mass 2.0×10^{-8} N kg^{-1}

← **P** ⟶

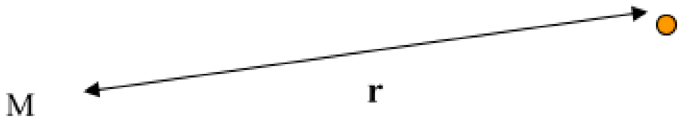
Both vectors are in the opposite direction, so resultant = 1.67×10^{-8} N kg^{-1} to the right

P ⟶

<u>Gravitational potential</u> V_g (units: Jkg^{-1}) at a point in space due to the existence of a point mass nearby. A mass has potential energy at a given point in a gravitational field.

V_g: the work done per unit mass in bringing a mass from infinity to that point. This is a *scalar* quantity.

$$V_g = -\frac{G\,M}{r}$$



Units. $V_g \rightarrow Jkg^{-1}$ $m \rightarrow kg$ $r \rightarrow m$

What the '-' sign? Gravity is always attractive, and so work will be done **by** any mass moving from that mass towards infinity (rather than work done **on** the mass if it were repulsive).The potential at infinity is zero and it decreases as we move towards the mass which is the source of the field. If the potential increases as you move towards infinity the value/number must be a negative to increase to zero.

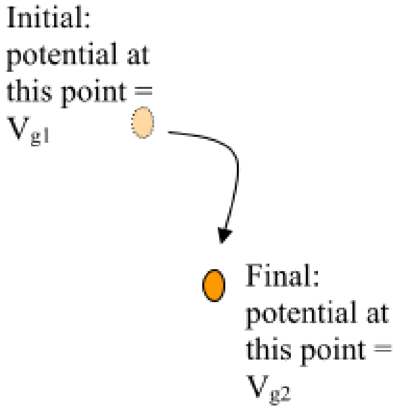
Potential energy

If a mass 'm' is placed in a gravitational field then it will have potential energy 'PE'. This is calculated by multiplying the magnitude of your mass (in kg) by the gravitational potential V_g.

$PE = -m\,V_g$ or if we substitute the equation for V_g then, **$PE = -\frac{G\,M_1 M_2}{r}$**

(Not given in data booklet)

The work done **W** is equal to the change in **PE.**



Change in potential energy	$= PE_2 - PE_1$
	$= m\,V_{g2} - m\,V_{g1}$
	$= m\,(V_{g2} - V_{g1})$
	$= m\,\Delta V_g$

$W = m\Delta V_g$ **(unit Joule, J)**

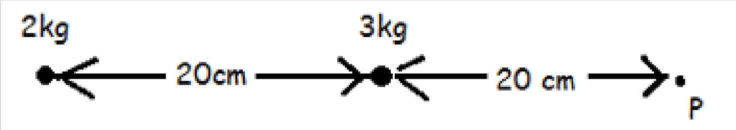
If we are very close to the surface of the Earth then the value of 'g' will not change very much so we can use the equation:

$$\Delta U_P = m\,g\,\Delta h$$

Only for distances over which the variation of *g* is negligible

This equation is the same as $\Delta E = m\,g\,\Delta h$ you used in Unit 1.

Net Gravitational potential for several point masses.
Example.



Calculate the net potential at P.

V_g for 2kg $= -\frac{G\,M}{r}$ = -3.34x10^{-10} Jkg^{-1} V_g for 3kg -1.00x10^{-9} Jkg^{-1}

Net potential = -3.34x10^{-10} Jkg^{-1} + (-1.00x10^{-9} Jkg^{-1}) = -1.33x10^{-9} Jkg^{-1}

Potential is scalar so no direction.

Escape velocity: the minimum velocity at which a projectile is thrown upwards, never to return to Earth (i.e. it goes to infinity).

M → mass of Earth m → mass of object.

From the Earth the potential at the surface, $V_g = -\frac{G\,M}{r}$

The potential energy of the projectile at the surface PE = $-\frac{G\,M\,m}{r}$

The potential at infinity = 0

The **P**otential **E**nergy at infinity is = 0

So gain in PE or work done in going to infinity =

$$0 - -\frac{G\,M\,m}{r} \rightarrow \frac{G\,M\,m}{r}$$

The E_k supplies this energy $= \frac{1}{2} mv^2$

Assume E_k at surface = gain in P.E. in going to infinity

$$\frac{1}{2} mv^2 = \frac{G\,M\,m}{r} \rightarrow \frac{1}{2} \not{m}v^2 = \frac{G\,M\,\not{m}}{r}$$

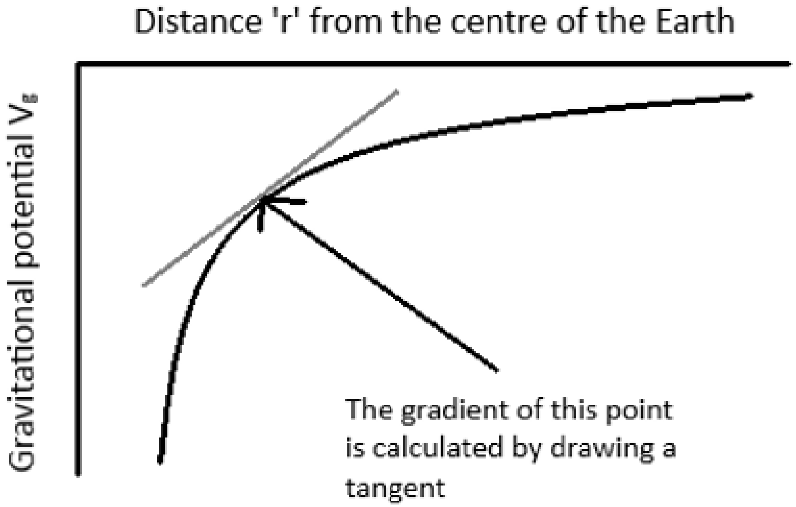
$$\mathbf{v} = \sqrt{\frac{2\,G\,M}{r}}$$

The escape velocity is 11,200 ms^{-1}.

Animation of the escape velocity of rocket.
http://highered.mheducation.com/olcweb/cgi/pluginpop.cgi?it=swf::800::600::/sites/dl/free/0072482621/78778/Escape_Nav.swf::Escape%20Velocity%20Interactive

The graph of gravitational potential (V_g) against distance

The graph below shows the result of plotting gravitational potential V_g against the distance from the centre of th Earth. Gravitational potential is calculated using the equation → $V_g = -\frac{G\,M}{r}$



By drawing a tangent and taking the negative of the gradient at that point you can calculate a value for the gravitational field strength 'g' at that point.

In summary:

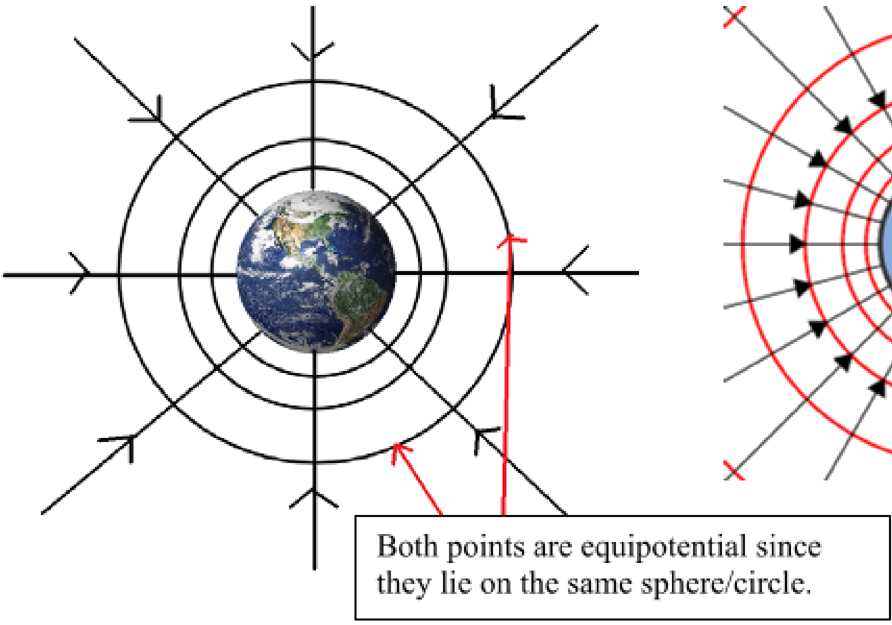
g = - slope of the $V_g - r$ graph at that point

$$g = -\frac{\Delta V_g}{\Delta r} \quad \text{(not in data booklet)}$$

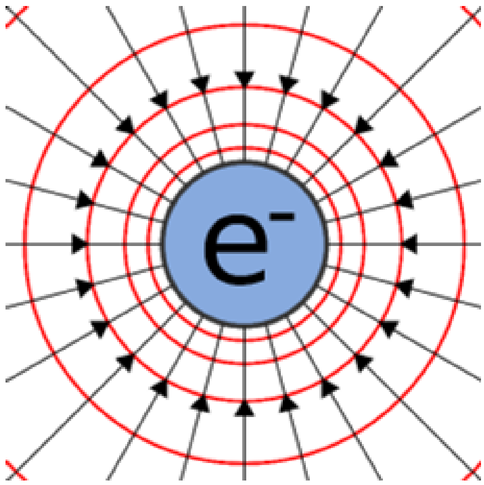
Equipotential surfaces

These are surfaces where the gravitational potential or electric potential is the same. Equipotential surfaces join points of equal potential and are therefore spherical for a point charge and for planets if you consider the Earth to be a perfect sphere and of even density.

Earth



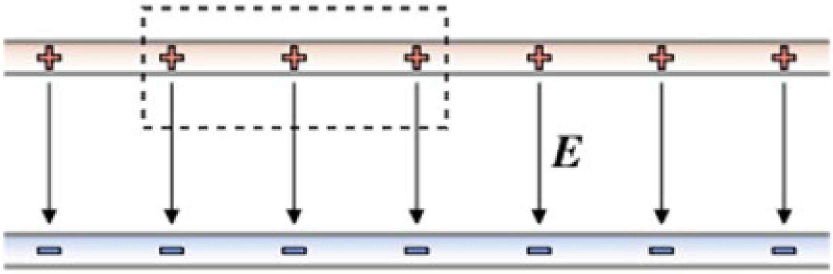
Point charge



Electrostatic fields

Electric fields

An electric field exists in the region around a charged particle. If another charged particle is placed within this field then that particle will experience a force (push or pull) due to the presence of the first particle.

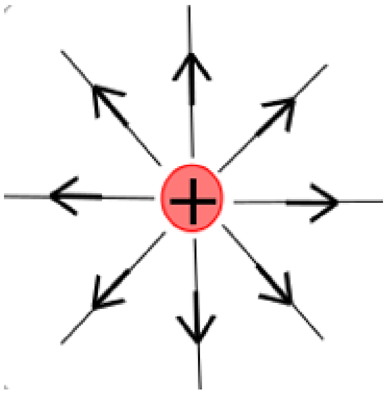


Arrows are drawn on the lines to show the direction in which a *positive* charge would move if it was placed at that point i.e. the arrow points from positive to negative.
The electric field between the 2 plates is uniform and this is shown by positioning the field lines with equal spacing between them. The closer the lines are the greater the strength of the electric field.

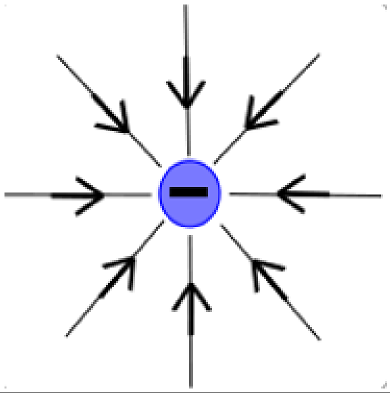
The electric field lines gives you two pieces of information about the electric field.

1. Strength of the field. ***2 .Direction of the field.***

The field lines of point charges.



For a ***positive charge*** the field lines are radially outward, since a positive charge placed in the field would be repelled and thus move away.



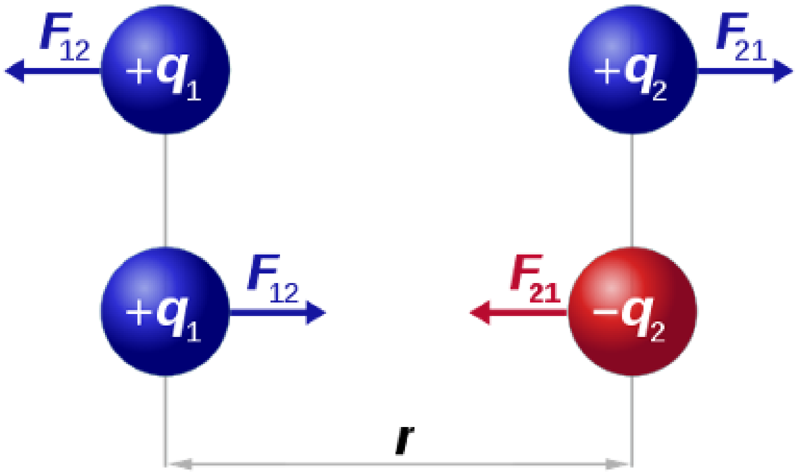
For a ***negative charge*** the field lines are radially inward, since a positive charge placed in the field would be attracted and thus move toward the negative charge.

