

Learning Objectives:

At the end of this topic you will be able to;

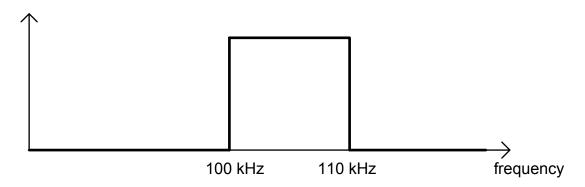
- □ recognise and sketch characteristics for a simple band pass filter;
- ☑ draw the circuit diagram for a band pass filter based on a parallel LC circuit;
- $\ensuremath{\square}$ select and use the formula $X_L = 2\pi f L$;
- \square recall that resonance occurs in a parallel LC network when $X_C = X_L$ and hence calculate the resonant frequency;
- $oxed{\square}$ select and use the formula $f_o \approx \frac{1}{2\pi\sqrt{LC}}$ where f_o is the resonant frequency;
- \square appreciate that in practical inductors, their resistance, r_L , has the effect of lowering the value of f_o ;
- $oxed{\square}$ select and use the formula for dynamic resistance, R_D , to calculate the output voltage of an unloaded filter at resonance where $R_D = \frac{L}{r_r C}$;
- \square be able to calculate the Q-factor, either from the frequency response graph, or component values;
- $oxed{\square}$ select and use the formulae $Q=rac{2\pi f_o L}{r_L}$ and $Q=rac{f_o}{bandwidth}$ for an unloaded circuit.



Resonant Filters

In the previous section we considered the design and operation of low pass and high pass filters. In this section we are going to consider one of the most important circuits that are used in communication circuits today, especially radio communication. This circuit is the **band pass filter** or **resonant filter**.

The ideal band pass filter as discussed in Topic 4.2.1 has the following characteristic.



The band pass filter is used specifically to allow only a narrow range of frequencies through it, as we will see later in Topic 4.4.1, this range of frequencies will correspond to a particular radio station. For now we will concentrate simply on how the circuit works and how to calculate the necessary component values.

The circuit we will be using is once again quite straight-forward in its construction, comprising of just two components, a capacitor which we have met before and also a new component called an **inductor**.

An inductor is a coil of wire, wrapped around a small ferrite rod. They come in many different shapes and sizes as shown by the photograph opposite. Its symbol is shown below.







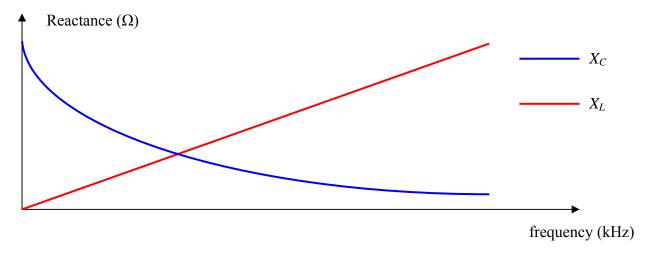
Inductance is measured in Henries, but more usually mH and μ H are used since the Henry is a very large unit.

In our last topic we discovered that a capacitor behaved differently in an a.c. circuit compared to a d.c. circuit. We defined a value for the **reactance** of the capacitor to be given as $X_c = \frac{1}{2\pi fC}$. For any given capacitor, this reactance value decreased rapidly as the frequency of the a.c. signal increased.

It will probably not come as a surprise to you that the inductor also behaves differently in an a.c. circuit compared to a d.c. one, again to signify its use in an a.c. circuit we call it's 'resistance' to the flow of current **reactance**. The reactance of an inductor is given by the formula $X_L = 2\pi f L$.

In an a.c. circuit with very low frequency the presence of the inductor is hardly noticeable since it is effectively a piece of wire with very low reactance. However, with higher frequency the reactance becomes significant, and at very high frequencies this can approach infinity.

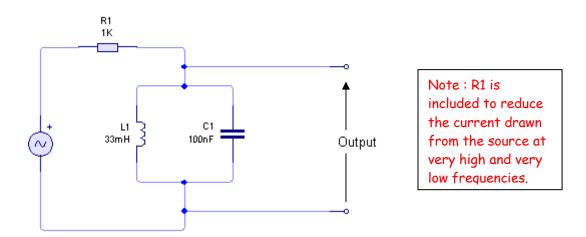
The following graph summarises the reactance of both a capacitor and inductor as the frequency of an a.c. signal is increased.



We can see from the graph that the reactance of the inductor increases linearly with frequency while the reactance of the capacitor decreases non-linearly.



We can now examine the band pass or resonant filter circuit in its simplest form, which is as shown below:



This basic circuit will allow us to consider very simply what will happen when different frequency signals are applied to it. You will need to refer to the previous graph which shows how the reactance changes with frequency.

Case 1: When the input frequency is very low.

In this case the reactance of the inductor will be very low, and the reactance of the capacitor will be very high since f is small. Current will therefore flow through the inductor rather than the capacitor. This means the output voltage will be negligible.

Case 2: When the input frequency is very high.

In this case the reactance of the inductor will be very high and the reactance of the capacitor will be very small, since f is high. Current will this time prefer to flow through the capacitor rather than the inductor as it has a lower reactance. Once again the output voltage at high frequencies will be negligible.

Case 3: At mid range frequencies.

At a mid range frequency the reactance of both the inductor and capacitor will be significant values and there will be some reactance in both parts of



the circuit. At one very special frequency, called the **resonant** frequency the reactance of the inductor and capacitor will be equal. At this frequency the maximum possible output voltage will be obtained.

Unfortunately the calculation of the effective reactance in parallel is very complex, and beyond the scope of this syllabus. However there is one calculation we can perform on this circuit which is to calculate the resonant frequency of the circuit. Remember we said that the maximum output should occur when $X_L = X_C$. The frequency at which this happens is called the **resonant frequency**, f_o

Therefore:

$$X_{L} = X_{C}$$

$$2\pi f_{o}L = \frac{1}{2\pi f_{o}C}$$

$$so \quad f_{o}^{2} = \frac{1}{4\pi^{2}LC}$$

$$or \quad f_{o} = \frac{1}{2\pi\sqrt{LC}}$$

So if we now apply our values for L and C from the circuit we achieve the following:

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

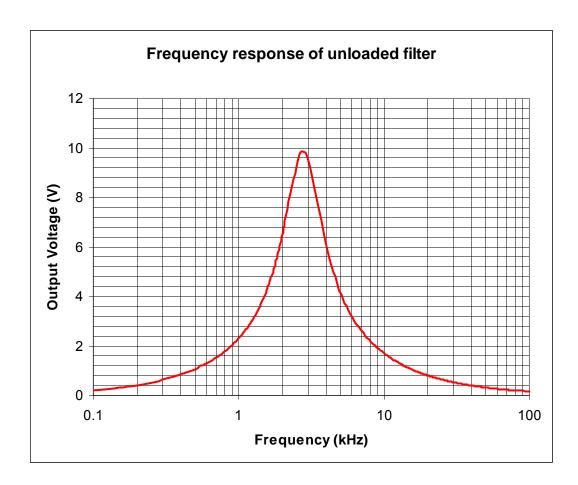
$$= \frac{1}{2\pi\sqrt{33\times10^{-3}\times100\times10^{-9}}}$$

$$= 2770 Hz$$

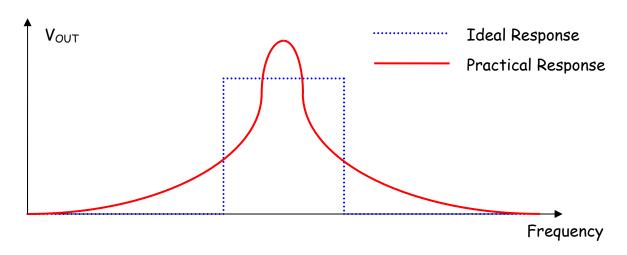
An Excel worksheet is provided for you to plot the frequency response of a resonant filter. The spreadsheet allows you to change the parameters of the circuit so that you see their effect. Your teacher may demonstrate this to you now, but before you use it there are a couple of other things that need to be considered.



Using this spreadsheet the graph below was obtained for this circuit, which clearly shows a peak response at approx 2.7kHz.



We can see that the response from this simple filter is not exactly what we would like from an ideal band pass filter as the following illustration shows.

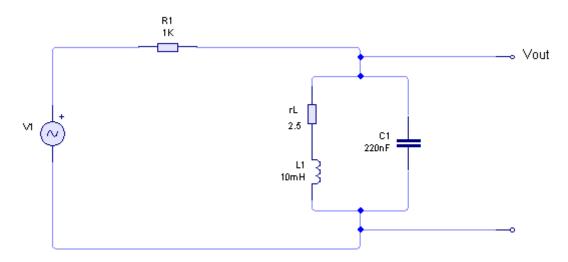


However given that this has been created from very simple components, it is not a bad start. This type of band pass filter is called a 'First Order' filter.



If you continue with your studies in electronics, and communication systems in general you will find much better band pass filters can be designed, however they need some sort of amplification to improve their operation and are outside the scope of this introductory unit. You will be introduced to the start of this area of work in ET5 when you look at Active Filters.