We will assume that charges are point charges i.e. all of the charge is concentrated at a single point, (so in the diagrams above the particles should have an infinitesimally small radius i.e a dot '•' and not a circle).

Coulomb's law

There are many similarities between gravitational fields and **electric fields**. In 1784 Coulomb showed that the force between two point charges was proportional to the magnitude of the charges and inversely proportional to the square of the distance between them. This is another example of an inverse square law, just like Newton's law of gravitation.

There will be a force of repulsion or attraction between two charges a distance r apart.



$$F \; \alpha \; \frac{Q_1 Q_2}{r^2} \qquad\qquad F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

Units: F → N, Q → Coulomb, C r→ m ε_0= 8.85 x 10^{-12} Fm^{-1}

The force F is **positive** when both charges are positive or both negative; this indicates that the force is **repulsive**. If one charge is positive and the other is negative, the force F is **negative** and therefore **attractive**.

Permittivity ε_0 - of a material is a measure of how well it *allows* an electric field to pass through. The lowest possible value is that of a vacuum (ε_0= 8.85 x 10^{-12} Fm^{-1}). This term ε_0 is called the **permittivity of free space**.

$$\frac{1}{4\pi\varepsilon_0} = 8.99\times10^9\ F^{-1}\,m \qquad \approx \qquad 9\times10^9\ F^{-1}\,m$$

(This information is given on the front of the data booklet)

Instead of having to calculate a value for $\frac{1}{4\pi\varepsilon_0}$ each time, you can use the approximate value of $9\times10^9\ F^{-1}m$. This large value of the constant tells us why electrostatic forces are very strong when compared with gravitational forces, even when the charges are quite small.

Example

Calculate the force between a charge, q_1 = + 5μC and a second charge, q_2 = - 2.5μC, if they are separated by a distance of 1cm.

$$F = \frac{9\times10^9 \times 5\times10^{-6} \times -2.5\times10^{-6}}{0.01^2} = -1125\ N \text{ (attractive)}$$

Electric field strength 'E' : the force experienced per unit charge by a small positive charge placed at that point.

Electric field strength = $\frac{Force}{Charge}$ **E** = $\frac{F}{Q}$ (Not in data booklet)

Units: NC^{-1} or Vm^{-1}

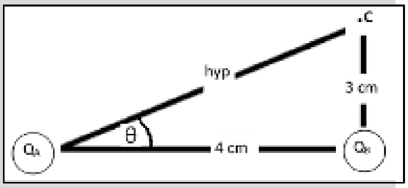
E is a vector quantity, it can be negative (attractive) or positive (repulsive).
We can obtain another expression for the field strength 'E' at any point in space by simply using this definition with Coulomb's law.

E = $\frac{F}{Q}$ **and substitute F for →** $\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$

$$E = \frac{\frac{1}{4\pi\varepsilon_0}\frac{Q\,Q}{r^2}}{Q} \rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$ (given in data booklet)

Calculating resultant electric field strength 'E'.

Two charges, Q_A = $+2\times10^{-9}$ C ,and Q_B = -7×10^{-10} C, and are placed at the corners of a right angled triangle, as shown below.
Calculate , (i) the resultant electric field at **C** (due to A and B),
(ii) the resultant force on a charge placed at C, where $Q_C = -6\times10^{-12}$ C.



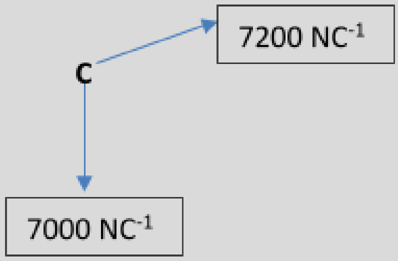
(i) The 1st step is to calculate the angle 'θ' and a value for the hypotenuse.

$$hyp = \sqrt{3^2 + 4^2} = 5 \text{ cm} \qquad \sin\theta = \frac{3}{5} = 36.9°$$

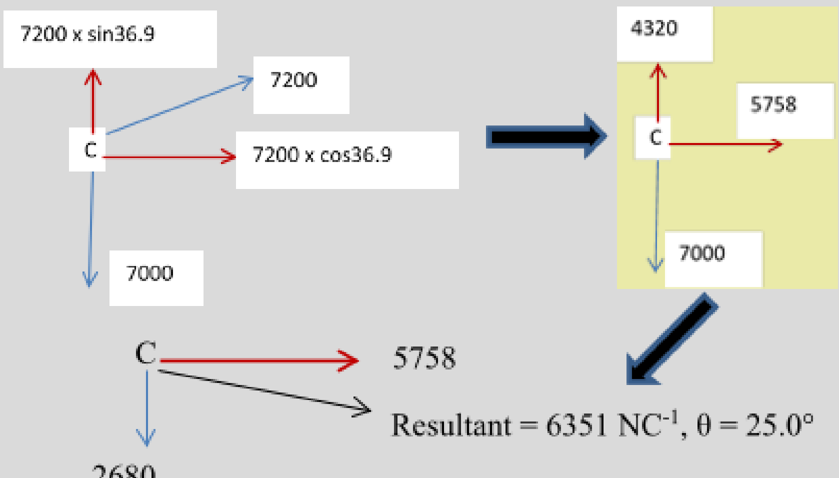
The 2nd step is to calculate the electric field strength due to both charges. Remember that point C is imagined as a positive charge.

Due to Q_A → E = $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ → E = $\frac{9\times10^{9} \times 2\times10^{-9}}{0.05^2}$ = 7200 NC^{-1}

Due to Q_B → E = $\frac{9\times10^{9} \times -7\times10^{-10}}{0.03^2}$ = -7000 NC^{-1}



The 3rd step is to resolve the vectors (vectors not to scale).



(ii) The resultant force 'F' = EQ = 6351 x -6×10^{-12} = -3.81×10^{-8} N (in the opposite direction to the resultant field strength vector arrow.).

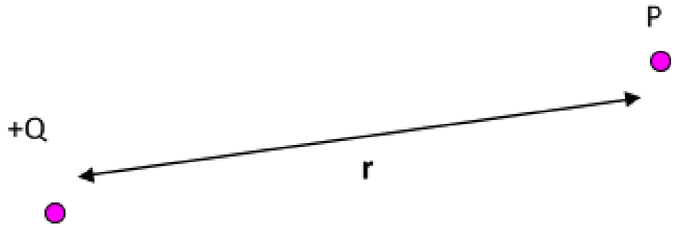
Electric potential, V_E.

Electric potential at a point is the work done per unit charge in bringing a positive charge from infinity to that point. This is a *scalar* quantity.

The electrical potential at point P:

$$V_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Units: **JC^{-1}** or V



The potential at infinity is zero. For practical reasons the potential of the Earth is taken as zero. This is possible since we deal with the potential difference.

This means that once the potential is known for a particular point in space, we can find the potential energy of a charge 'Q_2' placed at that point.

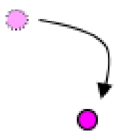
Potential energy, PE = Q V_E → **PE =** $\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r}$ (This version in data booklet)

Electric potential energy

If a charge 'q' is placed in an electric field then it will have potential energy 'PE'. If the charge is moved then it follows that there will be a change in the PE of the charge that has moved.

The work done **W** is equal to the change in **PE.**

Change in PE = $q\Delta V_E$ Work done, W = $q\Delta V_E$ (unit Joule, J) (version in data booklet)



Work done= $PE_2 - PE_1$

$W = Q V_{E2} - Q V_{E1}$

$W = Q (V_{E2} - V_{E1})$

$W = Q\Delta V_E$

Example: calculate the work done when a +2.0nC point charge moves from P to R.

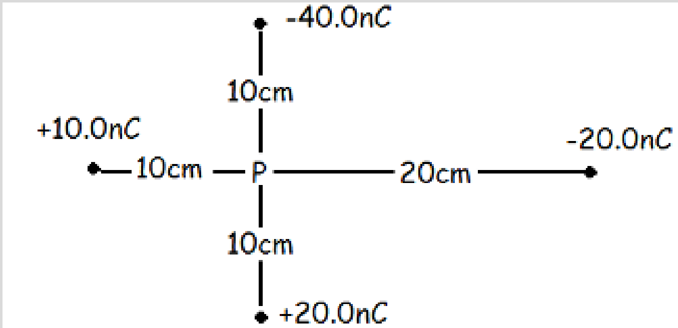
Potential at P = $V_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ = 2250-1800 = +450V

Potential at R = $V_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ = 900-900 = 0V

p.d between R and P ΔV_E = 0 – (+450) = -450V

$W = q\Delta V_E$ = 2×10^{-9} x -450 = -9×10^{-7} J If it went from R to P = $+9\times10^{-7}$ J

The net electric potential.
Example.
(a) Calculate the net electric potential at point P.



1st step is to calculate the potential due to each charge as shown below:

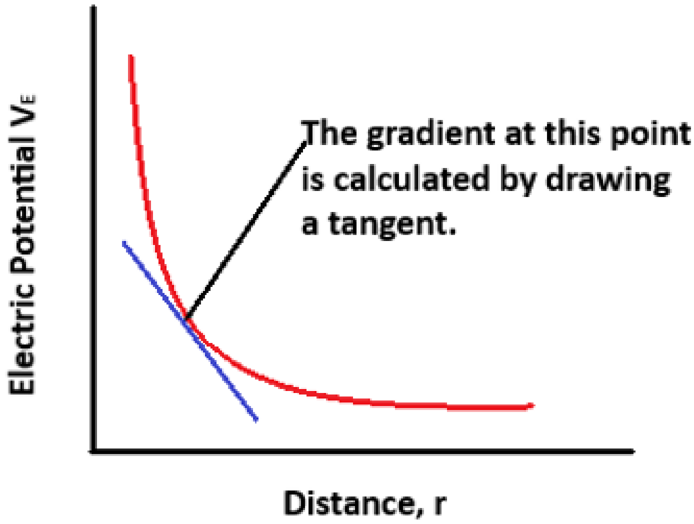
$V_E = \frac{9\times10^9 \times -40\times10^{-9}}{0.10^2}$ = -3600 JC^{-1}

2nd step is to add all the potentials. Remember to include the charge (+) or (-).

Net potential, V_E = (-3600) + (– 900) + (+1800) + (+900) = -1800 JC^{-1}

The graph of electric potential (V_E) against distance

The graph below shows the result of plotting electric potential 'V_E' against distance 'r' from a charge 'Q'. The electric potential is calculated using the equation → $V_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$



By drawing a tangent and taking the negative of the gradient at that point you can calculate a value for the electric field strength 'E' at that point.
In summary:

E = - slope of the $V_E - r$ graph at that point

$E = -\frac{\Delta V_E}{\Delta r}$ (not in data booklet)

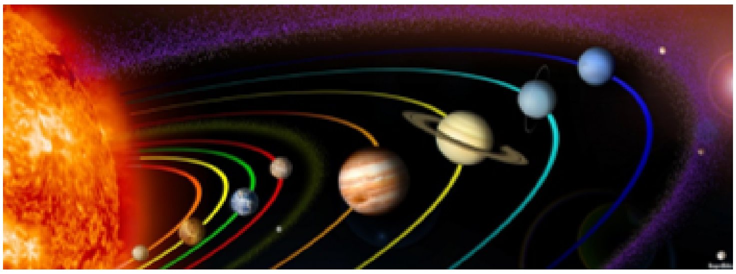
Summary

You can see many similarities between electric and gravitational fields.

ELECTRIC FIELDS	GRAVITATIONAL FIELDS
Electric field strength, E, is the force per unit charge on a small positive test charge placed at the point	Gravitational field strength, g, is the force per unit mass on a small test mass placed at the point
Inverse square law for the force between two electric charges in the form $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ (Coulomb's law)	Inverse square law for the force between two masses in the form $F = G\frac{M_1M_2}{r^2}$ (Newton's law of gravitation)
F can be attractive or repulsive	F is attractive only
$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$ for the field strength due to a point charge in free space or air **Unit: N C^{-1} or V m^{-1}**	$g = \frac{GM}{r^2}$ for the field strength due to a point mass **Unit: N kg^{-1} or m s^{-2}**
Potential at a point due to a point charge in terms of the work done in bringing a unit positive charge from infinity to that point **This definition can be used to calculate escape velocities.**	Potential at a point due to a point mass in terms of the work done in bringing a unit mass from infinity to that point **This definition can be used to calculate escape velocities.**
$V_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ **Unit: J C^{-1}** and $PE = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r}$ **Unit: J**	$V_g = -\frac{GM}{r}$ **Unit: J kg^{-1}** and $PE = -\frac{GM_1M_2}{r}$ **Unit: J**
Change in potential energy of a point charge moving in any electric field $= q\Delta V_E$ **Unit: J**	Change in potential energy of a point mass moving in any gravitational field $= m\Delta V_g$ **Unit: J**
Field strength at a point is given by E = - slope of the $V_E - r$ graph at that point	Field strength at a point is given by g = - slope of the $V_g - r$ graph at that point
Note that $\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\ F^{-1}m$ is an acceptable approximation	

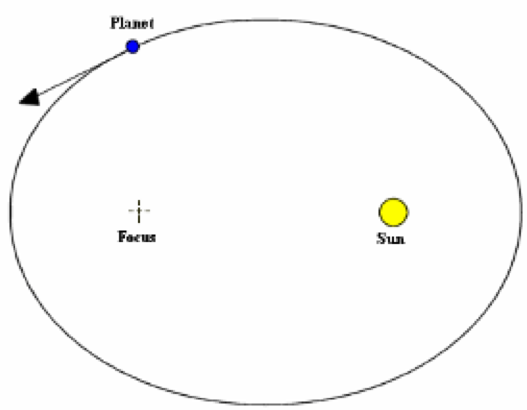
Unit 4.3 – Orbits and the wider Universe.