So far we have assumed that all the components we have used are 'ideal'. In practice of course there will be some slight difference between the stated value of the capacitor and inductor, and their actual value. In the same way that we had a tolerance for resistors we also have a similar tolerance for both inductors and capacitors. Inductors also have a small resistance from the wire used to make the coil. The formula will always give a value of resonant frequency slightly higher than the actual value as the formula assumes that $r_{\rm L}$ is negligible. In practice $r_{\rm L}$ effectively lowers the resonant frequency slightly. A more realistic circuit for the band-pass filter is therefore as shown below.



If the problem of calculating the output voltage was complex before it has now been made even more complex, fortunately we do not have to concern ourselves with this for this introduction, but we are able to calculate the output voltage at one specific frequency – the resonant frequency. This is due to a special case that occurs at resonance which allows the calculation of a quantity called the **dynamic resistance**, R_D .

This allows us to replace the parallel combination with a single resistance R_D which reduces the circuit to a simple potential divider as used in ET2.



It is important to remember that this simplification can **only** be applied to the circuit **at resonance**, and when it unloaded. In deriving the formula for R_D several assumptions have been made, one of which is that r_L is small, and so the formula works well for values of r_L <25 Ω .

The formula to calculate the dynamic resistance, R_D is as follows;

$$R_D = \frac{L}{r_L C}$$

If we apply this formula to the circuit given earlier, then R_D becomes

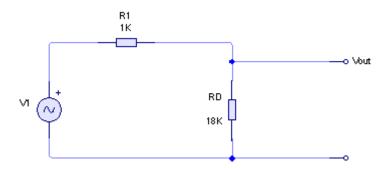
$$R_D = \frac{L}{r_L C}$$

$$= \frac{10 \times 10^{-3}}{2.5 \times 220 \times 10^{-9}}$$

$$= 18181\Omega$$

$$\approx 18k\Omega$$

The circuit can now be reduced to the following, with the tuned circuit being replaced by the equivalent resistor R_D .



So at resonance the output voltage of the previous circuit, using the approximate value for \mathbf{R}_D will be

$$V_{OUT} = \frac{V_{IN} \times R_D}{R1 + R_D}$$
$$= \frac{10 \times 18}{1 + 18} \approx 9.47V$$



This shows that at the resonant frequency the output voltage is very high which is exactly what we want to happen at this frequency.

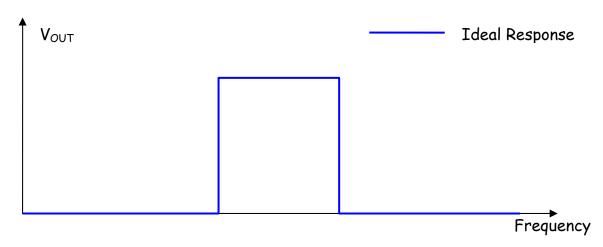
So to recap on what we have covered so far for resonant circuits:

- i. a resonant filter consists of an inductor and capacitor in parallel,
- ii. the reactance of an inductor is given by $X_L = 2\pi f L$,
- iii. at resonance $X_L = X_C$, and the resonant frequency is given by $f_o = \frac{1}{2\pi\sqrt{LC}}$
- iv. in practical circuits the inductor will have a small resistance $r_{L_{r}}$
- v. at resonance (only) the output voltage can be calculated by using the dynamic resistance R_D of the parallel RLC circuit, given by $R_D = \frac{L}{r_c C}$

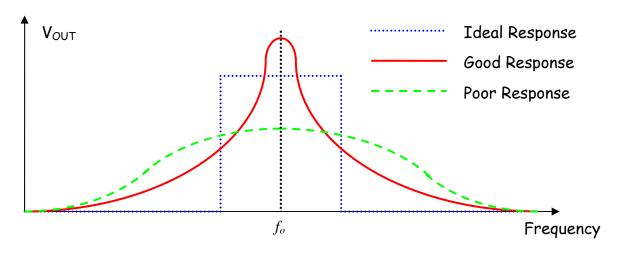


Q-factor and Selectivity

In the previous session we have been concentrating our work on trying to make a band pass filter that gave a good approximation to the ideal band-pass filter shown below.



We have seen that the response from the simple resonant filter built from an inductor and capacitor doesn't quite match this ideal characteristic. However if we choose inappropriate values for R and C we can affect the shape of the frequency response of the filter quite dramatically, as shown below:



From the graph above the red (solid) trace shows a much more focused response to the resonant frequency f_{\circ} and the frequencies either side of the resonant frequency will be very quickly attenuated.