Kepler's laws.



Johannes Kepler was a German mathematician and astronomer who worked with Tycho Brahe. Kepler used observational data acquired by Tycho to develop new mathematical theories that described planetary motion. He developed three scientific laws known as "Kepler's laws of planetary motion". These laws can be applied to **any** two objects which are orbiting each other.

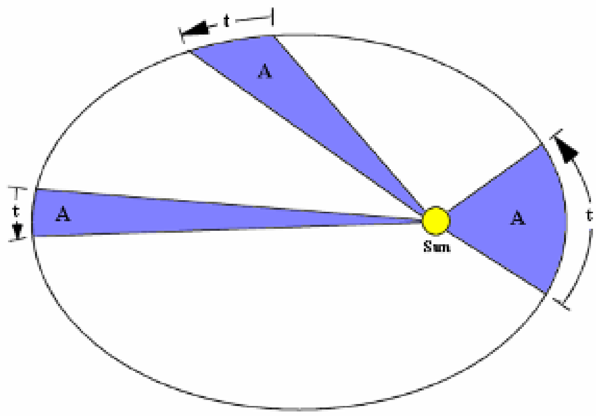
Kepler's 1st law: *each planet moves in an ellipse with the Sun at one focus.*



The planets do not orbit the Sun in a perfect circle, instead the orbits are **elliptical**. Most planets in our solar system have orbits that are very close to being a perfect circle and therefore we can apply the circular motion equations used in 'Unit 3' to their motion.

Video link explaining Kepler's 1st law.
https://www.youtube.com/watch?v=qDHnWptz5Jo

Kepler's 2nd law: the line joining a planet to the centre of the Sun sweeps out equal areas in equal times.

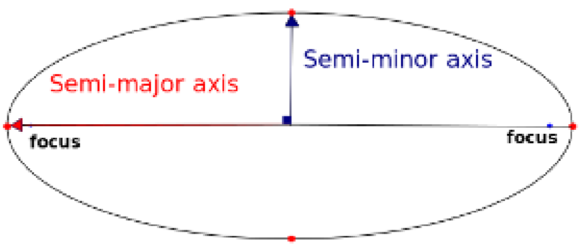


The time taken by the planet to move through all 3 blue shaded areas 'A' shown on the diagram are equal. When the planet is closer to the Sun it moves faster but when it is farthest away it moves slower and therefore the arc of the angle it moves through is smaller.

Video link explaining Kepler's 2nd law. https://www.youtube.com/watch?v=qd3dIGJqRDU

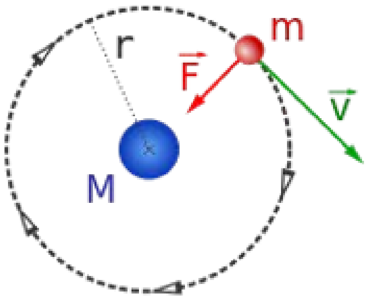
Kepler's 3rd law: the square of the period, T^2, of the planet's motion, is proportional to r^3, in which r is the semi-major axis of its ellipse.

$$T^2 \alpha\, r^3$$



The semi-major axis of an ellipse is half width of the long axis of the ellipse.
Video link explaining Kepler's 3rd law.
https://www.youtube.com/watch?v=KbXVpdlmYZo

Derivation of Kepler's 3rd law.



You need to know how to derive Kepler's 3rd law. Consider an object of mass *m* in a circular orbit of radius r about a much more massive object of mass *M*. This equation can be used for satellites, moons, planets or stars orbiting a black hole.

The equation for the centripetal force (unit 3) necessary for the circular motion is:

F = m ω^2 r but we can substitute **'ω' with** $= \frac{2\pi}{T}$ which becomes $\frac{4\pi^2}{T^2}$

So now the centripetal force, $F = \frac{m\,4\pi^2\,r}{T^2}$

T is the orbital period.

This centripetal force is provided by the gravitational force.

Gravitational force $F = \frac{GMm}{r^2}$

We can now combine the 2 equations to derive Kepler's 3rd law:

Centripetal force = Gravitational force

$$\frac{\cancel{m}\,4\pi^2\,r}{T^2} = \frac{G\ M\ \cancel{m}}{r^2}$$

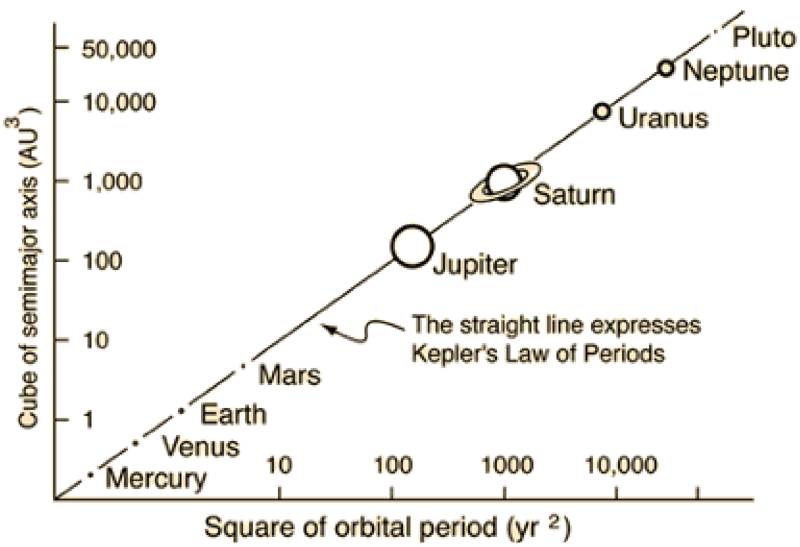
$$\frac{4\pi^2\,r^3}{T^2} = G\ M$$

$$4\pi^2\,r^3 = G\ M\ T^2 \quad \rightarrow \quad T^2 = \frac{4\pi^2\,r^3}{G\ M}$$

The mass of the Sun M (for example), G and $4\pi^2$ are all constants so we can say that;

$$T^2 \propto r^3$$

A graph showing the relationship between T^2 and r^3 for the planets in out solar system. The graph shows that the relationship is directly proportional.



Example. Calculating the mass of the central object.

Use the data below to calculate an approximate value for the mass of the Earth :

Radius of Moon's orbit = 384400 km = $3{\cdot}844 \times 10^8$ m

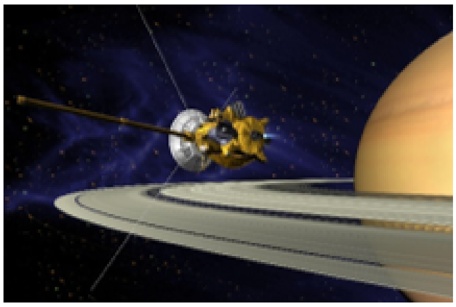
Period of Moon's orbit = 27·32 days x (60 x 60 x 24) = $2{\cdot}36 \times 10^6$ s.

$$\frac{m\,4\pi^2\,r}{T^2} = \frac{G\,M\,m}{r^2} \rightarrow M = \frac{4\pi^2\,r^3}{G\,T^2}$$

$$M = \frac{4\pi^2 \times \left(3.844\times10^8\right)^3}{6.67\times10^{-11} \times (2.36\times10^6)^2} = 6.04\times10^{24}\ \text{kg}$$

Velocity for weightlessness

Astronauts appear to be weightless but in reality are in free fall, accelerating towards the Earth all the time. If an object (e.g. satellite) is thrown sideways fast enough, it will fall continuously but never hit the Earth (without air resistance). An object can only be truly weightless if there is no gravitational field. The object would have to be infinitely far away from any other body.



Video - http://www.latestlesson.org/a-simple-animated-explanation-of-free-falling-and-zero-gravity/

In terms of equations,

the Centripetal force, $F = \frac{mv^2}{r}$ is provided by the gravitational force, $F = \frac{GMm}{r^2}$

The two equations are equal: $\frac{m\,v^2}{r} = \frac{GMm}{r^2} \rightarrow v^2 = \frac{GMr}{r^2}$

$$v^2 = \frac{GM}{r} \rightarrow v = \sqrt{\frac{GM}{r}}$$

Using this equation we can calculate the velocity a satellite must be travelling at to stay in orbit at a certain height above the Earth. Remember that a satellite in geostationary orbit remains above the same point on the Earth surface and has a 24hr orbit.

Example:

Calculate the (a) orbital speed and (b) the period in minutes of a satellite that is only 100km above the Earth's surface. Mass of Earth is $6.0\text{x}10^{24}$ kg and the radius of Earth is $6.4\text{x}10^6$ m.

1st step is to realise that the satellite is effectively orbiting the centre of the Earth. Therefore the total radius = $6.4\text{x}10^6 + 100{,}000 = 6.5\ \text{x}10^6$ m

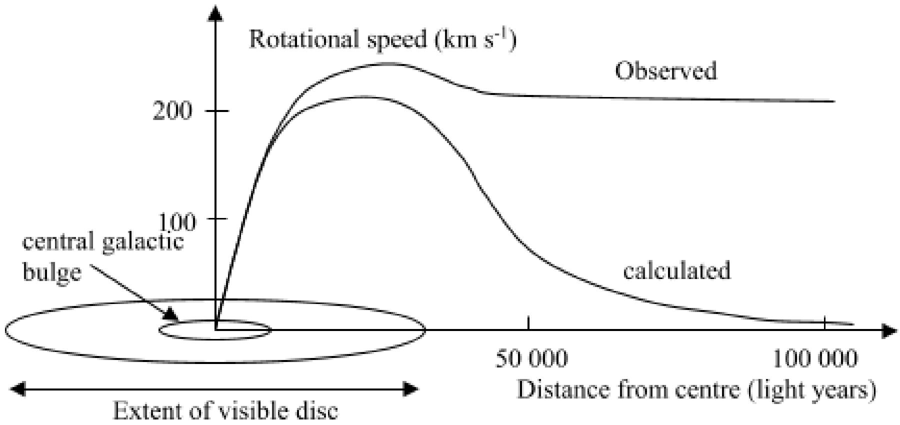
(a) $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11} \times 6.0\times10^{24}}{6.5\ \text{x}10^6}} = 7846\ \text{ms}^{-1}$

(b) $T = \frac{2\pi r}{v} = \frac{2\pi \times 6.5\ \text{x}10^6}{7846} =$ 5205 s divide by 60 → 87 minutes

Orbital speeds of galaxies

Spiral galaxies like our Milky Way are flattened disc, composed of millions of stars which all rotate in the same plane about a central point. Spiral galaxies also contain vast amounts of gas and dust from which new stars are formed.

The graph shows the observed (simplified) rotational speed curve for a typical spiral galaxy:



The calculated curve takes into account all the observable matter (stars and dark gas and dust nebula) in a galaxy.

Where the radius from the centre of the galaxy is low the rotational speed is roughly proportional to it and therefore all the matter in this region of the galaxy rotates with the same angular velocity (like a bicycle wheel). This implies that the density of matter is constant within this region of the galaxy.

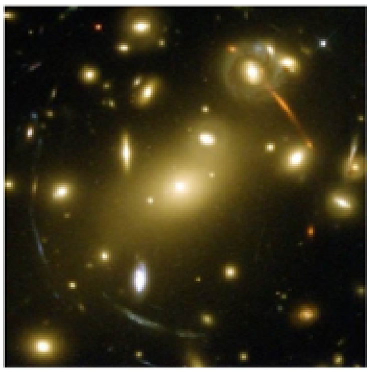
As the radius increases beyond 30,000 light years from the centre of the galaxy, (where the observed matter density is very low) you would expect the orbital speed, *v*, to decrease with $r^{-1/2}$.

This is the same relationship as we observe for the planets in the Solar System, since $v = \sqrt{\frac{GM}{r}}$.

The difference between the observed and calculated rotational speeds implies that there must be much more mass/matter in a galaxy than we can 'visibly' see. The rotational speed is approximately constant and does not 'tail off' as expected. This unaccounted for mass is known as **DARK MATTER.**

In summary: the measured rotational velocity of stars further out from the centre of the galaxy is higher than expected. This implies that the **mass is greater** than what we can see. This suggests the existence of the so called "dark matter".

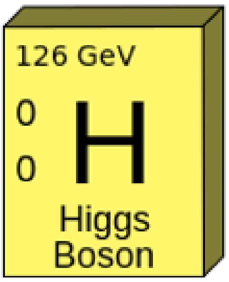
Dark Matter



Dark matter is one of the greatest unsolved mysteries of the universe. Its presence can be detected from the gravitational pull it exerts on visible matter that we can see, the fact that it does not emit or absorb any radiation makes it next to impossible to detect.

Dark matter is matter which we can't see, or detect by any sort of radiation, but whose existence we infer/detect from its gravitational effects.

The Higgs boson



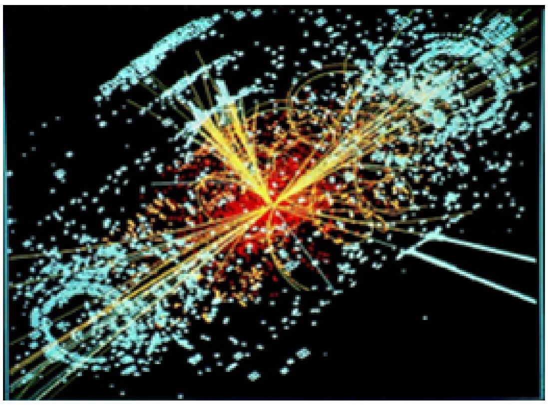
The Higgs boson is thought to be linked with the mass of particles including dark matter. Further experiments at the Large Hadron Collider may reveal links between the Higgs boson and dark matter and thus the nature of dark matter itself.
A useful link is:
http://blogs.cardiff.ac.uk/physicsoutreach/resources/heavens-kitchen/

Excerpt from:
http://www.symmetrymagazine.org/article/august-2013/using-the-higgs-boson-to-search-for-clues

One of the big questions concerns dark matter, the invisible 'stuff' that astrophysicists estimate makes up over 80% of the mass of the Universe. We have finally discovered the Higgs boson: this special particle, a particle that does not carry any spin, might decay to dark matter particles and may even explain why the Universe is matter dominated.

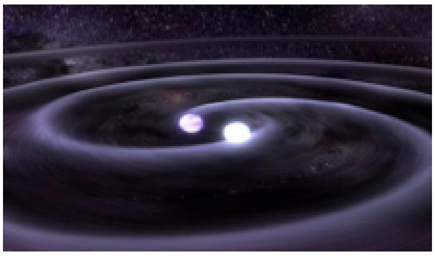
Because of the Higgs the electron has mass, atoms can be formed, and we exist. But why do elementary particles have such difference masses? Further study at the LHC will enable us to study, with higher precision, the decays of the Higgs boson to quarks. It will also enable us to search for other particles similar to the Higgs and determine if the Higgs decays to dark matter.

The reason we proposed the concept of dark matter is because we cannot explain the total mass of the universe. And the only way we know how fundamental particles acquire mass is through the Higgs mechanism. So if dark matter is fundamental, it has to interact with the Higgs to acquire mass, at least in our known framework. When the Higgs is produced at the LHC, it quickly decays into lighter, more stable particles. Currently LHC scientists are studying all the possible ways the Higgs can decay into other particles to search for any unexplained decays that could be hints of something new, like dark matter.



A Higgs event at the Large Hadron Collider.

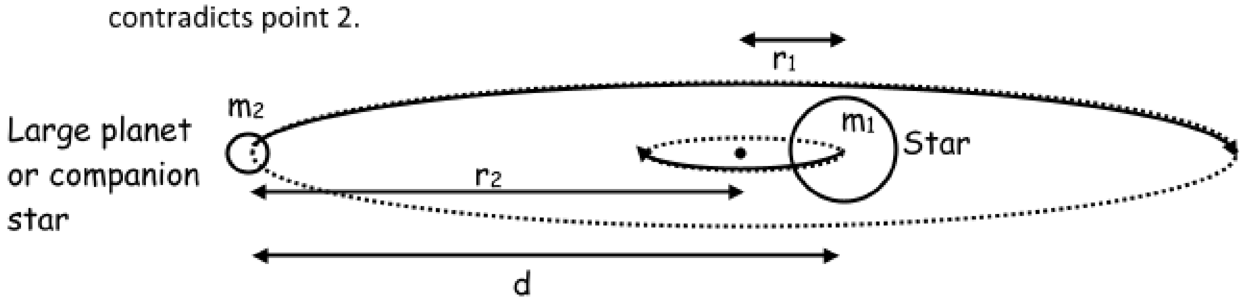
Binary systems and Exoplanets



When using Kepler's 3rd law we have assumed that a small mass is orbiting a larger mass. In fact the planet and its parent star both orbit the centre of mass of the system and the star will not remain stationary.

Centre of Mass: In a binary system, both bodies orbit around a point which is called the centre of mass. It is used when orbiting bodies have similar mass like a binary star system. Let us consider two spherically-symmetric objects, of comparable mass, orbiting about their centre of mass. We can infer three things about the system:

1. The centre of mass must be on the line joining the centres of the two objects.
2. The centre of mass must be between the objects as the direction of the centripetal acceleration must be towards it.
3. The angular velocities (ω) of the objects must be identical – if this were not the case, the objects would sometimes be on the same side of the Centre of Mass, which clearly contradicts point 2.



Consider two bodies, of mass m_1 and m_2 orbiting around their Centre of Mass, C. Each body exerts an attractive force upon the other and, by Newton's 3rd Law, these are equal and in opposite directions.

$$r_1 = \frac{m_2}{m_1+m_2}d \quad \text{(given in data booklet)}$$

We can use Newton's Law of Gravitation to work out the orbital characteristics of the binary system. T is the orbital period (this is same for both masses).

$$T = 2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}} \quad \text{(given in data booklet)}$$

In the case of a planet orbiting a star then we can assume that the mass of the star is much greater than the mass of the planet ($m_1 >>> m_2$).

1. $r_2 = \frac{m_1}{m_1+m_2}d$ (m_1 is negligible) so, $r_2 = \frac{\cancel{m_1}}{\cancel{m_1}}d$ and therefore, $r_2 = d$

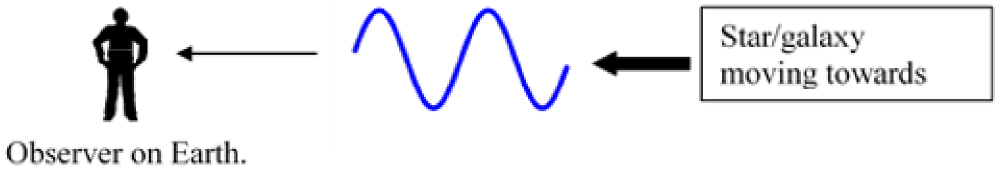
2. $r_1 = \frac{m_2}{m_1}d$ Rearranging gives → $m_1 = \frac{r_1\, m_1}{d}$

3. $T = 2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}}$ (m_1 is negligible) → $T = 2\pi\sqrt{\frac{d^3}{G(m_1)}}$

Measuring velocities using the Doppler Effect

We can determine the radial and orbital velocities of stars from the shift in their spectral lines. If such an object is moving towards or away from us, the wavelength of the radiation which we receive is shifted.

If a star/galaxy is moving **towards** us, the wavelength of the light will decrease, this is known as blue shift.



Recession - If a star/galaxy is moving **away from** us, the wavelength of the light will increases, this is known as redshift.

Doppler shift equation.

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

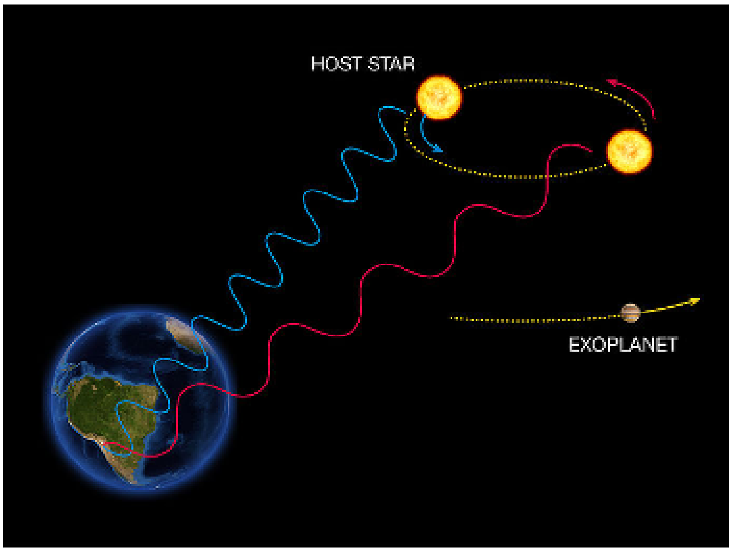
v – radial velocity (ms^{-1}) c - speed of light ($3x10^8\ ms^{-1}$)

$\Delta\lambda$ – observed (shifted) wavelength (m) λ – emitted (unshifted) wavelength (m)

Radial velocity: the velocity **v** in this equation is the component of the objects/star's velocity along the line joining the observer to the object.

In questions we would always assume that we see such a system edge-on. In reality, the situation is more complicated. We cannot measure any sideways movement using this method.

The equation can be used to calculate the orbital velocity of a star which has a massive planet in orbit. We cannot see the planet, but we can detect/infer its presence from data about the velocity of the star.



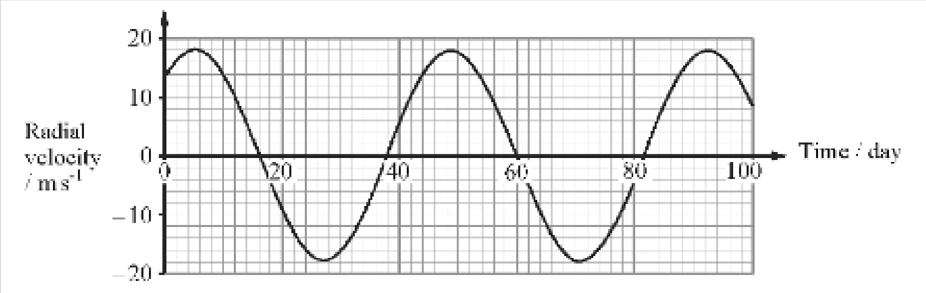
The Radial Velocity Method

ESO Press Photo 22a/07 (25 April 2007)

The Host star and the exoplant are both orbiting the same point. The planet doesn't emit enough light to be detected here on Earth. Therefore we can only detect its presence by the shift in the wavelength of the light emitted by the star. The shift will be greatest in the two positions shown on the left. We must assume that this system is viewed edge on.

Example: past paper (PH4 January 2013)

The light spectrum from a star 41 light years from Earth was analysed. It was found that this light was Doppler shifted due to the star orbiting the mutual centre of mass of the star and a nearby planet. The following graph is derived from the data obtained.



(a) From the graph, calculate the period of the orbit in **seconds**.

Time period = 44 days
Time period in seconds = 44 x 60 x60 x 24 = 3.83x10⁶ s

(b) Show that the radius of orbit of the star about the centre of mass is approximately 1.1×10^7 m.

Read 'v' from graph = 18ms^{-1}

$v = \frac{2\pi r}{T}$ Rearrange → $r = \frac{vT}{2\pi} = \frac{18 \times 3.83\times10^6}{2\pi} = 1.097x10^7m \approx 1.1x10^7m$

(c) The mass of the star is 1.9×10^{30} kg. Calculate the distance between the star and the planet, ensuring that you state any approximation that you make.

$T = 2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}}$ Rearrange → $d^3 = \frac{T^2G(m_1+m_2)}{4\pi^2}$

Assume that the mass of the star is much greater than the mass of the planet $m_1 >> m_2$,
So, $(m_1 + m_2) \approx m_1$

$d^3 = \sqrt[3]{\frac{T^2G(m_1)}{4\pi^2}} = \sqrt[3]{\frac{(3.83\times10^6)^2 \times6.67\times10^{-11}\,(1.9\times10^{30})}{4\pi^2}} = 3.6\,x10^{10}m$

(d) Using the centre of mass equation, or otherwise, estimate the mass of the planet and compare its mass with that of the Earth (ME = 5.9×10^{24} kg).

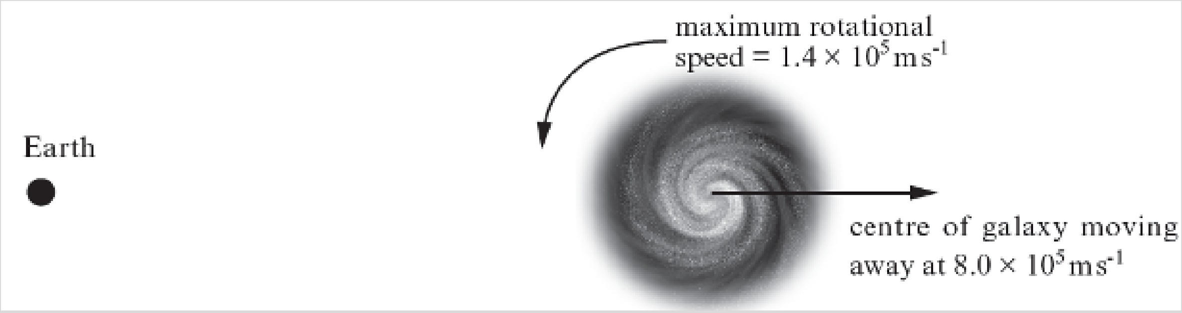
$r_1 = \frac{m_2}{m_1+m_2}d$ Assume $(m_1 + m_2) \approx m_1$, and rearrange to give →

$m_2 = \frac{r1\,m_1}{d} = \frac{1.097\times10^7 \times 1.9\times10^{30}}{3.6\times10^{10}} = 5.79 \times 10^{26}$ kg

Comparing: the mass of the planet is roughly 100 times the mass of the Earth.