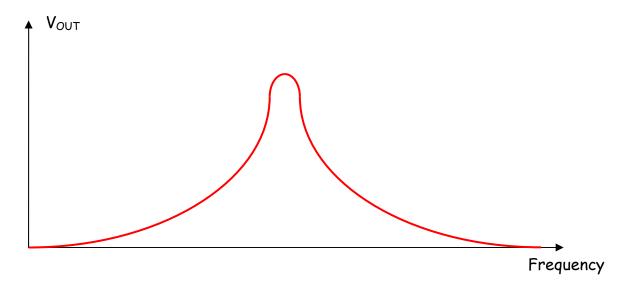
However the green (dashed) trace shows a much flatter response with a less well defined peak and a much wider range of frequencies before they start being attenuated.



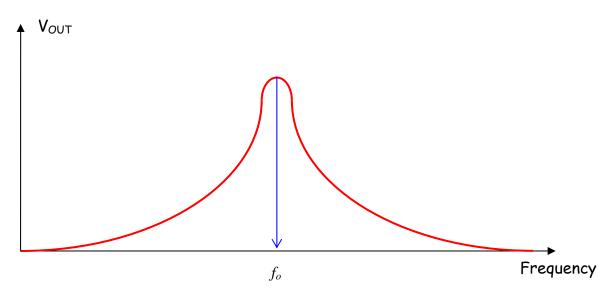
As we will see later in section 4.4 we will need the response of this resonant filter to be very focused on one frequency and the flatter response shown here with the green (dashed) trace will be completely inappropriate.

The correct way of describing this feature is the selectivity or Q-factor of the filter. The Q-factor of a circuit can be determined in one of three ways.

- 1. Using the formula $Q = \frac{2\pi f_o L}{r_L}$, where f_o is the resonant frequency, L is the value of the inductor, and r_L is the resistance of the inductor.
- 2. Using the formula $Q = \frac{f_o}{bandwidth}$, where f_o is the resonant frequency.
- 3. From the graph of the frequency response of the filter.



i. First find f_o , which is at the peak of the response curve.

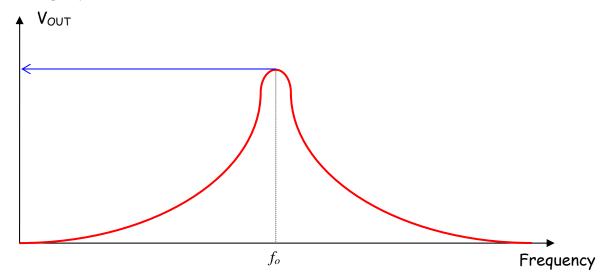


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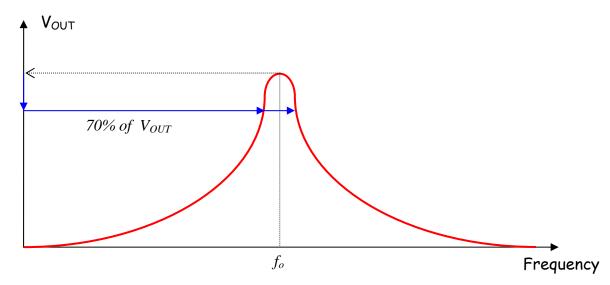
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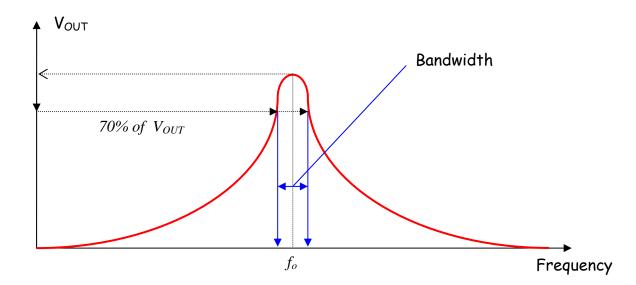
ii. Find the maximum output Voltage (or Gain) depending on how the graph is drawn.



iii. Calculate 70% of the maximum output voltage (or gain).



iv. Find bandwidth from frequency response.





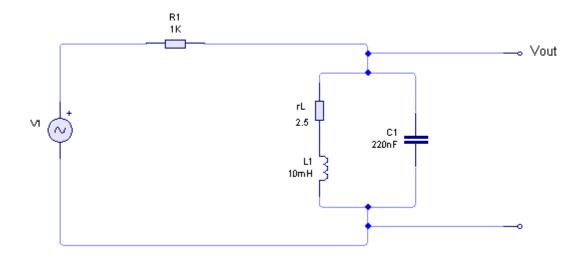
v. Use equation 2 from page 11 to calculate Q.

A good filter will have a high Q-factor, but a high Q-factor results in a very narrow bandwidth. The design of a band pass filter is not an easy task, since it is relatively easy to obtain a high Q factor, or large bandwidth, but not both at the same time. A compromise usually has to be reached, especially with this simple type of filter. So now for a couple of examples.

Note: The Q-factor of a circuit is dimensionless and has no units, just like gain.

Example:

Let us consider our previous circuit once again.



We will now calculate the Q-Factor and bandwidth for this circuit.

First we need the resonant frequency, this is calculated using the following formula.

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{10\times10^{-3}\times220\times10^{-9}}}$$

$$= 3393Hz \approx 3.4kHz$$



Now we can find the Q-factor of the circuit using equation 1 from page 11, i.e.

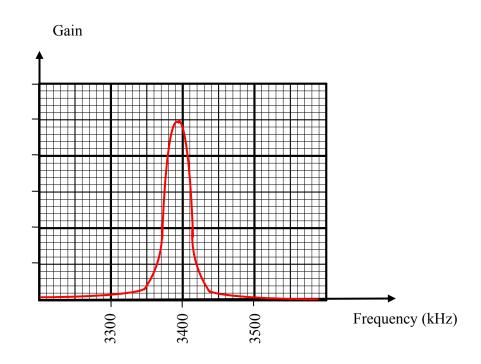
$$Q = \frac{2\pi f_o L}{r_L}$$
=\frac{2\pi \times 3393 \times 10 \times 10^{-3}}{2.5}
= 85.28

Finally, we can calculate the bandwidth of the filter using equation 2 from page 11, since we now know f_o and Q-factor.

$$Q = \frac{f_o}{Bandwidth}$$
so
$$Bandwidth = \frac{f_o}{Q}$$

$$= \frac{3393}{8528} = 39.78Hz$$

This example demonstrates clearly how it is quite easy to achieve a high Q-factor (85 in this case), but the impact on the bandwidth is severe as this has dropped to just 40 Hz approximately, so the characteristic would look something like this.





Student Exercise 1.

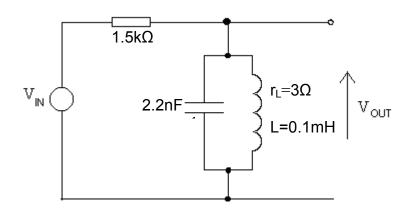
Information:

The formulae required for these questions are as follows.

$$Q = \frac{2\pi f_o L}{r_L} \qquad Q = \frac{f_o}{Bandwidth} \qquad R_D = \frac{L}{r_L C}$$

$$f_o \approx \frac{1}{2\pi\sqrt{LC}} \text{ which can also be rearranged into } C \approx \frac{1}{4\pi^2 f_o^2 L} \text{ or } L \approx \frac{1}{4\pi^2 f_o^2 C}$$

1. The following circuit shows a band pass filter connected to a signal generator with $V_{\rm IN}$ set to 8V. The inductor has a resistance $r_{\rm L}$ of 3Ω . $V_{\rm IN}$ is kept at 8V and the frequency increased to give the maximum value of $V_{\rm OUT}$.



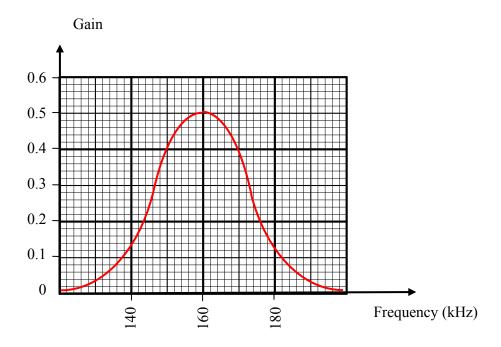
(a)	Calculate the frequency at which V_{OUT} is a maximum.



(b)	By calculating the Dynamic Resistance R_D of the filter, determine the maximum value of the voltage V_{OUT} with V_{IN} set to 8V.
(c)	Determine the bandwidth of this filter.
(d)	Sketch the frequency response of the filter using the axes below. Label all important values.
	V _{OUT}



2. The following graph was plotted by a student investigating the behaviour of a band pass filter.



(a) Determine the resonant frequency of the filter which has the response shown above.

.....

(b) Determine the maximum gain that can be obtained from this filter.

.....

(c) Determine the bandwidth of the filter shown above.

.....

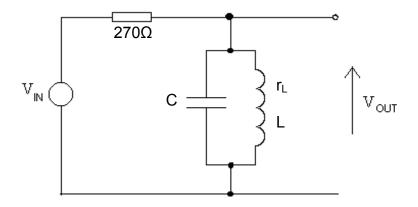
(d) Hence calculate the Q-Factor for the filter.