Example: past paper (PH4 January 2013)

A spiral galaxy is moving away from the Earth and rotating as shown.



a) Calculate the maximum and minimum red shift measured by an Earth observer when light of wavelength 656 nm is analysed from this spiral galaxy. [3]

The maximum resultant radial velocity $= 8.0\times10^5 + 1.4\times10^5 = 9.4\times10^5\ \text{ms}^{-1}$

The minimum resultant radial velocity $= 8.0\times10^5 - 1.4\times10^5 = 6.6\times10^5\ \text{ms}^{-1}$

Using the Doppler equation: $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$ Rearrange → $\Delta\lambda = \frac{\lambda v}{c}$

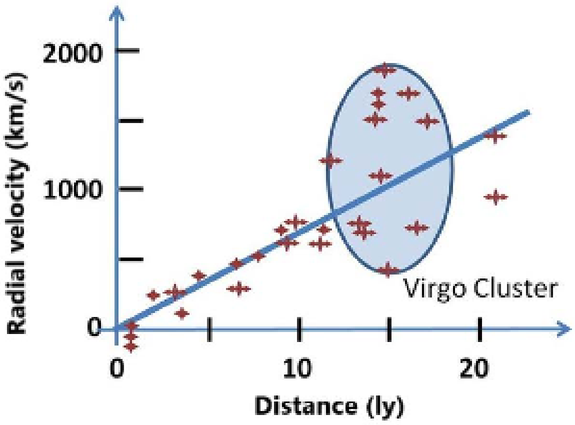
Maximum red shift → $\Delta\lambda = \frac{\lambda v}{c} = \frac{656\times10^{-9}\times 9.4\times10^5}{3\times10^8} = 2.06\times10^{-9}$ m or 2.06 nm

Minimum red shift → $\Delta\lambda = \frac{656\times10^{-9}\times 6.6\times10^5}{3\times10^8} = 1.44\times10^{-9}$ m or 1.44 nm

Hubble's Law

Hubble collected data about the redshift of the light from stars in galaxies at various distances away from Earth. He noticed that the further away the galaxies were, the greater the redshift.

A graph of the data was plotted to show the relationship:



The gradient of the graph is $H_0 = \frac{v}{d}$

Hubble's Law: The speed of the recession of a galaxy is directly proportional to its distance from the Earth.

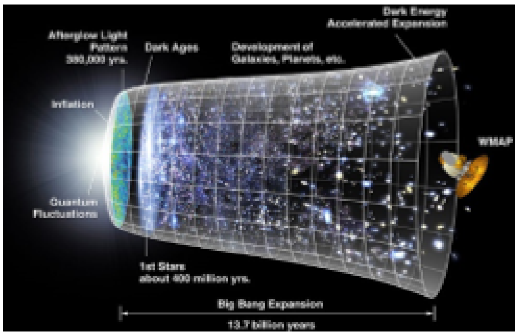
The gradient of the graph gives us the Hubble constant. The Hubble constant (H_0) relates the galactic radial velocity (v) to distance (D) and it is defined by $\boldsymbol{v = H_0 D}$.

The accepted value for the Hubble constant is $\mathbf{2.2\times10^{-18}\ s^{-1}}$, although the Hubble constant is normally given a value of 68 $\text{kms}^{-1}\ \text{Mpc}^{-1}$.

This constant gives a measure of the expansion rate of the universe. The Hubble Constant describes how fast objects appear to be moving away from our galaxy as a function of distance.

The age of the Universe. (T)

We can use the Hubble constant to calculate the size and age (t_H) of the universe. If we assume that the radial velocity of the galaxies has been approximately constant since the beginning of the universe we can work out how the distance they have travelled.



Distance = speed x time → $D = v\,t_H$

we know that $v = H_0 D$ so,

Substitute v for '$H_0 D$' to give → $\cancel{D} = H_0 \cancel{D} t_H$

Rearrange → $t_H = \frac{1}{H_0}$

So we can deduce that the age of the universe '$\mathbf{t_H}$' = $\frac{1}{H_0}$

Problems with the Hubble constant.

1. Uncertainty in the value of the Hubble constant since it is determined using the redshift and the distance to stars.
2. The assumption that the radial velocity of the galaxies has been constant.

Example

Calculate the value for the age of the universe in years from the following data. A galaxy at a distance of 1.0×10^{10} ly receded from the Earth with a speed of 2.0×10^{8} ms^{-1}.
1 light year = 9.46×10^{15}m.

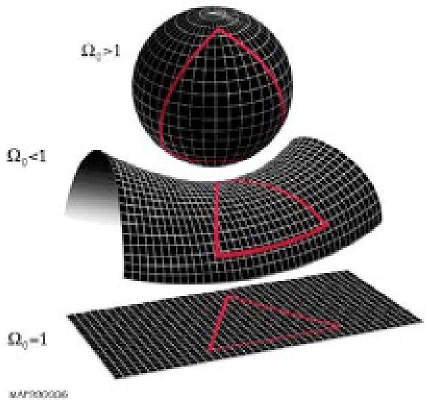
Change light years to metres. → $1.0 \times 10^{10} \times 9.46\times10^{15} = 9.46\times10^{25}$ m

Time = $\frac{distance}{speed} = \frac{9.46\times10^{25}}{2.0\times10^{8}} = 4.73 \times10^{17}$ s To change this value to years →

$\frac{4.73\times10^{17}}{(60\times60\times24\times365.25)} = 1.5 \times10^{10}$ years. This is 15 billion years.

$= 4.7\times10^{17}$ s / $3.15\times10^{7} = 1.5\times10^{10}$ years.

What is the fate of the universe?



Will the universe continue to expand forever? Some suggest that it will collapse back in on itself in what is known as the 'big crunch'? Most scientists believe that the rate of expansion of the universe is increasing.

The 3 possible fates of the universe are open, closed and flat.

https://www.youtube.com/watch?v=R5orcCuprG4 (Fate of universe)

https://www.youtube.com/watch?v=oCK5oGmRtxQ (Shape of universe)

For a flat universe, the radial velocity of galaxies becomes zero when the time is infinite i.e. the radial velocity of galaxies is equal to the escape velocity.

Critical Density - ρ_c

If the present density of the universe is less than the critical density, **ρ_c** then the universe will keep on expanding forever. If the density is greater than **ρ_c** the universe will at some point in the future stop expanding and collapse in a big crunch. According to the cosmological principle we know that the universe is very uniform.

Deriving an equation for the critical density of a 'flat' universe using conservation of energy.

For the galaxies to continue moving further away and never return (infinity), therefore preventing it collapsing in on itself (Big Crunch) then the radial velocity of the galaxies must be greater than the escape velocity.

The gain in potential energy of the galaxies in moving to infinity = 0 - - $\frac{G\,M\,m}{r}$ = $\frac{G\,M\,m}{r}$

This potential energy is supplied by the kinetic energy, kinetic energy = ½ mv^2

So, $$\frac{1}{2}\cancel{m}v^2 = \frac{G\,M\,\cancel{m}}{r}$$

$$\frac{1}{2}v^2 = \frac{G\,M}{r}$$ **Equation 1**

Assume universe is a sphere and the mass of the universe inside the sphere upon whose outer surface lies the galaxy:

Volume of sphere = $\frac{4}{3}\pi r^3$ → Mass = density x volume

Mass of sphere = $\rho_c \frac{4}{3}\pi r^3$ Substitute for 'M' into equation 1

. $$\frac{1}{2}v^2 = \frac{G\,\rho_c\,\frac{4}{3}\pi r^3}{r}$$ **Equation 2**

The velocity of the galaxy is given by Hubble's law v = $H_0 D$ or H_0r (since D=r), which can be substituted into equation 2.

$$\frac{1}{2}(\mathrm{H_0 r})^2 = \frac{G\,\rho_c\,\frac{4}{3}\pi r^3}{r} \rightarrow \frac{1}{2}\mathrm{H_0}^2\cancel{r^2} = \frac{G\,\rho_c\,\frac{4}{3}\pi\,\cancel{r^3}}{\cancel{r}}$$ (r cancel)

$$\frac{1}{2}\mathrm{H_0}^2 = G\,\rho_c\,\frac{4}{3}\pi$$ Rearrange → $$\rho_c = \frac{3\,\mathrm{H_0}^2}{4 \times 2 \times G\,\pi}$$

$$\rho_c = \frac{3\,\mathrm{H_0}^2}{8\,\pi\,G}$$

ρ_c – density (kgm^{-3})

By using this equation we can try to calculate the critical density of universe and hence attempt to determine the fate of the universe.

Example

Calculate a value of the critical density of the universe and estimate the number of hydrogen atoms per m³ that this density corresponds to.

$$\rho_c = \frac{3\ H_0{}^2}{8\ \pi\ G} = \frac{3\ x\ \ 2.2x10^{-18\ 2}}{8\ \pi\ G} = 8.66x10^{-27}\ \text{kg m}^{-3}$$

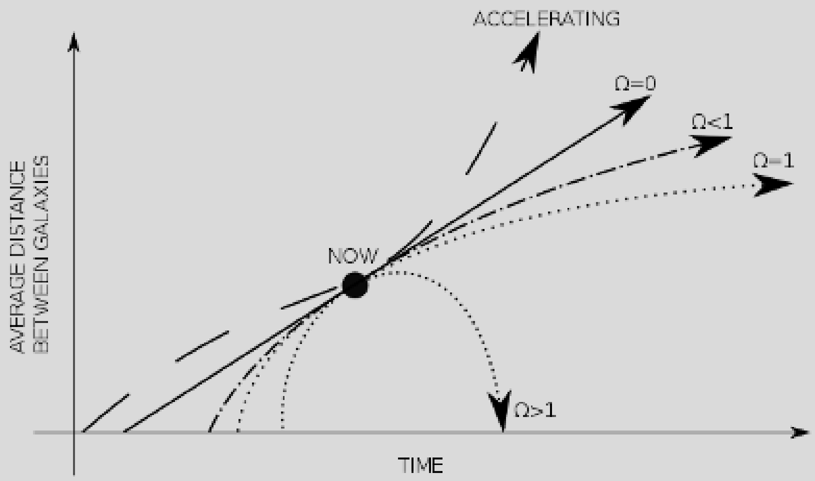
1 H atom ≈ $1.66x10^{-27}$ kg

Number of H atoms = $\frac{8.66x10^{-27}}{1.66x10-27}$ = 5.2 ≈ 5 H atoms per m³

Note.

If average density of the Universe is less than critical density then it will be too small to stop it expanding an therefore it will go on expanding forever.

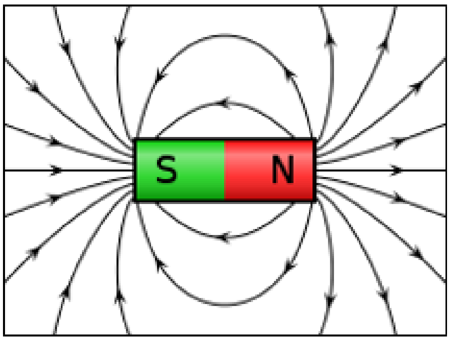
If the average density of the Universe is greater than the critical density value it will cause the universe to collapse back in on itself (big crunch).



An accelerating universe will have the longest lifetime since it will keep on expanding at an ever increasing rate.

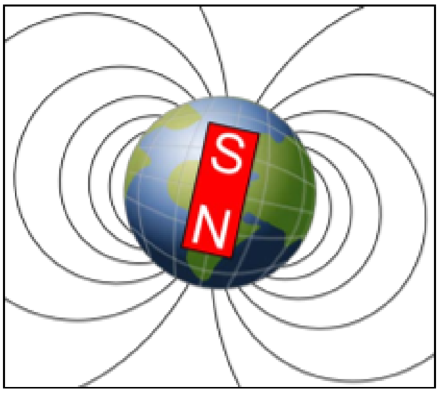
Unit 4.4 – Magnetic Fields (B-fields)

Magnetic fields can be shown by field lines, which go from North to South. The field lines in a strong magnetic field are more closely packed than in a weak field.
The magnetic field lines of a bar magnet →



Recap of magnetic theory.

- Unmagnetised materials are attracted to either pole.
- Like poles repel; unlike poles attract.
- In the Earth's magnetic field, the north pole of a magnet will align itself to point to the north.
- The Earth has a magnetic field like a bar magnet. The South-pole is under the geographic North pole.



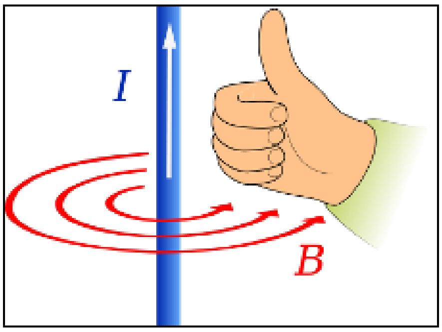
The Earth has a magnetic field thought to be caused by currents in the Earth's molten core.