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3. The following circuit shows a band pass filter connected to a signal generator with $V_{\rm IN}$ set to 6V. $V_{\rm IN}$ is kept at 6V and the frequency increased to give the maximum value of $V_{\rm OUT}$.



The filter is required to have a Q-Factor of 10, and a bandwidth of 10kHz.

(a)	Calculate th	e resonant	frequency	y of	the	filter.

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		• • • • • • • • • • • • • • • • • • • •					 	
		• • • • • • • • • • • • • • • • • • • •					 	

(b) Three inductors A, B and C with the following properties are available.

Inductor A L=50mH, r_L = 2.45 Ω .

Inductor B L=50 μ H, r_L = 3.15 Ω .

Inductor C L=50 μ H, r_L = 2.45 Ω .

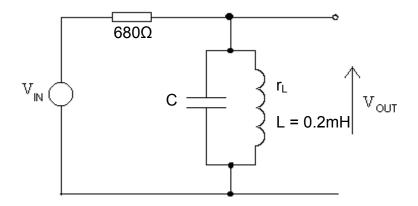
Show by calculation which is the correct inductor to use for this circuit.



(c)	Using your answer to (b) calculate the value of the capacitor ${}^{\backprime}\!\mathcal{C}{}^{\backprime}$ to
	meet the specification.
(d)	Hence calculate the value of R_D at resonance.
` '	
(0)	Llance calculate the output voltage at regeneres
(e)	Hence calculate the output voltage at resonance.



4. The following circuit shows a band pass filter connected to a signal generator with $V_{\rm IN}$ set to 12V. $V_{\rm IN}$ is kept at 12V and the frequency increased to give the maximum value of $V_{\rm OUT}$.



The filter is required to have a Q-Factor of 85, and a bandwidth of 2.4kHz.

(a)	Calculate the resonant frequency of the filter.
(b)	Use your answer to (a) to calculate the value of the capacitor
	required to achieve this resonant frequency.



(c)	Using your answer to (a), (b) and the Q-factor to calculate the
	value of $r_{\!\scriptscriptstyle L}$, the resistance of the inductor.
. 1	
(d)	Hence calculate the value of R_D at resonance.
(e)	Hence calculate the output voltage at resonance.



Solutions to Student Exercises

Exercise 1.

1. a

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.1\times10^{-3}\times2.2\times10^{-9}}} = 339319Hz \approx 339kHz$$

b

$$R_D = \frac{L}{r_L C}$$

$$R_D = \frac{0.1 \times 10^{-3}}{3 \times 2.2 \times 10^{-9}} = 15152\Omega$$

$$V_{OUT} = \frac{10 \times 15152}{1500 + 15152} = 7.28V$$

c.

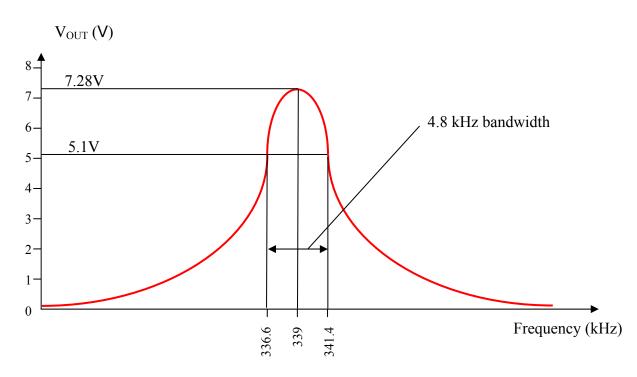
$$Q = \frac{2\pi f_0 L}{r_L}$$

$$Q = \frac{2\pi \times 339319 \times 0.1 \times 10^{-3}}{3} = 71.1$$

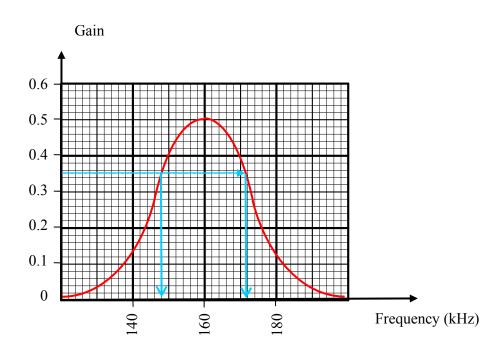
$$bandwidth = \frac{f_0}{Q} = \frac{339319}{71.1} = 4772Hz \cong 4.8kHz$$



d.



- 2. (a) Resonant Frequency = 160kHz
 - (b) Maximum gain = 0.5
 - (c) 70% of max gain = $0.7 \times 0.5 = 0.35$



Bandwidth = 172 - 148 = 24kHz.