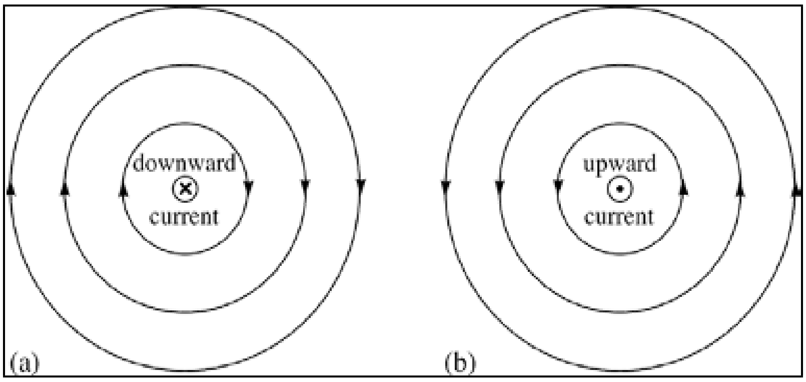
Shapes of magnetic fields.

A moving charge or a current creates a magnetic field in the surrounding space (in addition to an electric field). The magnetic field exerts a force on any other moving charge or current that is present in the field.

Magnetic field of a long straight current carrying conductor

The field lines circle the wire. The field strength is strongest near the wire. The circles get further apart as the distance from the wire increases. Using the 'right hand thumb rule' you can predict the direction of the field: the thumb should point in the direction of the current (+ to -) and the fingers of the right hand point in the direction of the magnetic field.



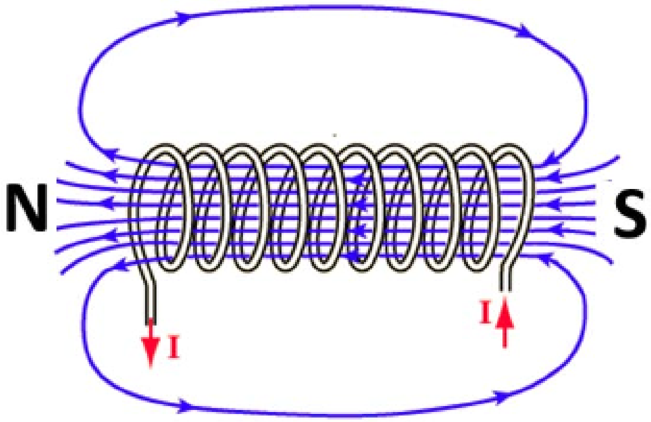


Arrow convention: an 'x' shows that the current is going into the paper (like the back of an arrow), whereas a '•' (like the head of an arrow) shows the current coming out of the paper.

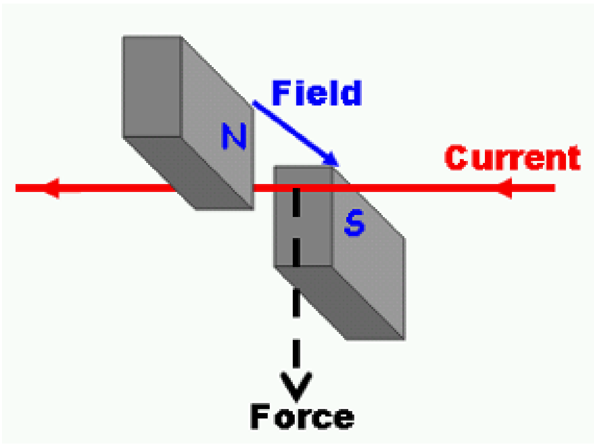
Magnetic field of a long solenoid/coil.

Inside the solenoid the field is very uniform, this shown with parallel evenly spaced lines. The field lines go from north to south (N → S).

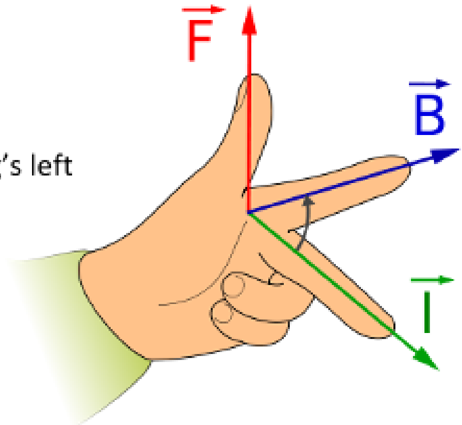
If a cylinder of **iron** is placed inside the solenoid the magnetic field is increased.
An electromagnet is a long coil of wire with a current passing through it with an easily magnetised core which is usually made of iron.



Movement of a current carrying conductor in a magnetic field (motor effect).



If a long straight wire carrying a current is placed in a magnetic field it will experience a force. In the diagram shown the wire experiences a downward force.

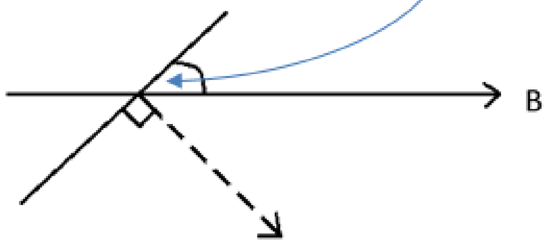


We can predict/determine the direction of the force using 'Fleming's left hand rule'. The digits must be at right angles.
The first finger gives the direction of the magnetic field, N → S.
The Second finger gives the direction of the current, + → - .
The thumb gives the direction of the force on the wire.

Magnetic field or magnetic flux density**, B:** is defined by the equation,

$$F = B\,I\,L\sin\theta$$

F – force on wire (N)
L – length of wire (m)
B – magnetic field strength – Tesla (T)
θ - angle of wire to the field (°)



The angle **θ,** is related to the direction of the magnetic field 'B' as shown in the diagram.

The Tesla, T. (vector quantity)

The magnetic field strength 'B' is defined by *(one Tesla): when a wire of length 1m carrying a current of 1A at a right angle (perpendicular) to the field experiences a force of 1N.*

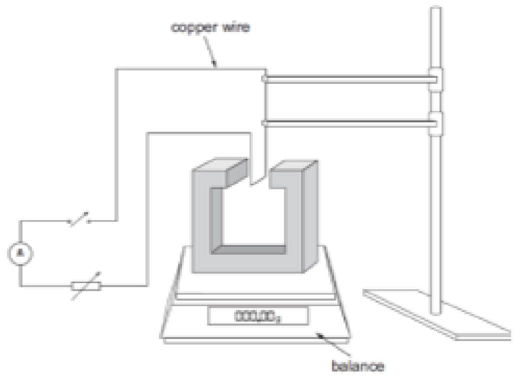
$$B = \frac{F}{Il}$$

1 Tesla is a strong magnetic field. The Earth has a magnetic field strength of 5×10^{-5} T.

INVESTIGATION OF THE FORCE ON A CURRENT IN A MAGNETIC FIELD.

The force on a current carrying wire in a magnetic field is described by the relationship: $F=B$IL $\sin\theta$. In this practical arrangement, the value of θ=90 so the equation can be simplified to $F=B$IL. The value of *F* is determined by the weight of the magnet placed on a balance.

In effect $F=\Delta m$g where Δm is the apparent change in mass as *F* varies due to the magnitude of the current. The current can be varied and a graph of *F* against *I* can be plotted which should be linear. The length of the wire can be measured and the magnetic flux density of the magnet can be determined from the gradient of the graph and the value of length of wire within the pole pieces of the magnet.



Force on a charged particle moving in a magnetic field.

We know that a magnetic field and an electric current interact to produce a force. Since current is the flow of charge, the magnetic field exerts a force on the charge carriers (electrons). In a magnetic field, the force acts on the electrons always at **90°** to the direction of the movement. Therefore the path is circular (look back at the circular motion unit).

Consider a charge '*q*' moving through a magnetic field B at a constant velocity, '*v*'. The charge forms a current that moves a certain distance, *l*, in a time *t*.

- Velocity, $v = \frac{l}{t}$ → $l = v\,t$
- Current, $I = \frac{q}{t}$
- $Force,\ F = B\,I\,l$

So we can substitute l and I to give: $F = \frac{B\,q\,v\,\cancel{t}}{\cancel{t}}$

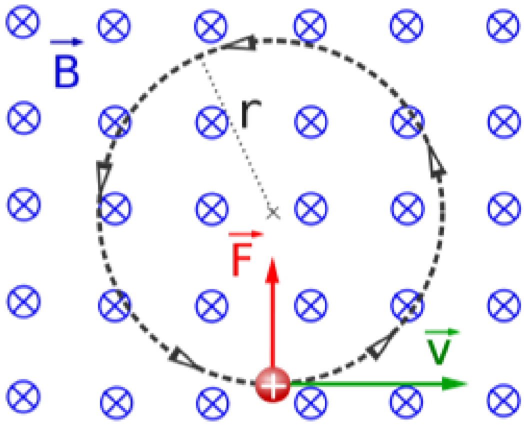
To give the equation→ $\boldsymbol{F = B\,q\,v \sin\theta}$

The charge 'q' is usually the electron charge 'e' = 1.6×10^{-19} C.

The 'sinθ' is included due to the component of the 'B' field as it is for a wire.

The direction of the force is given by Fleming's left hand rule but remember that if we are talking about an electron then the current is in the opposite direction to which it is travelling i.e the direction of the current flow (+ → -).

The path of charged particles in a magnetic field.



A magnetic field can alter the direction of the particle but it cannot change the speed of the particle. This is because the force applied is always at 90° to the direction of motion of the particle

We have seen that the force always acts on the wire at 90°, and that gives us the condition for **circular motion**.

The magnetic field provides the centripetal force:

$$F = \frac{mv^2}{r}$$

Try to use 'Fleming's left hand rule' to predict the force acting on the proton moving in a magnetic field. Remember that the magnetic field direction is into the paper and the current is in the direction of the green arrow (v).

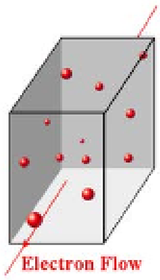
Equating the magnetic and centripetal force → $Bqv = \frac{mv^2}{r}$ *to give* → $Bq = \frac{mv}{r}$

Example. An electron passes through a cathode ray tube with a velocity of 3.7 × 10^7 ms^{-1}. It enters a magnetic field of strength 0.47 mT at a right angle. What is the radius of curvature of the path in the magnetic field?

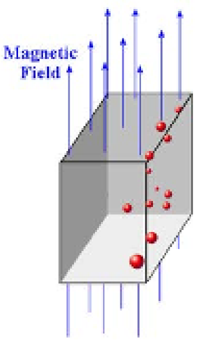
The mass and charge of the electron are on the front of the data booklet. Be careful with the magnetic field strength 0.47 mT → 0.47x10^{-3} T.

$Bq = \frac{mv}{r}$ Rearrange → $r = \frac{mv}{Bq} = \frac{9.11\times10^{-31} \times 3.7\times10^{7}}{0.47\times10^{-3} \times 1.6\times10^{-19}}$ = 0.45m

The Hall voltage.

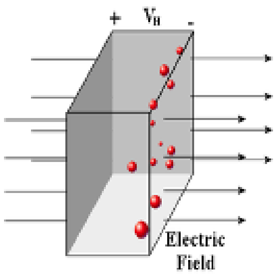


Consider a conductor carrying a current inside a magnetic field. You can use Fleming's left hand rule to predict the direction of the force on the electrons (red) and therefore determine which side of the conductor/ semiconductor slice is positive and which is negative.



When a current flows in the conductor the electrons will experience a force given by: **F = Bqv sin θ** . The electrons enter field at 90˚ so sin θ = 1.

As the electrons move through a piece of conductor / semiconductor through which a magnetic field passes, then the electrons experience a force which moves them to one side of the material. This means that there is a p.d. (voltage) between both sides of the probe - V_H. An electric field builds up giving a **force in the opposite direction.**

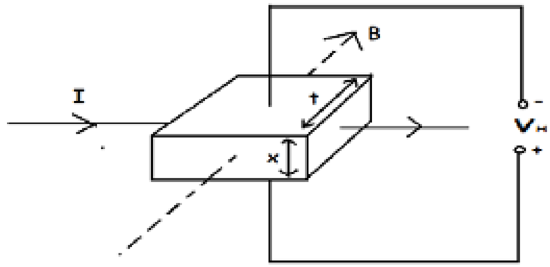


The Hall voltage.

When a magnetic field, ***B***, is applied to conductor carrying a current *I*, at right angles to the field direction, a so-called *Hall voltage* appears across the specimen, at right angles to the *B* and *I* directions.

The electric field strength is given by:

$$E = \frac{V}{d} \quad \rightarrow \quad E = \frac{V_H}{x}$$



Due to this field, the force on an electron is: $F = E\,q$

(Field strength is force per unit charge)

(substitute) $F = \frac{V_H}{x} q$

This force is directed downwards i.e. towards the positive surface (see diagram).