(d)
$$Q = \frac{f_o}{bandwidth} = \frac{160}{24} = 6.67$$

$$Q = \frac{f_o}{bandwidth}$$

$$f_o = Q \times bandwidth$$

$$= 10 \times 10kHz$$

$$= 100kHz$$

(b)

Inductor A:
$$Q = \frac{2\pi f_o L}{r_L} = \frac{2\pi \times 100 \times 10^3 \times 50 \times 10^{-3}}{2.45} = 12822$$

Inductor B:
$$Q = \frac{2\pi f_o L}{r_L} = \frac{2\pi \times 100 \times 10^3 \times 50 \times 10^{-6}}{3.15} = 9.97$$

Inductor C:
$$Q = \frac{2\pi f_o L}{r_L} = \frac{2\pi \times 100 \times 10^3 \times 50 \times 10^{-6}}{2.45} = 12.8$$

Inductor B is therefore the most suitable one to use as this matches the Q-factor required.

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$100 \times 10^3 = \frac{1}{2\pi\sqrt{50 \times 10^{-6} \times C}}$$

$$\sqrt{50 \times 10^{-6} \times C} = \frac{1}{2\pi \times 100 \times 10^3}$$

$$\sqrt{50 \times 10^{-6} \times C} = 1.59 \times 10^{-6}$$

$$50 \times 10^{-6} \times C = (1.59 \times 10^{-6})^2$$

$$C = \frac{(1.59 \times 10^{-6})^2}{50 \times 10^{-6}} = 50 \times 10^{-9} = 50nF$$



(d)
$$R_D = \frac{L}{r_r C} = \frac{50 \times 10^{-6}}{3.15 \times 50 \times 10^{-9}} = 317\Omega$$

(e)
$$V_{OUT} = \frac{6 \times 317}{270 + 317} = 3.24V$$

4. (a)

$$Q = \frac{f_o}{bandwidth}$$

$$f_o = Q \times bandwidth$$

$$= 85 \times 2.4kHz$$

$$= 204kHz$$

(b) Either

$$f_o = rac{1}{2\pi\sqrt{LC}}$$
 $204 imes 10^3 = rac{1}{2\pi\sqrt{0.2 imes 10^{-3} imes C}}$
 $\sqrt{0.2 imes 10^{-3} imes C} = rac{1}{2\pi imes 204 imes 10^3}$
 $\sqrt{0.2 imes 10^{-3} imes C} = 7.80 imes 10^{-7}$
 $0.2 imes 10^{-3} imes C = \left(7.8 imes 10^{-7}\right)^2$
 $C = rac{\left(7.8 imes 10^{-7}\right)^2}{0.2 imes 10^{-3}} = 3 imes 10^{-9} = 3nF$

or directly

$$C = \frac{1}{4\pi^2 f_o^2 L}$$

$$= \frac{1}{4 \times 3.142^2 \times (204 \times 10^3)^2 \times 0.2 \times 10^{-3}}$$

$$= 3 \times 10^{-9} = 3nF$$

(c)

$$Q = \frac{2\pi f_o L}{r_L}$$

$$r_L = \frac{2\pi f_o L}{Q} = \frac{2\pi \times 204 \times 10^3 \times 0.2 \times 10^{-3}}{85} = 3.01\Omega$$

(d)
$$R_D = \frac{L}{r_L C} = \frac{0.2 \times 10^{-3}}{3 \times 3 \times 10^{-9}} = 222222\Omega$$

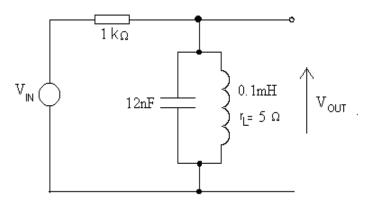
(e)
$$V_{OUT} = \frac{12 \times 22222}{680 + 22222} = 11.64V$$

Now for some Examination style questions.



Examination Style Questions:

1. The following circuit shows a band pass filter connected to a signal generator with V_{IN} set to 10V. The inductor has a resistance r_L of 5Ω . V_{IN} is kept at 10V and the frequency increased to give the maximum value of V_{OUT} .