The flow of electrons between the upper and lower surfaces will stop when the two forces are equal:

$$B \cancel{q} v = \frac{V_H}{x} \cancel{q} \quad \rightarrow \quad B\ v = \frac{V_H}{x} \quad \rightarrow \quad V_H = B\,v\,x \quad \text{(equation 1)}$$

From Unit 2 electricity the equation relating the drift velocity to the current is:

$I = nAve$ Rearrange → $v = \frac{I}{nAe}$

Substitute 'v' in equation 1 →

$$V_H = \frac{BIx}{nAe}$$

A (area of slice) = xt so we can substitute 'xt' for A,

$$V_H = \frac{BI\cancel{x}}{n\cancel{x}te} \quad \rightarrow \quad V_H = \frac{BI}{nte}$$

So V_H is proportional to B for a constant I (current) → $V_H \propto B$

V_H is proportional to 1/n. The value of n in metals is about 10^4 times that in semi-conductors. For metals the typical value for V_H is 1μV whereas for semi-conductors under the same conditions V_H is about 10mV, this is why semiconductors are used.

The Hall Probe

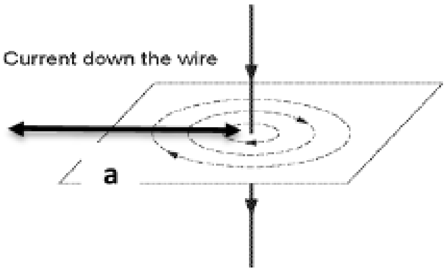
The flux density (B) of a magnetic field can be measured by using the Hall probe. One type of Hall probe has a small slice of germanium (a semi-conductor) on the end of long thin handle. The slice is now placed with its face at right angles to the magnetic field. A current is passed through the slice and the value of this current and the value of the Hall voltage is measured.

The value of the constants 'n','e' and 't' can be found by placing the probe in a magnetic field of known flux density and reading the value of the current and the Hall voltage .

Magnetic field strength in a long wire.

The equation for the magnetic field strength, B is:

$$B = \frac{\mu_0 I}{2\pi a}$$



I – Current (A) a – Perpendicular distance from wire (m)

μ_0 – Permeability of free space ($4\pi \times 10^{-7}$ Hm^{-1})

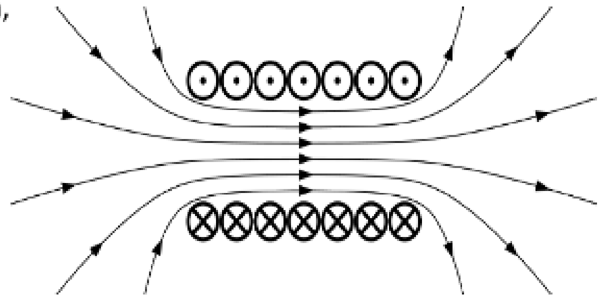
Example. A current of 5A flows through a long straight wire. Calculate the magnetic field strength at a right angle distance of 20cm to the wire.

$$B = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.20} = 5.0 \times 10^{-6}\ T$$

Magnetic field strength in a long solenoid.

The equation for the magnetic field strength, B is:

$$B = \mu_o n I$$



I – current (A) n – number of turns per m (m^{-1})

μ_0 – Permeability of free space ($4\pi \times 10^{-7}$ Hm^{-1})

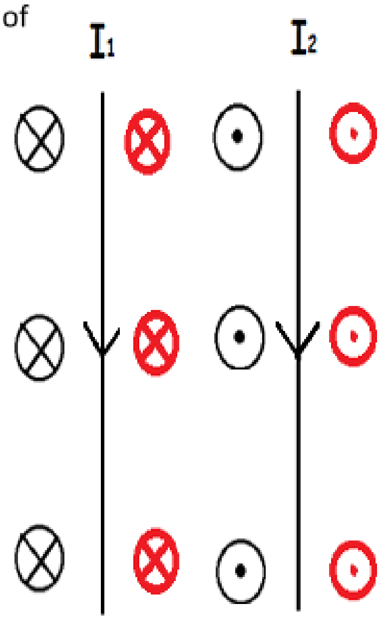
Example. A long solenoid of length 1.45m has 9560 turns. Calculate the magnetic field strength (B) inside the solenoid when it carries a current of 320 mA. (PH5 June 2014 Q4)

1st step is to calculate n: $n = \frac{9560}{1.45}$ = ***6593 turns per m***

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times 6593 \times 320 \times 10^{-3} = 2.65 \times 10^{-3}\ T$$

Why do current carrying conductors exert a force on each other?

The current flowing in wire I_1 creates a magnetic field. This magnetic field reaches wire I_2. The wire I_2, is also carrying current and so it will experience a force. The direction of this force is given
by Fleming's left hand rule. There will be an equal but opposite force on wire I_1 (Newton's 3rd law).
The black 'x' and '•' show the magnetic field due to wire I_1 and the red 'x' and '•', are the magnetic field due to the wire carrying the current I_2.

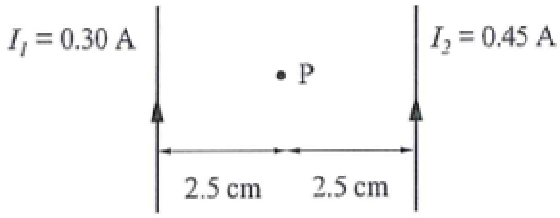


Explanation.

1. One wire is in the magnetic field of another
2. Field due to I_2 into paper at I_1
3 The force on the wire I_1 is to the right, due to Fleming's left hand rule. (The force is to the left for wire I_2).

If the currents flow is in the same direction the two wires will attract. If the currents flow in opposite directions the wires repel.

Example. Two long, straight wires carry currents as shown. Calculate the resultant magnetic field strength at point P in the diagram and state its direction. (PH5 June 2012 Q4)



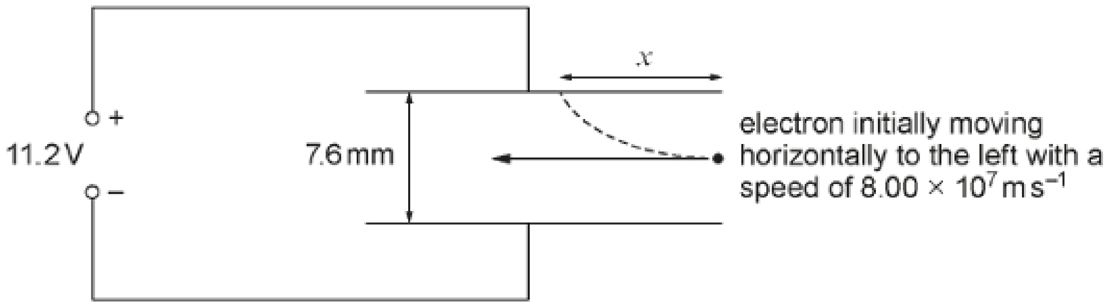
1st step is to calculate B for each wire.

For current I_1, B = $\frac{\mu_0 I}{2\pi a} = \frac{4\pi\times10^{-7}\times 0.30}{2\pi\times0.025}$ = 2.4x10^{-6} T $\qquad$ I_2 = 3.6x10^{-6} T

Using the right hand grip rule we can deduce that the magnetic field of wire I_1 at P is directed into the paper and the magnetic field of wire I_2 at P, is out of the paper. Since the two field are in opposite direction the resultant magnetic field =(3.6x10^{-6} - 2.4x10^{-6}) = 1.2 x10^{-6} T, out of the paper.

Motion of charged particles in magnetic and electric fields.

A charged particle like an electron will be deflected in a parabola if it enters a uniform electric field at **right angles (as shown in the diagram below, PH5 June 2014 Q5).**



The electric field strength is given by: E = $\frac{V}{d}$

From our definition of electric field we can say that the force on the electron is given by: F = EQ

Therefore the force is given by combining these two equations: $E = \frac{VQ}{d}$

Using Newton's 2nd law we can calculate the **acceleration**. If we wanted to know the upwards velocity, we would work out the **time interval**, and then use: acceleration = change in velocity/time to calculate the **upwards velocity**. Since the electrons are in a vacuum, the horizontal velocity is constant. Once we know the upwards velocity, then we can do a **vector addition** to calculate the **resultant velocity**.

Calculating the acceleration, using Newton's 2nd law: F=ma, rearrange → a = $\frac{F}{m}$

Substitute F=EQ for 'F' → a = $\frac{EQ}{m}$

Substitute E, since E = $\frac{V}{d}$ → $a = \frac{VQ}{d\,m}$

Input the data/numbers (from the above diagram). The mass and charge of the electron are found on the front of the data booklet.

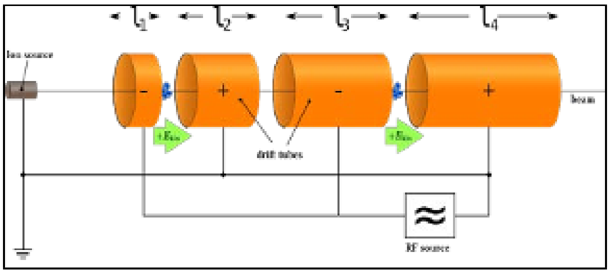
$$a = \frac{11.2\times1.6\times10^{-19}}{7.6\times10^{-3}\times 9.11\times10^{-31}} = 2.59\times10^{14}\ \text{ms}^{-2}$$

Particle accelerators. (The working of linear accelerators, cyclotrons and synchrotrons is not required.)

Linear accelerator.

The ions are accelerated in a straight line by an alternating p.d. The acceleration occurs in the gaps between the drift tubes. The drift tubes must increase in length as the speed of the ions increases.

The force, F on the charged particle, F = Eq which is equal to ma. → Eq = ma



The Cyclotron.

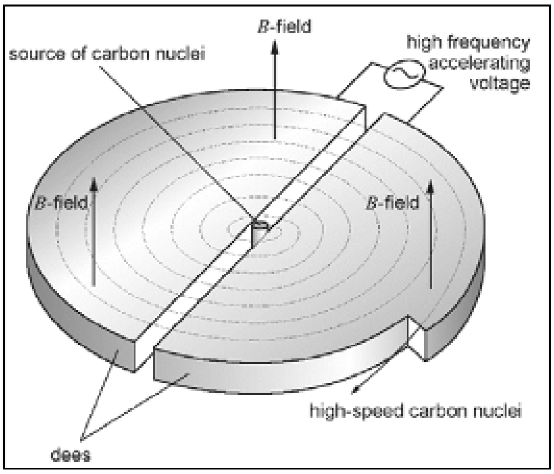
In a cyclotron a uniform magnetic field (B) provides a centripetal force. So we can write:

$$Bqv = \frac{mv^2}{r}$$

An electric field accelerates the charged particle as they cross between the dees. The resulting motion is a spiral as shown in the diagram. This means that the radius (r) increases with the velocity.

We can relate the velocity (v) to the frequency (f) of the a.c. supply using the equation:

$$v = 2\pi rf$$

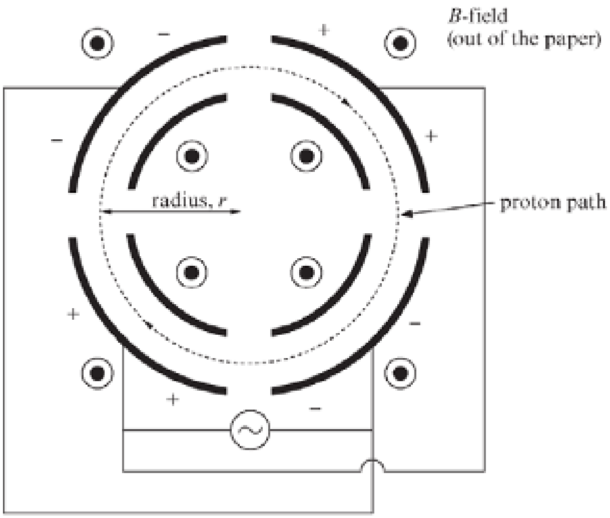


'r' is the radius of the path the particle follows.

As the speed of the particles gets close to the speed of light, the **relativistic mass** increase becomes significant and so the motion of the particle is out of sync with the alternating p.d.

The Synchotron.

In a synchrotron, protons are accelerated and their path kept circular by a magnetic field which has to increase as the speed of the protons increases. The protons are accelerated by the alternating p.d. applied to the quarter circle plated (see + and – in the diagram). The frequency of the alternating p.d. can be increased to overcome the problem of the increase in relativistic mass of the particle as its speed increases. This means that higher energies can be achieved compared with a cyclotron.



As for the cyclotron the magnetic field provides the centripetal force. So we can use:

The radius (r) of the path is constant and the value of (m) and (q) are constant so (B) is proportional to (v).

The kinetic energy gained by the particle is due to the alternating electric field and is given in electron volts (eV). So we can write:

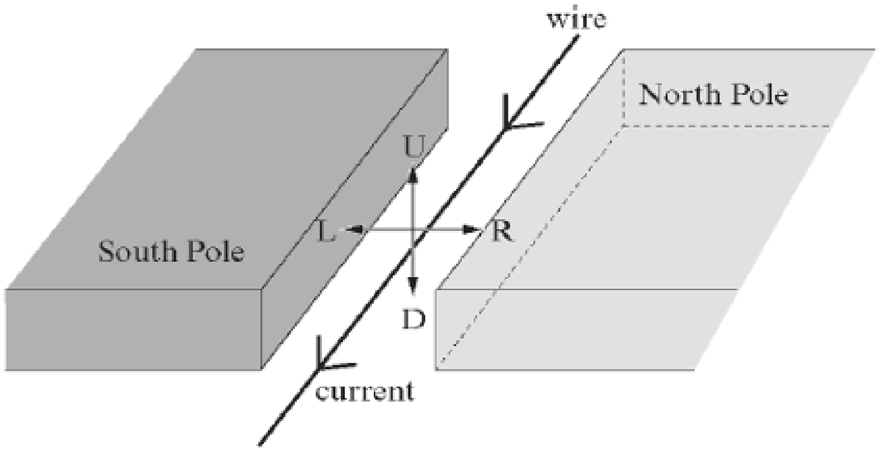
$$eV = \frac{1}{2}mv^2$$

Unit 4.5 – Electromagnetic Induction

Electromagnetic induction.

Michael Faraday thought that if an electric current produced a magnetic field then a magnetic field could generate an electric current, this can be done by moving or changing a magnetic field.

To induce (make) an electric current the magnet or wire must be moved. If the wire in the diagram shown below is moved side to side (L or R) then no current will be induced as no magnetic field lines are being cut. If the wire is moved up or down (U or D) then the magnetic field lines are being cut and therefore an e.m.f. is induced which causes a current to flow in the wire.



Fleming's right-hand rule.

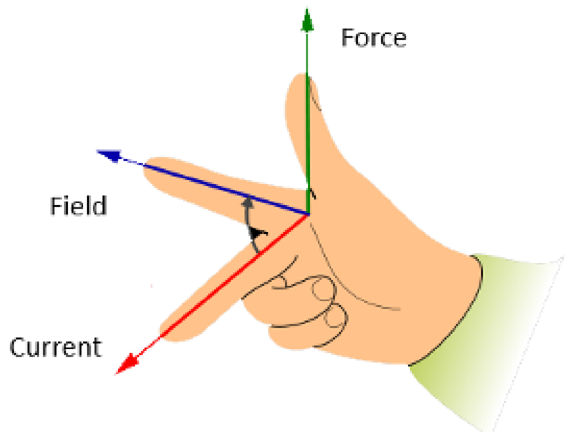
This is used to predict the direction of flow of the current when a conductor moves in a magnetic field.

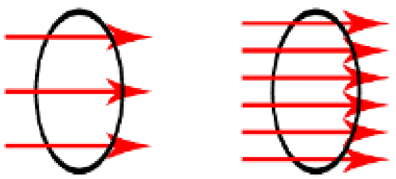
The ***thumb*** shows the direction of motion of the wire.

The ***first finger*** shows the direction of the magnetic field, always from north to south (N → S)

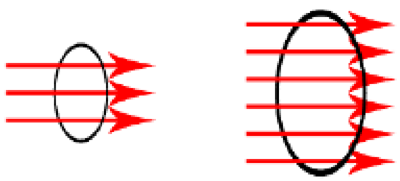
The ***second finger*** shows the direction that the current will be induced.

Use the right hand rule to check that the wire in the diagram above must be moved in direction **'U'** for the current to be induced in the direction shown.





Flux is proportional to the density of flow.



Flux is proportional to the area within the boundary.

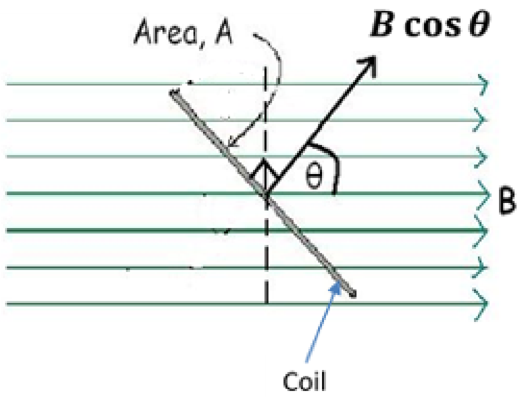
Magnetic flux. ϕ

Definition: the product of the magnetic flux density B when the field is at right angles to the area. ϕ = AB

The magnetic flux (ϕ) is a measure of the magnetic field passing through the area (loop of wire). Flux density (B) is a measure of the strength of the magnetic field and corresponds to the density of the magnetic field lines.

Magnetic flux. ϕ

If the area of the coil is not at 90˚ to the flux then you must use the component of the flux density B.
The magnetic flux through the coil is given by:



$$\phi = A\,B\cos\theta$$

The maximum value of ϕ occurs when θ = 0˚, since cos 0˚ = 1.

Units of ϕ: **Weber, Wb =Tm²**

One Weber is the magnetic flux when a magnetic field of flux density 1 tesla passes at right angles through an area of 1m².

Flux density

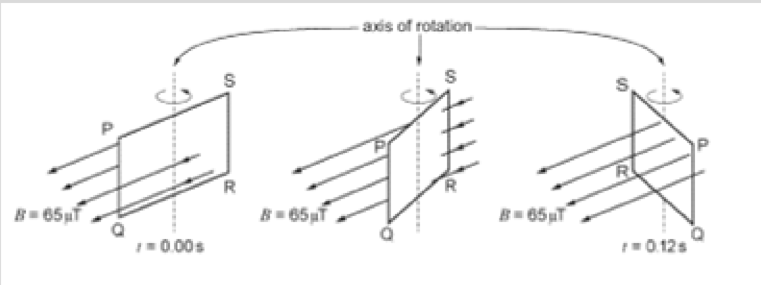
The magnetic flux only takes into account one coil/loop of a wire. The amount of magnetic field cut by a coil in a generator depends on the number of turns on the coil.

Definition: the product of the magnetic flux of the coil and the number of turns on the coil.

Flux linkage = $N\,\phi$

Units: **Wb or Wb turn**

Example. A large loop has sides of length 0.815m and is rotated through 90˚ in a uniform magnetic field of 65 µT. The diagram show the same loop at different times.



(a) Determine the magnetic flux through the square loop:

(i) When t= 0.00s (sides QR and SP are parallel to the B-field);

Answer: zero (no magnetic field lines pass through the coil)

(ii) And when t= 0.12s (PQ, QR, RS and SP are perpendicular to the B-field).
Use $\phi = A\,B\cos\theta$ The area of the coil, A = 0.815 x 0.815 = 0.664 m²
ϕ =0.664 x 65x10^{-6} = 4.32x10^{-5} Wb

(b) Calculate the flux linkage if the number of coils was increased to 90.
Flux linkage = $N\,\phi$ = 90 x 4.32x10^{-5} = 3.88 x10^{-3} Wb turn

Electromagnetic Induction.

An emf (voltage) will be induced in a coil whenever the magnetic flux ϕ through the coil changes. The name for this effect is ***electromagnetic induction.***

If the coil is part of a complete circuit this induced emf will cause a current to flow. This effect was discovered by Faraday in 1831. Magnetic flux through the coil can be changed in several ways.

1. Increasing the strength of the magnetic field / magnetic flux density.

2. Increasing the area of the coil in the field / rotating the coil.

3. Moving the coil or the field

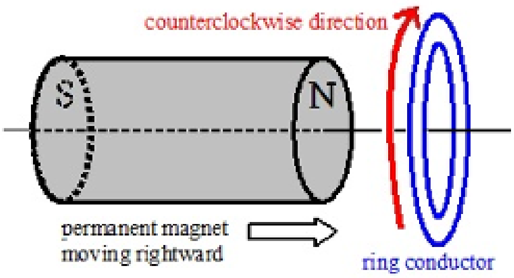
There are 2 laws which govern the induced emf.

Faraday's Law: ***the magnitude of the induced emf is equal to the rate of change of flux linkage.***
https://phet.colorado.edu/sims/html/faradays-law/latest/faradays-law_en.html

Lenz's Law: the direction of any current resulting from an induced emf is such as to oppose the change in flux linkage that is causing the current.
http://micro.magnet.fsu.edu/electromag/java/lenzlaw/index.html

Demonstrating Lenz's law

The magnet moves towards the coil inducing an emf, which gives rise to current that flows in such a way as to make the face of the coil facing the magnet a north pole, thus opposing the motion. When the magnet is moved away the current will flow in the opposite direction making the face of the coil a south pole thus attracting the magnet and again opposing the motion.



The law follows from the conservation of energy law. Work has to be done in order to move the magnet and cause the current flow. The work done is converted to electrical energy which is dissipated as heat. This principle can be used for braking systems on lorries, buses, trains and roller coasters. Moving a conductor through a magnetic field will cause a current to be induced in a direction that will oppose the direction of motion. This system reduces wear and tear on brakes so less maintenance.

Calculating the magnitude of the emf.

The 2 laws of electromagnetic induction can be expressed in an equation

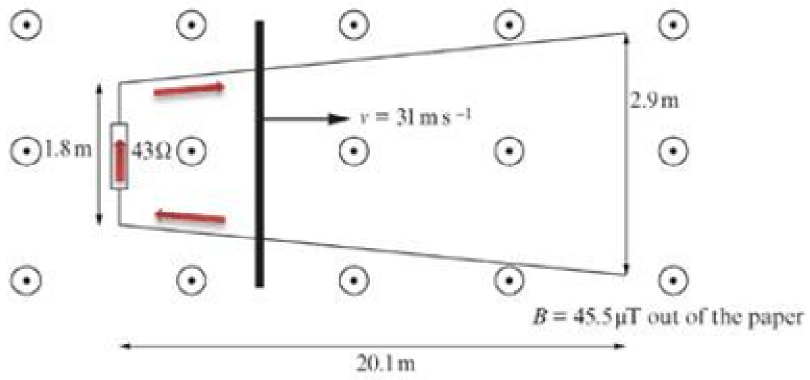
$$E \text{ (emf)} = -\frac{\Delta N\Phi}{\Delta t} \quad \textbf{(not given in data booklet)}$$

Unit: Volt, V

The negative sign follows from Lenz's law. The induced emf opposes any forward emf

The emf induced in a linear conductor moving at right angles to a magnetic field.

A thick conducting bar is move with constant speed over non parallel conducting rails as shown below. The rails have negligible resistance and the B-field is uniform.



An emf will be induced in the wire since there is a change in the flux. A current will flow in a clockwise direction (red arrows on diagram) which can be determined using Fleming's right hand rule. The magnitude of the emf increases as the bar moves to the right because the area is increasing at an increasing rate.

Calculating the mean current flow when the conducting bar has travelled the full 20.1m length of the track. **(N = 1, since there is only 1 coil)**

Use → E (emf) =- $\frac{\Delta N\Phi}{\Delta t}$ = $\frac{\Delta NAB}{\Delta t}$

Time, t = $\frac{d}{s} = \frac{20.1}{31} = 0.648\ s$

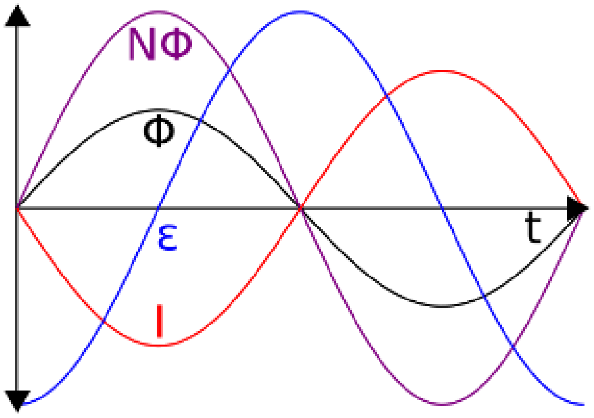
Area of coil, A = $\frac{2.9+1.8}{2} \times 20.1 = 47.2\ m^2$

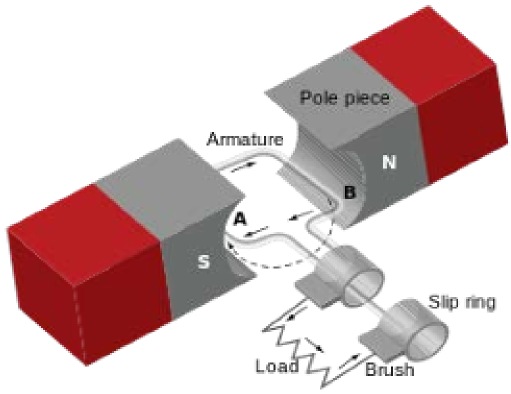
E (emf) = $-\frac{1 \times 47.2 \times 45.5 \times 10^{-6}}{0.648}$ **=-** $3.31 \times 10^{-3}\ V$

Current, $I = \frac{V}{R} = \frac{3.31 \times 10^{-3}}{43}$ **=** 77.1×10^{-3} **A → 77 μA**

Relating the position of a coil to the emf.

The graph below shows how emf (ϵ), magnetic flux (ϕ), flux linkage (N ϕ) and current (I) are related.





The instantaneous emf induced in a coil rotating at right angles to a magnetic field is related to the position of the coil, flux density, coil area and angular velocity.

Position of coil. When the coil between the poles of the magnet is in the horizontal position (as shown in diagram) then rate of change of flux linkage is at a maximum and so the emf is also at its maximum. When the coil is in the vertical position then the rate of change of flux linkage is zero and therefore no emf is induced.

Flux density (B): the flux density (B) and the emf are directly proportional.

Coil area (A): if the area of coil is increase then the emf increases, they are directly proportional.

Angular velocity (ω): if the angular velocity increases the induced emf increases. This is due to the decrease in time. (Remember $\omega=2\pi/T$ or $\omega=2\pi f$)

Authors: Penri Jones & Andrew Sharrock

Acknowledgments

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4.5	AC generator	