<i>(a)</i>	Calculate the frequency at which V_{OUT} is a maximum.	
		[2]
<i>(b)</i>	By calculating the Dynamic Resistance R_D of the filter, determine the maximum valof the voltage V_{OUT} with V_{IN} set to $10\mbox{V}$.	
		[4]



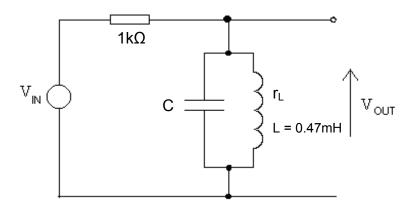
(c)	Determine the bandwidth of this filter.	3]
		•••••
(d)	Sketch the frequency response of the filter using the axes below. Label all important values.	
	V _{OUT}	
	frequency /Hz	

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[4]



2. The following circuit shows a band pass filter connected to a signal generator with V_{IN} set to 9V. V_{IN} is kept at 9V and the frequency increased to give the maximum value of V_{OUT} .



The filter is required to have a Q-Factor of 11, and a bandwidth of 2.4kHz.

(a)	Calculate the resonant frequency of the filter.
	[2]
(b)	Use your answer to (a) to calculate the value of the capacitor required to achieve this
	resonant frequency.
	[2]
(c)	Use your answer to (a), (b) and the Q-factor to calculate the value of r_L , the resistance of the inductor.

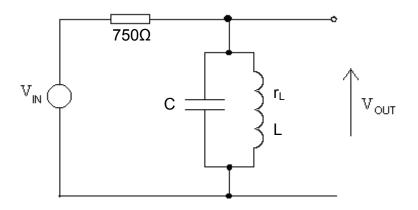


(d)	Hence calculate the value of R_D at resonance.
	[2]
(e)	Hence calculate the output voltage at resonance.
	[2]

WJEC CBAC

Module ET4 - Communication Systems

3. The following circuit shows a band pass filter connected to a signal generator with V_{IN} set to 9V. V_{IN} is kept at 9V and the frequency increased to give the maximum value of V_{OUT} .



The filter is required to have a Q-Factor of 14, and a bandwidth of 5kHz.

nt frequency of the filter.	
and C with the following properties are available.	[2
L=1.5mH, $r_L = 6.4\Omega$.	
L=0.30mH, $r_L = 2.8\Omega$.	
L=0.15mH, $r_L = 4.7\Omega$.	
which is the correct inductor to use for this circuit	·
	•••••
	and C with the following properties are available L=1.5mH, $r_L=6.4\Omega$. L=0.30mH, $r_L=2.8\Omega$. L=0.15mH, $r_L=4.7\Omega$.

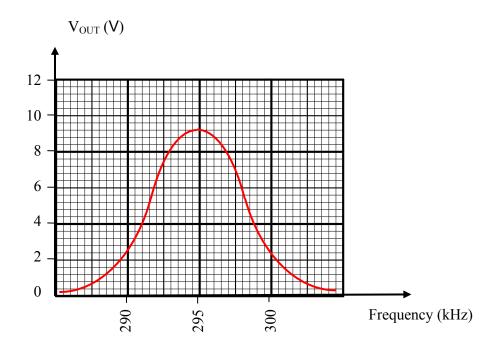
[2]



(c)	Calculate the value of the capacitor 'C' to meet the specification.	
		[3]
(d)	Hence calculate the value of R_D at resonance.	
		••••
		[2]
(e)	Hence calculate the output voltage at resonance.	
		 [2]



4. The following graph shows the practical response of a band pass filter.



(a)	Determine the resonant frequency of the filter which has the response shown above.	
		[1]
(b)	Determine the maximum output voltage that can be obtained from this filter.	[+]
		Г1 7
(c)	Determine the bandwidth of the filter shown above.	[1]
(d)	Hence calculate the Q-Factor for the filter.	[1]
		[2]



Self Evaluation Review

	My personal review of these objectives:			
Learning Objectives	\odot			
recognise and sketch characteristics for a simple band pass filter;				
draw the circuit diagram for a band pass filter based on a parallel LC circuit;				
select and use the formula $X_L = 2\pi f L$;				
recall that resonance occurs in a parallel LC network when X_C = X_L and hence calculate the resonant frequency;				
select and use the formula $f_opprox rac{ extbf{1}}{ extbf{2}\pi\sqrt{LC}}$				
where f_o is the resonant frequency;				
appreciate that in practical inductors, their				
resistance, r_L , has the effect of lowering the				
value of f_o ;				
select and use the formula for dynamic				
resistance, R_D , to calculate the output voltage				
of an unloaded filter at resonance where				
$R_D = \frac{L}{r_L C} \; ;$				
know that the $\mathcal{Q} ext{-factor}$ is a measure of the				
selectivity of the band pass filter;				
be able to calculate the Q -factor, either from				
the frequency response graph, or component				
values;				
select and use the formulae $Q=rac{2\pi f_o L}{r_L}$ and				
$Q = \frac{f_o}{bandwidth}$ for an unloaded circuit.				

Targets:	1.	
	2.	